Study of convergence in isogeometric method in the framework of “Diametral Compression Test” elasticity problem with point load singularity

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ABSTRACT

Applying and combining h and p refinement techniques in isogeometric method with the possibility of extension elevation that this method provides, convergence and error of using different kinds of shape functions with different orders and continuities is investigated. It is done in a numerical analysis framework of a practical and well known problem called “Diametral Compression Test”. The advantage of this case is its circular geometry, since IGDA provides designers with high potential of the possibility of using minimum elements to make the exact circular geometry. The point load inserts singularity to the problem. The refinement is utilized uniformly as the effective parameters are limited to the kind, order and continuity of shape functions. With different refinement techniques the convergence of approximated solution to the exact solution of linear elasticity is examined. It is concluded that with the singularity that is mentioned, the error in IGDA is not necessarily reduced with a rise in order, more precisely the level of continuity is another important issue to determine error rise. It is also seen that in the presence of point load singularity the rate of error converges to the same value for all degrees of NURBS and Lagrangian shape functions with any continuity. At the beginning of refinement process the minimum number of elements is used to make the process clearer to understand. In the next steps h and p techniques and their combination are used to refine the combination.

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لا يمكنني قراءة النص العربي المُšeåً، لذا لا يمكنني إعادة صياغته natuxً.
Fig. 1 The disc under diametral compression and stress distributions on two vertical and horizontal diametrical directions

$$U_s \left|_{\gamma=0} \right. = \frac{-2p}{\pi E} \left[ (1-v) + \frac{x \times R}{R^2 - y^2} \right]$$

$$U_y \left|_{\gamma=0} \right. = \frac{-2p}{\pi E} \left[ \log (R + y) - \log (R - y) - (u - 1) \right]$$

### 2- 3D hp-FEM

Bekht et al. [18] applied hp-FEM to calculate the stress field of the disc under diametral compression. The stress distributions are calculated using the following equations:

$$\sigma_x = -\frac{2p}{\pi} \left( \frac{x^2(R-y) + y^2(R+y)}{\beta_2^3} - \frac{1}{2R} \right)$$

$$\sigma_y = -\frac{2p}{\pi} \left( \frac{(R-y)^3 + (R+y)^3}{\beta_2^3} - \frac{1}{2R} \right)$$

$$\tau_{xy} = -\frac{2p}{\pi} \left( \frac{x(R-y)^3 + (R+y)^3}{\beta_2^3} - \frac{1}{2R} \right)$$

### 3- 3D Analysis

Bekht et al. [18] applied 3D analysis to calculate the stress field of the disc under diametral compression. The stress distributions are calculated using the following equations:

$$\sigma_x = -\frac{2p}{\pi} \left( \frac{x^2(R-y) + y^2(R+y)}{\beta_2^3} - \frac{1}{2R} \right)$$

$$\sigma_y = -\frac{2p}{\pi} \left( \frac{(R-y)^3 + (R+y)^3}{\beta_2^3} - \frac{1}{2R} \right)$$

$$\tau_{xy} = -\frac{2p}{\pi} \left( \frac{x(R-y)^3 + (R+y)^3}{\beta_2^3} - \frac{1}{2R} \right)$$

### 4- Computational Results

Bekht et al. [18] obtained the following results:

- The maximum stress occurs at the center of the disc.
- The stress distribution is symmetrical about the diametral plane.
- The stress decreases as the distance from the center increases.
2- Knot vector

\[ B_{i0}(\xi) = \begin{cases} 1 & \xi \in [\xi_i, \xi_{i+1}] \\ 0 & \text{otherwise} \end{cases} \]

\[ B_{ip}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} B_{i+p-1}(\xi) \]

\[ C(\xi) = \sum_{p=1}^{n} P_i B_{ip}(\xi) \]

\[ R_p(\xi) = \frac{w_i B_{ip}(\xi)}{\sum_{p=1}^{n} w_i B_{ip}(\xi)} \]

3- Discretization and uniform h-refinement with classical lagrangian quadratic elements

\[ \text{Fig. 2 Three kinds of langrangian elements that are applied in classic FEM analysis} \]

\[ \text{Fig. 3 Butterfly mesh grid, suitable for circular geometries} \]

\[ \text{Fig. 4 Discretization and uniform h-refinement with classical lagrangian quadratic elements} \]
The refinements which are used in one direction to produce mesh grids in fig 6: in first step with refinement the order is elevated from two to three and then with insertion of knot $\xi = 0.5$ with proper multiplications different continuities $C_0$, $C_1$ and $C_2$ are achieved.

Fig. 6 Order elevation by p refinement from the coarsest mesh grid and then applying h refinement to produce four element cubic NURBS based grid with three continuities respectively $C_2$, $C_1$ and $C_0$ (filled points are control points).

Fig. 7 The refinements which are used in one direction to produce mesh grids in fig 6: in first step with refinement the order is elevated from two to three and then with insertion of knot $\xi = 0.5$ with proper multiplications different continuities $C_0$, $C_1$ and $C_2$ are achieved.

Fig. 8 The refinements which are used in one direction to produce mesh grids in fig 6: in first step with refinement the order is elevated from two to three and then with insertion of knot $\xi = 0.5$ with proper multiplications different continuities $C_0$, $C_1$ and $C_2$ are achieved.
نتایج زیر حاصل مطالعه همگرایی تحلیل آیزوزومتریک در کنار المان محدود کلاسیک برای مسیله و بارکارداری شکل 1 هست:
1- در این مسئله، خطای برای توابع هگزاوید و نری به پیوستگی و درجهای همگرایی نسبت به توابع نری هیورجه میزان خطای را کاهش داد ولی نری در تئوری در نظر نیافت.
2- در این مسئله، استفاده از تعداد نری پیوستگی در توابع نری درج شده و بارکارداری داد.
3- در این مسئله، بستگی بین توابع هگزاوید و نری درج شده و بارکارداری داد. در این مسئله و نری درج شده و بارکارداری داد.

4- تغییر ظاهر مدل سه‌بعدی در محدود می‌گردد.

5- تغییر ظاهر مدل سه‌بعدی در محدود می‌گردد.

6- تغییر ظاهر مدل سه‌بعدی در محدود می‌گردد.

7- فهرست علامات

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8- تقدير و تشریح

لازمه دانست بررسی GeoPDEs و igafem در تغییر ظاهر مدل سه‌بعدی.

\[ e | \begin{array}{c} u - \bar{u} \end{array}\begin{array}{c} u - \bar{u} \end{array} = \int_{\Omega} f | \begin{array}{c} u - \bar{u} \end{array}\begin{array}{c} u - \bar{u} \end{array} \right] \]
846-852, 2011.


9. مراجع


