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Optimization of perforated composite plates under tensile stress using genetic algorithm

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Abstract

Discontinuities such as cutouts always cause stress concentration in the structure, which increases the local stress. Understanding the effective parameters on stress concentration and proper selection of these parameters enables the designer to achieve a reliable design. In this paper, the optimum values of the effective parameters on stress distribution around the cutout are determined by genetic algorithm. The fitness function for a genetic algorithm is defined by generalization of the analytical solution based on Lekhnitskii method for different cutouts. The finite element method is used to check the validity of the obtained fitness function. Also, the genetic algorithm was able to predict the optimal value of each effective parameter on stress distribution while keeping constant the values of the other parameters. The results showed that material properties, geometry of cutout and angle load have much effect on stress concentration.

Keywords

Perforated composite plates, stress concentration, analytical solution, genetic algorithm

Introduction

One of the most important cases of stress concentration is the study of the stress distribution around various cutouts in an infinite plate, which is subjected to uniaxial in-plane tension stress. Stress concentration in the vicinity of cutouts can frequently be considered as crucial factor for structural design. So, the study of it is always an important issue in the design.[1] Investigating the parameters affecting the stress concentration in the orthotropic material is very important because in some cases the value of the stress concentration is large.[2] Savin[3] was one of the first researcher to obtain the stress distribution around circular and non-circular cutouts by using the complex variable method and Schwartz's equation for isotropic and anisotropic perforated plates. Lekhnitskii[4] used the analytical solutions to investigate the boundary value problems by complex variable method based on Kolosov-Muskhelishvili formulas for anisotropic plates with circular and elliptical cutout. Rezaeepazhand and Jafari[5] studied the stress distribution around different cutouts in infinite composite plates. The stress distribution around rectangular and quasi-square cutouts in an orthotropic plate which subjected to uniaxial load was given by Jong.[6] Abuelfoutouh[7] formulated a relation for tangential stress around of different cutouts such as circular, elliptical and triangular and rectangular by a single equation. Rezaeepazhand and Jafari[8,9] used Lekhnitskii theory to study the stress analysis of composite and metallic plates containing a non-circular cutout. Asmar and Jabbour[10] also applied the same theory to investigate the stress distribution around the cutout in an anisotropic plate with a quasi-square cutout and subjected to uniaxial load. But this research studied only the effect of bluntness and rotation angle for very special cases. Rezaeepazhand and Jafari[11]also studied the stress concentration around several non-circular cutouts in isotropic plates. They investigated the effects of rotation angle and the bluntness of the square and triangular cutouts on the stress concentration. However, aspect

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ratio of the cutout was not investigated in this study. To date, few studies in the field of the application of genetic algorithms for optimizing all the parameters affecting the stress concentration in the orthotropic material have been fulfilled. Muc and Gurba[12] used a combination of genetic algorithm and finite element analysis in optimization of composite structures. In this study, stacking sequence, shape and size of structure were utilized as design variables. In addition, the optimization objective was to achieve the maximum buckling load. Kradinov et al.[13] showed application of genetic algorithm in optimal design of bolted composite lap joints. In this research, the laminate thickness, laminate lay-up, bolt location, bolt flexibility, and bolt size were considered as optimization variables for maximizing the strength of the joint. Narayana Naik et al.[14] performed the optimization of composites by using genetic algorithms and failure mechanism based failure criterion. The composite laminated structures were optimized by using genetic algorithms and finite element analysis.[15] The objectives of this research were to minimize the weight and deflection or weight and cost in order to maximize the stiffness of a composite shell under pressure load.

In this study, rotation angle of cut out, fiber angle, load angle, bluntness, material type and number of sides of the cutout are design variables. The main objective of this paper is to obtain the optimal design variables which minimize the maximum stress around different cutouts. The optimal values of these parameters are determined using genetic algorithm. The good agreement between analytical and numerical results indicates that the accuracy of the analytical solution presented in this study is very good. In the present study, the material behavior is supposed to be linearly elastic.

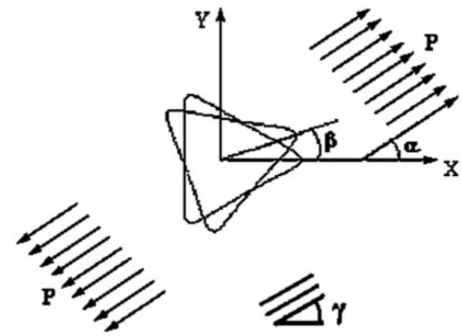

Figure 1. A plate with a central cutout under uniaxial tension, rotation angle (β), fiber angle (γ), load angle (α).

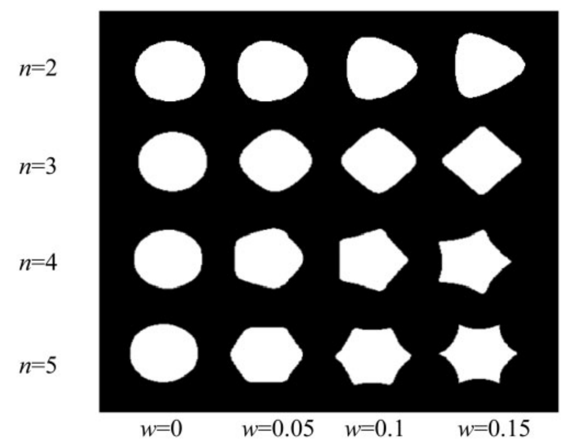

Figure 2. The effect of w and n on the hole shape.

Genetic algorithm (GA)

Many studies have been done for optimizing composite structures using the genetic algorithm.[16–19] However, no study has been done so far on the topic of this paper. To evaluate the effective parameters of the stress distribution around the cutout using genetic algorithm, the following steps are performed. First, it is assumed that an infinite orthotropic plate with a cutout subjected to a uniformly distributed tensile load at a large distance from the cut out. The cutout is located at the center of plate, as shown in Figure 1. The cutout size is small enough with respect to the plate dimensions. Therefore, its effect will be negligible at a distance of a few diameters from its edge. The load is applied at angle α (load angle) with respect to x-axis (Figure 1). The major axis of the cutout is directed at angle β (rotation angle) with respect to x-axis. As shown in Figure 1, γ is fiber angle for composite plates.

Second, the cost function for the genetic algorithm in this study is defined as

$$\text{cost\_fun} = f(\alpha,\ \beta,\ \gamma, w, n) \tag{1}$$

where α, β, γ, and w are the load angle, rotation angle, fiber angle, and bluntness respectively. As observed in Figure 2, integer n in the cost function represents the shape of the cut out. The cut out sides are given by $n+1$. Figure 2 also shows the effect of parameter w on the cutout shape. In the present study, the value range of design variables is defined as follows: number of sides of the cutout changes from 2 for triangular cut out to 7 for octagonal cut out; bluntness is the range of 0 to 1/n. load angle, rotation angle and fiber angle are the range of 0° to 90°.

The cost function for the genetic algorithm is the maximum normalized stress for each cut out boundary. The cost function is obtained by an analytical solution based on Lekhnitskii's theory. The normalized stress is defined as the ratio of the highest stress around the cutout to nominal or applied stress.

To find the optimal value of the parameter n, cutouts with odd and even number of sides are evaluated separately, because, as will be discussed later, the optimal point of these two cases will be different.

Each chromosome consists of four variables: load angle, rotation angle, fiber angle, bluntness

$$\text{chrom} = [\alpha,\ \beta,\ \gamma, w] \tag{2}$$

The cost of a chromosome is found by evaluation of the cost function cost_fun at variables

$$\text{cost} = f(\text{chrom}) = f(\alpha,\ \beta,\ \gamma, w) \tag{3}$$

The GA begins its work with a random population of chromosomes (solutions) according to following equation

$$\text{pop} = \text{rand}\left(N_{\text{pop}}, 4\right) \tag{4}$$

where N_{pop} is the number of chromosomes in the population from generation to generation.

The value of the maximum normalized stress is calculated for each member of the population. Then, the individuals in the population are arranged in descending order according to the fitness values. The population is divided into two equal parts and the top half with the higher fitness values will survive and be replicated into the next generation.[17,19] The new value of a variable (for example, bluntness parameter) is calculated as follows

$$w_{\text{new}} = \varphi w_m + (1 - \varphi) w_d \tag{5}$$

where φ is random number between zero and one, w_m and w_d are the chromosomes of the mother and father for bluntness parameter, respectively. The mutation operator is used to prevent trapping in local optimum and fast convergence.[20] The flowchart of genetic algorithm is shown in Figure 3. A computer code was also developed in MATLAB software to implement this flowchart.

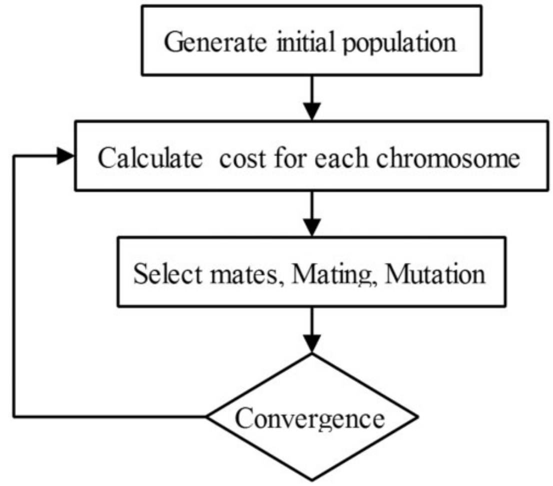

Figure 3. Flowchart of a GA.

Define fitness function for genetic algorithm

The fitness function is designed based on the theory of anisotropic elasticity presented by Lekhnitskii and Savin.[3,4] It is well known that, for the plane stress state, equilibrium equations are satisfied if we introduced a stress function $U(x,y)$ such that:[21]

$$\sigma_x = \frac{\partial^2 U}{\partial y^2} \quad , \sigma_y = \frac{\partial^2 U}{\partial x^2} \quad , \tau_{xy} = \frac{\partial^2 U}{\partial x \partial y} \tag{6}$$

By substituting equation (6) in the equilibrium equation, a fourth order differential equation in U is obtained

$$R_{22}\frac{\partial^4 U}{\partial x^4} - 2R_{26}\frac{\partial^4 U}{\partial x^3 \partial y} + (2R_{12} + R_{66})\frac{\partial^4 U}{\partial x^2 \partial y^2} - 2R_{16}\frac{\partial^4 U}{\partial x \partial y^3} + R_{11}\frac{\partial^4 U}{\partial y^4} = 0 \tag{7}$$

R_{ij} are the coefficient of the reduced compliance matrix that are function of material properties. For plane stress and plane strain conditions, the stress function is governed by the same differential equation with different coefficients which derived from corresponding assumption. Lekhnitskii[4] showed this equation can be transferred to four linear operators of the first order D_k

$$D_1 D_2 D_3 D_4 F(x, y) = 0 \text{ where } D_k = \frac{\partial}{\partial y} - \mu_k \frac{\partial}{\partial x} \tag{8}$$

where μ_k are the roots of the following characteristic equation

$$R_{11}\mu^4 - 2R_{16}\mu^3 + (2R_{12} + R_{66})\mu^2 - 2R_{16}\mu + R_{22} = 0 \tag{9}$$

It can be proved, in general, equation (9) has four distinct roots. These roots are mutually conjugate

$$\begin{aligned} \mu_{1,3} &= \alpha_1 \pm i\beta_1 \\ \mu_{2,4} &= \alpha_2 \pm i\beta_2 \end{aligned} \tag{10}$$

The general expression for the stress function U depends on the roots of characteristic equation μ_k. Since the roots of the characteristic equation are mutually conjugate, the general expression for the stress function involves the real part of some functions

$$U(x, y) = 2Re[\phi_0(z_1) + \psi_0(z_2)] \tag{11}$$

ϕ_0 and ψ_0 are arbitrary holomorphic function in terms of complex variable $z_k = x + \mu_k y$ for $k = 1,2$. With this approach, the problem is reduced to finding two complex functions ϕ_0 and ψ_0 such that the boundary conditions on the cutout edge are satisfied.

Finally, according to this theory, in-plane stress under the given loading condition, as shown in Figure 1, is calculated using the below formula[4]

$$\begin{aligned}\sigma_x &= P\cos^2\alpha + 2\mathrm{Re}[\mu_1^2\phi_0'(z_1) + \mu_2^2\psi_0'(z_2)]\\ \sigma_y &= P\sin^2\alpha + 2\mathrm{Re}[\phi_0'(z_1) + \psi_0'(z_2)]\\ \tau_{xy} &= P\sin\alpha\cos\alpha - 2\mathrm{Re}[\mu_1\phi_0'(z_1) + \mu_2\psi_0'(z_2)]\end{aligned}$$

where $z_k = x + \mu_k y$ and $k = 1, 2$. μ_1 and μ_2 are obtained from the characteristic equation of the anisotropic material.[4] P and α are the applied load and load angle, respectively. φ_0 and ψ_0 are holomorphic stress functions in terms of the complex variable, which are obtained by applying the boundary conditions.

The geometry of cutout for applying boundary conditions is defined according to the following equation

$$\begin{aligned}X &= \cos\theta + w\cos(n\theta)\\ Y &= -(c\sin\theta - w\sin(n\theta))\end{aligned} \tag{7}$$

where w is the bluntness parameter which changes the radius of curvature at the corner of the cutout. If w approaches zero, cutout converges to a circle and if w greater than $1/n$, the corners of the cutout are very sharp. Integer n determines the number of sides of the cutout. The number of sides of the cutout is odd and even when n is even and odd, respectively. The parameter c determines the aspect ratio of the cutout. The composite material properties used in this study are given in Table 1.

Knowing the optimal values of the design parameters is essential. Almost all of the researches have been done in this area until now have mainly been concerned about investigating some parameter affecting the stress concentration in the composite material.[6–11] While the purpose of this paper is to optimize the parameters that affect the stress concentration in the composite material using GA.

Table 1. Composite material properties.[9]

Material	E_1(GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}
CE9000 Glass/Epoxy	47.4	16.2	7	0.26
Woven Glass/Epoxy	29.7	29.7	5.3	0.17
Plywood	11.79	5.89	0.69	0.071
Carbon/Epoxy	181	10.3	7.17	0.28

Results

The normalized stress is used as an indicator of optimal response for GA. Figure 4 presents variations of normalized stress versus rotation angle for triangular cutout with different bluntnesses. The results of

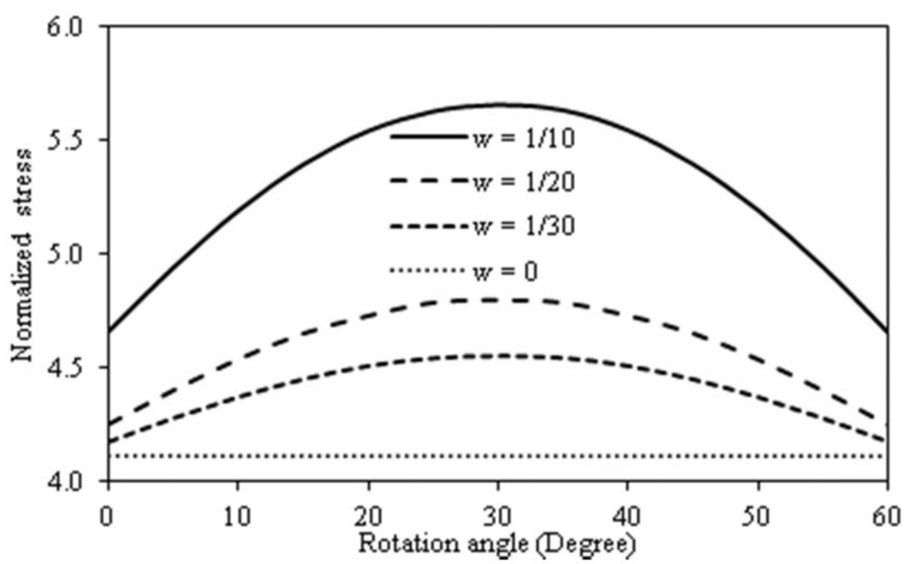

Figure 4. Variations of normalized stress versus rotation angle for triangular cutout.

Table 2. Statistical variables of FEM and ANA data and the corresponding p values.

Cutout shape	Method	Mean	*p-value*	Variance	*p-value*	Skewness	Kurtosis	*p-value*
Triangular	FEM	3.24	0.84	0.04	0.97	0.53	1.93	0.99
	ANA	3.22		0.04		0.61	2.01	
Hexagonal	FEM	5.52	0.68	1.90	0.73	0.76	2.36	1.00
	ANA	5.76		2.20		0.66	2.29	

Figure 4 are obtained only for the load angle and fiber angle 0°. The maximum and minimum normalized stresses can be seen in Figure 4. These two points are very important in the design. The point which shows the maximum normalized stress is called undesirable stress and the other is called desirable stress. If possible, the designer should avoid undesirable stress and is always trying to find the desirable stress.

Evaluation of the fitness function

In order to test the validity of the designed fitness function using the analytical solution as cost function for GA, numerical experiment was performed on the perforated plate with hexagonal and triangular shaped cutouts. A finite element model (FEM) was created for each cutout. Plane183 elements were chosen. For each case, mesh sensitivity was investigated and optimal mesh was chosen. Mean, variance, skewness and kurtosis values of the normalized stresses were calculated around of cutout by analytical method (ANA) and FEM. statistical comparisons between them have been reported in Table 2. The null hypothesis assumes that statistical parameters of both data series are not different. Each hypothesis was tested at the 0.05 level of significance. The hypothesis is rejected if the p value is less than 0.05. The t-test was applied to compare the means of two data sets. The obtained p-value was greater than 0.68. Therefore, the null hypothesis cannot be rejected. The homogeneity variance was checked using the F-test. The p value was greater than 0.73. Thus, the null hypothesis was confirmed. The Kolmogorov–Smirnov test also confirmed the similarity of the distribution for both of the two cutouts ($p > 0.99$). Therefore, differences between the results obtained by the two methods are not statistically significant.

By comparing the results obtained with ANA and other references (Table 3), the validity of the designed fitness function using the analytical solution is again approved.

Cutout with an odd number of sides

The results of optimization using GA for the composite material which have been given in Table 1 show that the optimal shape for all cutouts with an odd number of sides is circle (Table 4). The bluntness in the circle cutout is equal to zero. Although the optimal shape of the cutout for all materials is circle, but optimal amounts of normalized stress for each of the materials used in this study, are quite different from each other. This difference is due only to the difference in material properties. Therefore, the optimum values of design parameters are very dependent on the material properties. The results show that the order of decreasing normalized stress is *woven glass/epoxy*, *CE9000 glass/epoxy, plywood* and *carbon/epoxy*. However, the difference between the lowest and highest normalized stress is about 19%. The results show that, depending on the type of material used, the optimal amount of fiber angle can be varied in the range of 30° to 45°. Figure 5 shows the stress distribution around the circular cutout for different materials. As shown in this figure, the stress

Table 3. Comparison of the obtained normalized stresses on ANA with the others references.

w	Material	Reference 3	Reference 22	ANA
0.25	Isotropic	5.31	5.33	5.31
	CE9000	7.74	7.63	7.70
0.33	Isotropic	8.24	8.28	8.25
	CE9000	12.39	13.13	12.26

Table 4. Optimal values of the design parameters for the cutouts with an odd number of sides.

	Material			
Parameter	*CE9000 Glass/Epoxy*	*Woven Glass/Epoxy*	*Plywood*	*Carbon/Epoxy*
w	0.00	0.00	0.00	0.00
β	0.00	0.00	0.00	0.00
α	90	90	90	90
γ	30	45	40	31
Normalized stress	2.66	2.55	2.84	3.15

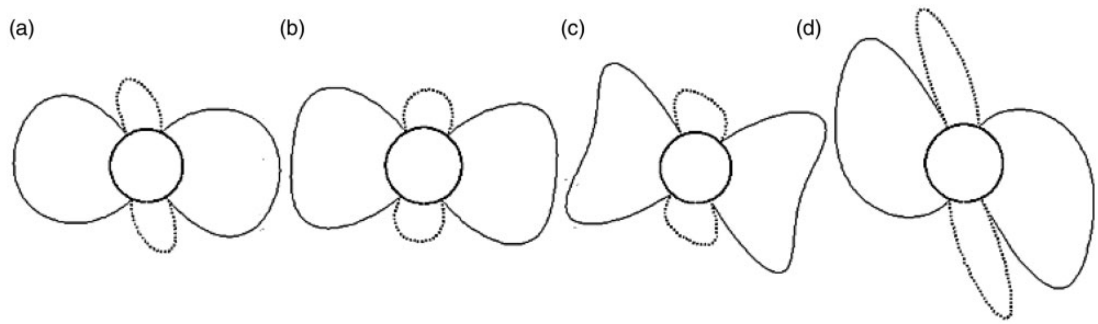

Figure 5. The stress distribution around the circular cutout. CE9000 Glass/Epoxy (a), woven glass/epoxy (b), Plywood (c), and carbon/Epoxy (d).

Table 5. Optimal values of the design parameters for the cutouts with an even number of sides.

Parameter	Number of sides	Material			
		CE9000 Glass/Epoxy	*Woven Glass/Epoxy*	*Plywood*	*Carbon/Epoxy*
w	4	0.045	0.040	0.062	0.065
	6	0.008	0.006	0.017	0.015
	8	0.0025	0.0018	0.0059	0.0083
β	4	25	60	45	0.00
	6	63	41	87	40
	8	40	35	20	18
α	4	70	90	74	70
	6	63	83	62	73
	8	63	80	64	75
γ	4, 6, 8	0.00	35	12	17
Normalized stress	4	2.39	2.50	2.66	2.75
	6	2.57	2.55	2.54	2.98
	8	2.61	2.52	2.78	3.07

distribution strongly depends on the material properties. In this figure, the solid and dotted lines, respectively represent the positive stress and negative stress.

Cutout with an even number of sides

The optimal values of design parameters for the cutouts with an even number of sides have been shown in Table 5. Unlike the previous case, the optimal shape for all cutouts with an even number of sides is not circle. By comparing the results in Tables 4 and 5, it can be concluded that the minimum normalized stress in this case is less than the previous case for all used materials. The optimal value of bluntness parameter decreases with the increasing number of sides of the cutout and the shape of cutout tends to circle. Also, it is seen that for CE9000 and carbon/epoxy, the optimal value of normalized stress increases with increasing the number of sides of the cutouts, but this is not true for the woven and plywood. Accordingly, the optimum shape of cutout for the CE9000, woven andcCarbon/epoxy is quasi-square cutout. While the optimum shape for the Plywood is hexagonal cutout. Among all material used, the lowest possible minimum normalized stress obtains for CE9000 with quasi-square cutout. The results of this table show that the optimal values of the fiber angle (γ) are independent of the number of sides of the cutout for all used material.

The optimal value of load angle (α) in all material varies in the range of 60°–90°. Figure 6 shows the stress distribution around the quasi-square, hexagonal and octagonal cutout for used material. It is seen that the patterns of stress distribution in various cutouts are the same for each individual material. Also, the pattern of stress distribution for carbon/epoxy is different than other materials. In Figure 6, the solid and dotted

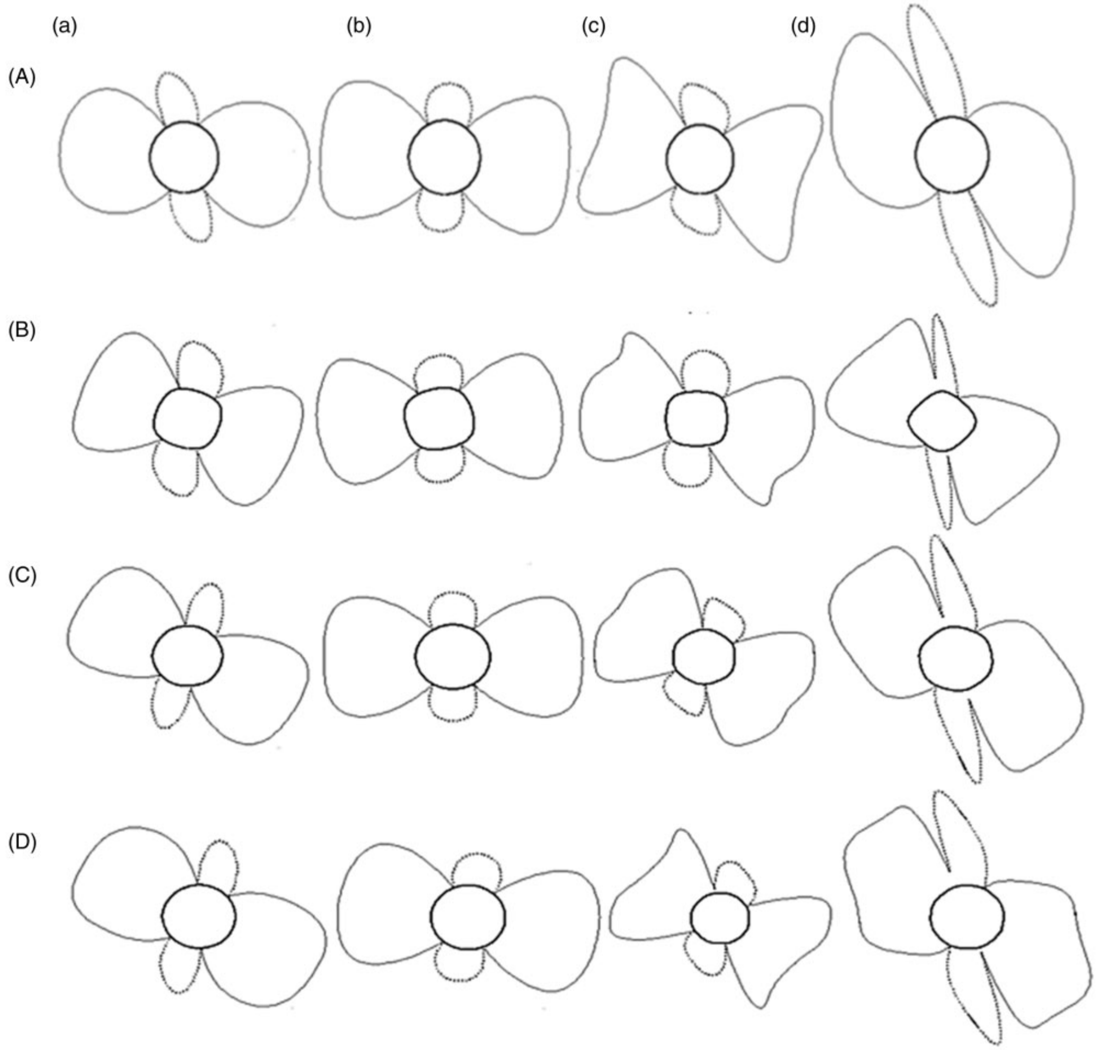

Figure 6. Pattern of stress distribution around the optimal cutout for cutout with an even number of sides. CE9000 glass/epoxy (a), woven glass/epoxy (b), plywood (c) and carbon/epoxy (d). quasi-square (A), hexagonal (B),octagonal (C) and circular cutout (D).

lines, respectively, represent the positive stress and negative stress.

Optimization of quasi-square cutout for different load angle

Our previous results showed that the quasi-square cutout with respect to the others is approximately optimal. Namely, the lowest possible minimum normalized stress is achieved for this cutout. Also, in many cases, the load angle may be predefined. Therefore, the goal of this section is to obtain the optimal effective parameters in different load angle for quasi-square cutout. The optimal values of the design parameters calculated by the genetic algorithm for various levels of load angle have been given in Table 6. It is found that the optimal values of normalized stress for the complementary load angles are equal to each other. In the most cases, there are two groups of results. The resulting angles of each group are complementary to corresponding angles of another group.

The normalized stress at a load angle of 0° and 90° for all used materials resulted in the lowest value of the optimal (Tables 5 and 6). Therefore, such a point normalized stress is known as a global minimum for the fitness function. An interesting result in the case of woven glass/epoxy is that the amount of normalized stress is unique for all optimal values of design variables.

This feature is not observed in other materials. However, in the case of carbon/epoxy composite, the optimal normalized stress for all values of load angle except 45 is similar. Moreover, for all used materials, the value of optimal normalized stress at load angle 45° is more than the other load angles.

Table 6. The optimal values of design parameters for a given levels of load angle in a perforated plate with quasi-square cutout ($n = 3$).

	Optimal values				
Material	α	w	γ	β	Normalized stress
CE9000 Glass/Epoxy	0 (90)	0.045	70 (20)	45	2.39
	30 (60)	0.035	90 (0)	80 (10)	2.43
	45	0.035	90	15	2.58
Woven Glass/Epoxy	0 (90)	0.040	55 (35)	30 (60)	2.50
	30 (45) <u>60</u>	0.016	75 (90) <u>15</u>	75 (0) <u>15</u>	2.50
Plywood	0(90)	0.066	63 (27)	30 (60)	2.66
	30 (60)	0.056	90 (0)	55 (35)	2.75
	45	0.060	83	25	2.84
Carbon/Epoxy	0 (90)	0.060	53 (37)	71 (19)	2.75
	30 (60)	0.063	83 (7)	10 (80)	2.75
	45	0.075	90	24	3.03

Table 7. The optimal values of design parameters for a given levels of rotation angle in a perforated plate with quasi-square cutout ($n = 3$).

	Optimal values				
Material	β	w	γ	α	Normalized stress
CE9000 Glass/Epoxy	0 (90)	0.025	0 (90)	55 (35)	2.50
	30 (60)	0.036	14 (76)	77 (13)	2.41
	45	0.045	20	90	2.39
Woven Glass/Epoxy	0 (90)	0.016	90 (0)	45	2.50
	30 (60)	0.040	55 (35)	0 (90)	2.50
	45	0.026	30	80	2.50
Plywood	0 (90)	0.023	58 (32)	12 (78)	2.85
	30 (60)	0.066	63 (27)	0 (90)	2.66
	45	0.062	78	16	2.66
Carbon/Epoxy	0 (90)	0.065	17 (73)	70 (20)	2.75
	30 (60)	0.007	85 (5)	26 (64)	3.15
	45	0.010	75	15	3.25

Optimization of quasi-square cutout for different rotation angle

Sometimes, depending on the specific needs of industry, the rotation angle of cutout is occasionally specified. The optimal values of the design parameters obtained by the genetic algorithm based on the fitness function for different rotation angle have been shown in Table 7. It is found that the optimal values of normalized stress for the complementary rotation angles are equal to each other. This behavior is similar to the results related to the previous discussion about influence of load angle on optimal normalized stress. Unlike previous findings about the load angle, the minimum optimal normalized stress obtains at rotation angle 45 for all materials except for carbon/epoxy. In the case of woven glass/epoxy, the optimal normalized stress for all values of rotation angle is unique. This may be by reason of the equality of the Young's modulus in different directions. Therefore, for woven glass/epoxy, the global minimum of fitness function can always be achieved under all conditions.

Conclusions

Stress concentration is one of fundamental and very important problems in structures. The stress distributions around cutouts are usually specified either experimentally or numerically using finite element methods. In this paper, the analytical solution was used to determine the stress distribution around different cutouts in perforated composite plates. The selection of optimal parameters is an important aspect to achieve the efficient design. In this study, the genetic algorithm was used to find the optimal effective parameters. The fitness function for a genetic algorithm was defined by the analytical solution based on Lekhnitskii method.

The results presented herein showed that the optimal value of normalized stress was significantly changed using proper cutout shape, material properties, bluntness, load and rotation angles. The optimal values of normalized stress for all cutouts with an odd number of sides were always more than the corresponding value of a circular cutout while, all cutouts with an even number of sides were more efficient than circular cutout. Almost for all materials, the optimal value of normalized stress was changed by varying the load and rotation angles.

Declaration of Conflicting Interests

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