Optimization Model of Hirmand River Basin Water Resources in the Agricultural Sector Using Stochastic Dynamic Programming under Uncertainty Conditions

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In this study, water management allocated to the agricultural sector was analyzed using stochastic dynamic programming under uncertainty conditions. The technical coefficients used in the study referred to the agricultural years, 2013-2014. They were obtained through the use of simple random sampling of 250 farmers in the region for crops wheat, barley, melon, watermelon and ruby grapes under the scenarios of drought, wet, normal, and water required in the most sensitive growth stages. Production function and profit function were obtained from the yield-water-product function of crops using Eviews software. Expected net profit of the system and optimal allocation of water were also calculated based on the GAMS economic analysis software. The results revealed that 14% of the cases over the past 30 years had wet years (high), 47% of the time and that 39% had experienced drought (low) and normal (average) years. In the best case, i.e. with high current levels, respectively at, 58, 67, 54, and 48% of water requirements for these crops and, in the worst case (with low current levels), 47, 35, 49, 53 and 48% of the water requirements provided during the most sensitive growth stages. Moreover, the results showed that the cultivation of the ruby grape was the best product with the highest expected profit in normal and rainfall conditions. In general, when the expected value of net profit is positive, managers would act optimistically and they would promise the optimal level of water provided to the farmers. Conversely, when the net value is negative they would prefer to be more conservative and would promise a lesser amount of water provided to the farmers. Hence, if the promised water to the farmer is not wasted, he will choose the loss incurred from a lesser harvest.

Abstract

Keywords: Expected value, Optimal allocation, Stochastic dynamic programming

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INTRODUCTION

From three perspectives, water plays a key role in sustainable development. First, it is consumed as a final product. Second, water is an important input element in many businesses. Third, it has a key role in biological organisms on Earth (Divakar, 2011). Sustainable socio-economic development in countries with low water is limited to the availability of water and its reduced quality (Ghaffour et al., 2013). In terms of water resource management, low water would provide high risk for different sectors of development programs (Salimfard and Mostafae Dowlatabad, 2013). According to the latest estimates by UNESCO of the water cycle on Earth, it can be inferred that the average annual rainfall in Iran was 251 mm, having a significant difference with the average rainfall of each continent (Kessler and Van Drop, 1998). The average rainfall in all lands and Asia was 831 and 732 mm, respectively (Thompson and Powell, 1998). In Iran, the average annual rainfall was 413 billion m$^3$; however, the area of Iran is 1.1 % of all lands and 3.35% of the land area of Asia. Iran’s rainfall volume comprises just 0.37% of all rainfall from the earth’s lands and 1.29% of rainfall volume in Asia. Also, the average annual evaporation in Iran is estimated at about 70–71% of annual rainfall. In this regard, just Africa and Australia, with 70% and 80% evaporation under undesirable rainfall conditions, respectively, are lower than Iran (Mihankhah, 2013). The Hirmand basin is located in the province of Sistan and Baluchistan. In all climate categories, the Sistan region had a hot and dry climate. Based on different calculation methods, its average annual temperature was 21$^{\circ}$ C, its annual rainfall was 61.4 mm, its relative humidity was 38%, and its potential evapotranspiration was 4196 mm (Ebrahimzade and Lashkaripoor, 2001). Of the total cultivated lands of the country, an area of 12 million hectares is located in Sistan and Baluchistan out of which 52.4% is located in the Sistan region (Mohamadghasemi, 2008). The purpose of this study was to estimate the optimization model of the Hirmand River Basin water resources in agriculture using stochastic dynamic programming under the conditions of uncertainty.

MATERIALS AND METHODS

The yield-water-product function was used to estimate crop production function. Under each irrigation condition, crops had their unique water-product functions estimated using regression methods (Tu et al., 2003). This function expressed the relationship between the actual yield and the effective irrigation, so the second-degree polynomial function for estimating the crop-water function recommended by (Divakar, 2011) was as follows:

$$\frac{Y_a}{Y_m} = f(w) = a_0 + a_1w + a_2w^2$$  \hspace{1cm} (1)

where $Y_a$ is the actual crop yield (tons / ha), $Y_m$ is the maximum potential yield (t/ha), $W = WA / ET_m$ is the ratio of total available water to the maximum potential seasonal evaporation in the crop, i.e. the ratio of actual evapotranspiration to the potential evapotranspiration.

Total water available for the crops was obtained through effective rainfall, irrigation and soil moisture (Mahan et al., 2012) Hence,

$$WA_{j,cp} = SM_{j,cp} + EP_{j,cp} + EI_{j,cp}$$  \hspace{1cm} (2)

where $WA_{j,cp}$ is the total available water for plants during the growing season, $SM_{j,cp}$ is soil moisture in the root zone at the beginning of the growing season, $EI_{j,cp}$ is effective rainfall, and $EI_{j,cp}$ is efficient water used for the crop. Depending on time and irrigation technology, actual evapotranspiration, is the sum of actual soil moisture, effective precipitation, and effective watering during the growing season (Mahan, 1997). In the studied region, the rate of effective rainfall for wheat, barley, melon, watermelon and ruby grapes was zero. Since there was no information available on soil moisture in the region, soil moisture was excluded from the calculations and it was assumed that it was hidden in the effective irrigation (Ghaifi Moghadam et al., 2012).

Therefore, the actual evapotranspiration included effective irrigation during the growing season. Effective irrigation and potential evap-
Evapotranspiration were also determined using monthly weather data for 26 years with Net-wat-Cropwat Software and the Penman-Monteith method, respectively. In this method, based on the types of available data, potential evapotranspiration was calculated daily and monthly. Of course, in the present study, the monthly evapotranspiration was applied. For calculating the amount of actual evapotranspiration, the following formula was used (Abbott and Cohen, 2009):

\[ ET_a = K_c \times ET_o \]  

where \( ET_a \) is the actual evapotranspiration, \( K_c \) is the plant factor varying in different crops, and \( ET_o \) is the potential evapotranspiration.

Based on the water-crop yield, the total profit yield of irrigation water is expressed as follows (Dorfman, 1969):

\[ \beta_j = \sum_{cp} pCP_{j,cp} \cdot Y_{a,j,cp} \cdot A_{f,j,cp} - \sum_{cp} V_{c,j,cp} \cdot A_{f,j,cp} \]  

where \( PCP_{j,cp} \) is the crop price, \( cC_{j,cp} \) is the cultivation cost, and \( V_{c,j,cp} \) is the variable costs of crop production. The target function can be written as:

\[ \max \beta_j \]  

s.t.:

\[ \sum_{cp} A_{f,j,cp} \leq A_j \]

\[ A_{f_{mx},j,cp} \leq A_{f,jc,cp} \leq A_{f_{mn},j,cp} \]

\[ \sum_{cp} E_{l,j,cp} \cdot A_{f,j,cp} \leq Q \]

where \( A_j \) is the total area under cultivation (ha) in the \( j^{th} \) area, and

\( A_{f_{mx},j,cp} \) and \( A_{f_{mn},j,cp} \) are respectively the maximum and minimum levels of cultivated areas (ha), \( E_{l,j,cp} \) is the rate of effective irrigation required during the growing season (m³/ha) and \( Q \) is the total amount of effective irrigation available in the \( j^{th} \) region.

Following slight changes, the shadow price, which includes any variation in the target function, will be placed on the right side of resource limitations. It will be considered as an indicator for the ultimate value of water. In the form of an algebraic expression, the shadow price is expressed as follows: (Mahan et al., 2012)

\[ MWV_j = \frac{\Delta \pi_j}{\Delta Q_j} \]

where \( MWV_j \) is the final value of water ($/m³) in the \( j^{th} \) region, \( \Delta \pi_j \) are changes in income (IRR) caused by slight variation of \( Q \) in the \( j^{th} \) region, and \( \Delta Q \) includes changes in the total amount of effective irrigation in the \( j^{th} \) region. In their research on economic allocation of water resources in Sistan region, (Sharaki and Mohamadghasemi, 2014), used the dynamic optimization models, analyzed its effect on sustainable agricultural development of the area and chose two farming patterns for okra and cucurbits. Karim et al. (2012), studied the effects of cultivating ruby grapes on the economy of the Sistan farmers (Mohamadghasemi, 2008). Also analyzed the cost-benefit performance of agricultural crops (wheat, barley and triticale) in Sistan and Baluchistan. These crops included wheat, barley, melon, watermelon and ruby grapes which were planted under drought, wet, normal, and water requiring scenarios at the most critical growth stages. It is worth to note that, the most sensitive stage of water for horticultural and gardening crops was at the time when they enter the reproductive stage (flowering) (Ghasemi et al., 2012).

Since the system manager always faces issues regarding water allocation between competing agricultural consumers (including various scenarios) and due to the fact that water supply tends be random in the future, the demand for water will also be estimated based on the needs of different scenarios and a logical period will be considered for all data (Zeng et al., 2014).

In cases where the agricultural sector is informed that it has little water available, it will change its activities so that it would need less water. When there is uncertainty, the manager is supposed to create a plan in which, despite allocation of water efficiency, the system benefits increase, and in turn, the system risk reduces (Homauonifar and Rastgaripour, 2011). Hence, the random variable of water supply with \( P_{tk} \) (probability scenario \( k \) in time period \( t \)) was used to design a set of scenarios with branching structure. This model can be formulated as follows (7)
where $F$ is the net system profit of the planning horizon, $NB_i$ is the net income of $i$th crop per allocated water unit, $W_t$ water promised for the product $i$, $C_{it}$ is the farmer’s losses per unit of water promised, but not delivered in period $t$, $D_{itk}$ is water scarcity for the crop $i$ under scenario $k$ in period $t$ (in other words, some of $W_{it}$ was not delivered at $q^k$), $q^k$ is random variable of water supply in period $t$, $W_{itmax}$ is the amount of water allocated for $i$th consumer at time $t$, $P_{tk}$ is frequency probability of scenario $k$ in period $t$, $k$ are total number of scenarios and $t$ is the most sensitive growth stage, $i$ is the type of crop ($i=1$ wheat, $i=2$ barley, $i=3$ melon, $i=4$ watermelon, and $i=5$, ruby grape).

Model 7 expresses uncertainty in the amount of water supplied by the probability level of $P_{tk}$, but it considers the parameters of $W_{it}$, $NB_{it}$, $C_{it}$ in their definite form. In the real world, however, these parameters may not be definite.

To solve this problem, the parameters of this model were considered periodically. The result of the model was as follows: (8)

$$\text{max } f^± = \sum_{i=1}^{m} \sum_{r=1}^{r} NB_{it} W_{it}^± \text{ - } \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{r} P_{itk} C_{it} D_{itk}$$
$$s.t. q_{it} \geq \sum_{j} (W_{it} - W_{it}^+) \quad \forall i, j$$
$$W_{itmax} \geq W_{it}^+ \geq D_{it} \forall i, t, k$$

where $f^±$ is the net profit of the system in the planning horizon, $NB_i^±$ is the farmer’s profit resulted from cultivation of the $i$th crop in period $t$ per unit of water allocation, and $W^±$ is the water promised to the farmer for cultivation at time $t$, $C_{it}^±$ is the farmer’s loss resulting from planting the crop per unit of water allocation promised but not delivered in period $t$, $D_{itk}$ is water scarcity for the $i$th crop under the scenario $k$ in period $t$ (in other words, some of $W_{it}$ which was not delivered in time $q^k$), $q^k$ is a random variable of water supply in period $t$, $W^±_{itmax}$ is the maximum amount of water allocated for consumer $i$ at time $t$, $P_{tk}$ is the frequency probability of scenario $k$ in period $t$.

Since $W^±_{it}$ is considered as a periodic parameter, equation 8 cannot be solved directly, so it needs to be oversimplified. To solve this problem $y_{it}$ is defined as a decision variable (9):

$$W_{it}^± = W_{it}^+ + \Delta W_{it} y_{it}$$
$$\Delta W_{it} = W_{it}^+ - W_{it}^-$$
$$y_{it} \in [0,1]$$

in this equation, $y_{it}$ is a decision variable used to define the optimal range $W^±_{it}$, when $y_{it}$ reaches its highest level, $y_{it}=1$. If the required water is delivered to the sectors, the system profit will reach its peak level. In case of losses, the reverse is true, too. When $y_{it}=0$ and the promised rate of water is delivered, the system profit will decrease dramatically, but if the promised water is supplied, it would have the least amount of loss for the system. Substituting model 9 for model 8, the following model is obtained: (10)

$$\text{max } f^± = \sum_{i=1}^{m} \sum_{r=1}^{r} NB_{it}^± (W_{it}^+ + \Delta W_{it} y_{it}) - \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{r} P_{itk} C_{it} D_{itk}$$
$$s.t. q_{it} \geq \sum_{j} (W_{it} - W_{it}^+) \quad \forall i, j$$
$$W_{itmax} \geq W_{it}^+ \geq D_{it} \forall i, t, k$$

where the $W^±_{it}$ interval is defined as the optimum case, model 10 is divided into two sub-models. After solving these two sub-models, the maximum and minimum rates of total system profit can be obtained. To obtain the highest profits of the whole system ($f^+$) in model 11, the upper limit of interest ($NB^+$) and the lower limit of losses ($C^-$) of the farmer were considered. Model 11 can be formulated as follows: (11)

$$\text{max } f^+ = \sum_{i=1}^{m} \sum_{r=1}^{r} NB_{it} (W_{it}^+ + \Delta W_{it} y_{it}) - \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{r} P_{itk} C_{it} D_{itk}$$
$$s.t. q_{it} \geq \sum_{j} (W_{it} - W_{it}^+) \quad \forall i, j$$
$$W_{itmax} \geq W_{it}^+ \geq D_{it} \forall i, t, k$$

In the model (11), low farmer income ($NB^-$) and low loss of water consumption are indicated. Farmer ($C^+$) was used for obtaining the lowest income limit of the system ($f^-$).

Model (12) is formulated as follows: (12)
the value \( F_{opt}^+ \), \( D_{ik}^+ \) is obtained from the model (12). Using the solutions of the models (11 and 12), the following equations are obtained:

\[
\begin{align*}
F_{opt}^+ &= [F_{opt}^- , F_{opt}^+] \\
D_{ik}^+ &= [D_{ik}^- , D_{ik}^+] \\
\end{align*}
\]  

(13)

as a result, the optimal water allocation for the planned period is calculated as follows:

\[
A_{ikop}^+ = w_{ikop}^+ - D_{ikop}^+ v i, t, k
\]  

(14)

**Data**

Technical factors used in the study corresponded to the farming years 2013-2014. Research was conducted through simple random sampling of 250 regional farmers. Moreover, in order to calculate the possibility of water flow rate (low, medium or high) based on rainfall data gathered from 3 decades ago to the present and using the standardized precipitation index, the percentages of dry, wet and normal years were obtained. These percentages were used to determine the frequency possibility of low, medium and high water flows (Sabouhi et al., 2008). The Standard Precipitation Index is defined as follows:

\[
\text{SPI} = \frac{(P_i - S)}{P}
\]  

(15)

where SPI is the standard precipitation index, \( P_i \) is desired annual rainfall, \( P \) is long-term average rainfall, and \( S \) is the standard deviation of long-term rainfall. If the index is greater and/or less than 1, it means that wet and drought conditions exist, respectively.

Rates between 1 and -1 represent a year with normal and average rainfall (Table 1). The results showed that the number of years with an amount of precipitation less than the average (i.e. drought) is higher than the number of years that are higher than the average amount of precipitation (wet years).

Over the past 30 years, 14% of cases had wet states (high), 47% had drought (low) and 39% had normal (average) cases.

**RESULTS AND DISCUSSION**

The rate of target allocation of water for agricultural crops was calculated by the gross irrigation requirement. Likewise, its maximum and minimum rates were also considered in terms of the highest and lowest water use efficiency in the region. The variable of maximum allocation of water to different crops was calculated based on the most unfavorable efficiency of the irrigation area. Table 2 summarizes this information.

Since water is the most limiting factor for agriculture in the region, any change in this variable would be a good way to estimate the relationship between profits and losses for the water used. This information is presented respectively in Tables 3 and 4.

In addition, based on the information mentioned in Tables 1, 2, 3, and 4, the optimal allocation...
of water under the drought scenario for wheat, barley, melon, watermelon and Ruby grapes are presented in Table 5.

The solution of the target function $f^+$ was positive at the final value of the expected net profit of the crop under a normal scenario. The results of the optimal allocation of water in drought conditions showed that using ruby grapes would lead to the lowest expected value. Based on the information mentioned in Tables 1, 2, 3, and 4, the optimal allocation of water under the normal scenario for wheat, barley, melon, watermelon and ruby grapes are presented in Table 6.

Solving the target function $f^+$ is within the final expected value of net profit of the crops and in accordance with the positive normal scenario. The results of the optimal allocation of water under normal conditions also showed that cultivation of ruby grapes could lead to the highest expected profit.

Table 5: The results of the model under drought scenario during the most sensitive time of irrigation

<table>
<thead>
<tr>
<th>Row</th>
<th>Crop</th>
<th>Percent of occurrence</th>
<th>Target water demand (m³)</th>
<th>Allocated water (m³)</th>
<th>Expected value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Wheat</td>
<td>47</td>
<td>3780</td>
<td>2018</td>
<td>(-1.86,-3.75)</td>
</tr>
<tr>
<td>2</td>
<td>Barley</td>
<td>47</td>
<td>2980</td>
<td>1938</td>
<td>(-1.84,-3.60)</td>
</tr>
<tr>
<td>3</td>
<td>Melon</td>
<td>47</td>
<td>13721</td>
<td>7031</td>
<td>(-1.88,-3.65)</td>
</tr>
<tr>
<td>4</td>
<td>Watermelon</td>
<td>47</td>
<td>15980</td>
<td>7555</td>
<td>(-1.89,-4.41)</td>
</tr>
<tr>
<td>5</td>
<td>Ruby grapes</td>
<td>47</td>
<td>2310</td>
<td>1182</td>
<td>(-1.82,-3.53)</td>
</tr>
</tbody>
</table>

Table 6: The solution of the model under a normal scenario during the most sensitive time of irrigation

<table>
<thead>
<tr>
<th>Row</th>
<th>Crop</th>
<th>Percent of occurrence</th>
<th>Target water demand (m³)</th>
<th>Allocated water (m³)</th>
<th>Expected value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Wheat</td>
<td>39</td>
<td>3610</td>
<td>1576</td>
<td>(3514,243)</td>
</tr>
<tr>
<td>2</td>
<td>Barley</td>
<td>39</td>
<td>2960</td>
<td>1007</td>
<td>(2564,324)</td>
</tr>
<tr>
<td>3</td>
<td>Melon</td>
<td>39</td>
<td>13690</td>
<td>6640</td>
<td>(8249,831)</td>
</tr>
<tr>
<td>4</td>
<td>Watermelon</td>
<td>39</td>
<td>15420</td>
<td>7850</td>
<td>(8117,361)</td>
</tr>
<tr>
<td>5</td>
<td>Ruby grapes</td>
<td>39</td>
<td>2298</td>
<td>1102</td>
<td>(9119,491)</td>
</tr>
</tbody>
</table>
Ultimately, based on the information in Tables 1, 2, 3, and 4, the optimal allocation of water under the wet scenario for wheat, barley, melon, watermelon and Ruby grapes are presented in Table 7.

The solution of the target function $f^+$ was positive at the final value of the expected net profit of the crop under normal scenario. The results of optimal allocation of water in wet conditions showed that cultivation of ruby grapes could provide the highest expected profits.

**CONCLUSIONS**

In this study, management of water allocated to agricultural sector was analyzed using randomly dynamic programming in the context of uncertainty. Technical factors used in the study referenced the farming years 2013-2014. It was conducted through a simple random sampling of 250 farmers in the region for crops wheat, barley, melon, watermelon, and ruby grapes.

![Figure 2: Target water demand and allocated water in normal scenario](image2)

![Figure 3: Target water demand and allocated water in wet scenario](image3)

Table 7: The solution of the model under a wet scenario during the most sensitive time of irrigation

<table>
<thead>
<tr>
<th>Row</th>
<th>Crop</th>
<th>Percent of occurrence</th>
<th>Target water demand ($m^3$)</th>
<th>Allocated water ($m^3$)</th>
<th>Expected value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Wheat</td>
<td>14</td>
<td>3520</td>
<td>1495</td>
<td>(451,558)</td>
</tr>
<tr>
<td>2</td>
<td>Barley</td>
<td>14</td>
<td>2920</td>
<td>978</td>
<td>(462,467)</td>
</tr>
<tr>
<td>3</td>
<td>Melon</td>
<td>14</td>
<td>13600</td>
<td>6554</td>
<td>(452,471)</td>
</tr>
<tr>
<td>4</td>
<td>Watermelon</td>
<td>14</td>
<td>15590</td>
<td>8031</td>
<td>(352,355)</td>
</tr>
<tr>
<td>5</td>
<td>Ruby grapes</td>
<td>14</td>
<td>2211</td>
<td>1024</td>
<td>(462,667)</td>
</tr>
</tbody>
</table>
By inserting the amounts of water scarcity and water allocation in the target function, the profit earned from optimal allocation of water was obtained. The results revealed that 14% of these cases occurred during the past 3 decades had a normal year (high), 47% experienced drought (low) and 39% had a wet (average) year. The results also showed that the rates of final water allocation in drought conditions for wheat, barley, melon, watermelon and ruby grapes were, respectively 2018, 1938, 7555,7031,1182 m3, in wet conditions, they were 2034, 1953, 7050,7570, and 1196m³, respectively, and in normal conditions, they were 2025, 1942, 7046, 7559, and 1189m³.

The results also showed that ruby grapes were the best crop with the highest expected profit in all conditions. In general, whenever the expected net income value of some crops is positive, the government will act optimistically and the high levels of water required are promised to the farmers. In turn, they would prefer to be more conservative and would promise the least amount of water provided to the farmers. Hence, if the promised water to the farmer is not wasted, he will choose the loss incurred from a lesser harvest.

RECOMMENDATIONS

It has been suggested to choose the type of crops based on the irrigated conditions. Moreover, if the farmers have enough freedom to choose and use different variables, the model can provide practical solutions in terms of establishing the amount of profit in farmers’ mental calculations. Since in this study, the expected profit was obtained in drought, wet and normal scenarios under the most sensitive water requirement conditions, it is wise to consider several measures so that sustainable water could be provided to the farmers to grow crops on time and earn the minimum rate of household income. It would reduce the migration of Sistani villagers to cities and neighboring provinces. It is significant to note that if the east of the country becomes haunted it will endanger the security of the area and the whole country.

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