The Conflation Of Gradient Method From Geoffrion With Fuzzy Logic, A New Approach for optimizing Multi-Objective Decision-Making Models

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Abstract

Gradient algorithm of Geoffrion can solve problems related to linear and non-linear optimizations, provided that decision maker can determine its utility function out of “n” existent goals. However, this diagnosis of utility function doesn’t need its objectivity and direct calculations for the function, but only needs position information of decision maker to continue calculations. Gradient method, due to high convergence speed, has lots of applications in optimization issues. In this article, firstly, gradient method from Geoffrion is described and then some changes are considered due to fuzzy logic. . fuzzy approach is as the mediator between the decision maker and analyst. A problem is solved through these two methods to enable results be compared with each other. Results show that these two are similar with each other in number of steps. However, in the second step of algorithm, using fuzzy logic causes higher speed. Estimation of W, by using fuzzy logic, was easier compared to its previous method.

Keywords: optimization, multi-objective problem, gradient algorithm of Geoffrion, fuzzy logic.

Introduction

One of the problems in optimizing is coming across with multiple objectives or criteria that most of real world problems are amid these models, single objective problems are rarely found. It is suggested to use exchange curve in two purpose problems. By creating exchange curve, the decision maker can select the best point based on one's own preferences (Winstone, 1994). But how is the solution at the time the estimate of exchange curve is difficult or the number of objectives is more than two? To answer this question, many solutions have been suggested that most of them don't have universality and are single objective. As an example (Johnsen, 1968) and (Roy, 1971) can be highlighted, presenting special multi-objective problems.

One of the possible ways answering multi-objective problems mentioned in this article is Gradient algorithm of Geoffrion. The idea for this method goes back to Frank-Wolf method used for non-linear problems. Frank-Wolf method changes non-linear problems to linear problems by linear approximations and does consecutive optimization (Geofrion et al., 1972).

Gradient advantages are: 1) its simple convergence and fast speed based on Frank-Wolf method 2) solving problems with non-linear objectives in case of having concave functions (Asgharpoor,2011). Because of these advantages, gradient algorithm of Geoffrion is suggested for solving multi-objective problems. Innovation in this article is in applying fuzzy logic in gradient algorithm of Geoffrion. The speed for estimation of utility function can be increased and made easier by using fuzzy logic. The difference between these two methods is shown through providing an example and solving that by gradient algorithm of Geoffrion method and incorporating that with fuzzy logic.

Models for decision making by multiple objectives

Multiple objective decision making (MODM) models is a branch of multiple criteria decision making (MCDM) that its another branch is multiple attribute decision making (MADM). In real problems, the opposition is among various objectives, in a way that access and movement along with some of them causes distancing other objectives, therefore, finding collection of variables that can optimize all objectives compared to the mood that only a single objective is followed is rare and difficult (Mehregan, 2013).

Multi objective decision making is like this:

$$optimize: F(X) = \{ f_1(x), f_2(x), ..., f_k(x) \} \quad (1)$$

$$s.t: g_i(X) \begin{bmatrix} \le \\ \ge \\ = \end{bmatrix} 0; i = 1, 2, .., m$$

$$X \in E^n$$

The scale measuring each objective might differ scale measuring other objectives and might not be able to simply add them, for example. The purpose in these design models might be optimizing estimation of utility function for decision making. This utility function in some evaluation methods is objectively calculated and optimized, and in some others is implicitly studied and optimized (Asgharpoor,2011). In most real problems, due to opposition of model objectives, an excusable answer that optimizes an objective function, cannot cause optimization of other objective functions; therefore, decision making in this situation requires a preferable and efficient reply (Mehregan, 2013).

Methods solving multi-objective problems can be categorized into four: 1) Methods that do not need collecting data from the decision maker. 2) Methods that need collecting primary data from the decision maker. 3) Methods that need interactively collecting data from the decision maker. 4) Methods that just need final data from the decision maker (Lai et al., 1994). According to the mentioned categorization, gradient algorithm of Geoffrion can be placed in the third category that will be discussed later.

Gradient algorithm of Geoffrion Method

Gradient model for solving a k-objective problem is obtained from this equation:

$$\max : U\{f_1(x), f_2(x), \dots f_k\} \tag{2}$$
$$s.t: X \in S,\ S = \{X \,|\, g_i(x) \le 0\ \ ;\ i = 1,2,\dots,m\}$$

fj(x) and gi(x) functions are explicitly definite, but U(F) utility function is only implicitly considered. The following supposition must come true to use Frank-Wolf conditions:

1. A set of limitations be convex and closed bordered.
2. U(F) be differentiable and concave on X.
3. Each fj(x) be concave.
4. $\frac{\partial U}{\partial f_1} > 0$

The first step: select the Xi∊S starting point and put: i=1

The second step: obtain the optimized solution of yi to navigate for an efficient movement from the following problem:

$$\max : \nabla_x U\left(f_1(x^i), f_2(x^i), \dots, f_k(x^i)\right).y^i \tag{3}$$
$$s.t: y^i \in S$$

Implicit estimation of $\nabla_x U(F)$ would be discussed by using data collected from decision making. Put $d_i = y^i - x^i$ and go to the third step.

The third step: calculate $\tilde{\lambda}^i$ optimum by the following optimization:

$$\max : U\{F(X^i + \lambda^i.d_i)\} = \{f_1(X^i + \lambda^i.d_i), \dots, f_k(X^i + \lambda^i.d)\} \tag{4}$$
$$s.t: 0 \le \lambda^i \le 1$$

The way to estimate $\tilde{\lambda}^i$ by the decision maker will be discussed later. Set $X^{i+1} = X^i + \tilde{\lambda}.d_i$, $i \to (i+1)$ and return to the second step. Continuously do the movements as mentioned to the point that criterion for end of algorithm is met, in a way that $X^i = X^{i+1}$.

Development of the second step: Main part of interaction with decision making is in the second step because U(F) is not explicitly known and needs the help of decision maker. It is assumed that decision maker can estimate the balance between both objectives of problem objectives in lieu of a definite solution. This substitution marginal rate between objectives can be used for estimation of gradient of U(F) for each definite solution.

For this purpose, the objective function of available model in the second step can be extended like this:

$$\nabla_X U\left(f_1\left(x^i\right),\dots f_k\left(x^i\right)\right).y^i = \frac{\partial U}{\partial\left(f_1, f_2,\dots f_k\right)}.\frac{\partial\left(f_1, f_2,\dots f_k\right)}{\partial\left(x_1,\dots x_n\right)}.y^i =$$

$$\left\{\frac{\partial U}{\partial f_1},\dots,\frac{\partial U}{\partial f_k}\right\}.\begin{Bmatrix}\frac{\partial f_1}{\partial x_1},\dots\frac{\partial f_1}{\partial x_n}\\ \\ \frac{\partial f_k}{\partial x_1},\dots,\frac{\partial f_k}{\partial x_n}\end{Bmatrix}.y^i = \left\{\sum_{j=1}^{k}\left(\left[\frac{\partial U}{\partial f_j^i}\right]\nabla_x f_j\left(x^i\right)\right)\right\}.y^i \qquad (5)$$

In a way that $\frac{\partial U}{\partial f_j^i}$ shows U partial derivation in proportion to the j^th objective that is evaluated in $\{f_1(x^i),\dots,f_k(x^i)\}$ point, and also $\nabla_x f_j(x^i)$ shows $f_j(x)$ gradient that is calculated in x^i point. Optimization of the available model does not change by multiplying that in a constant number, therefore, the mentioned objective function is divided by $\frac{\partial U}{\partial f_1^i}$ that consequently the problem of available navigation is equal by the following model:

$$\max : \sum_{j=1}^{k}\left[w_j^i.\nabla_x f_j\left(x^i\right)\right].y^i \qquad (6)$$
$$y^i \in S$$
$$\rightarrow w_j^i = \frac{\frac{\partial U}{\partial f_j^i}}{\frac{\partial U}{\partial f_1^i}}$$
$$j = 1, 2, \dots, k$$

w_j^i weights reflects the balance of decision making between f_1 objective (that is optionally considered as reference) and f_j objective in x^i, that's because:

$$\frac{\frac{\partial U}{\partial f_j}}{\frac{\partial U}{\partial f_1}} \approx \frac{\lim \frac{\Delta U}{\Delta f_j}}{\lim \frac{\Delta U}{\Delta f_1}} \approx \frac{\Delta f_1}{\Delta f_j} \qquad (7)$$

In order to estimate w_j^i, substitution marginal rate between f_1 and f_j is evaluated by the aid of decision making. In a way that an indifferent balance of really small change (Δf_1) in f_1 objective versus required objective change (Δf_j) be found. It means that change in f_1 objective to size (Δf_1), having other objectives except j^th objective fixed, how much change to size (Δf_j), for example, can amend of J^th objective? This estimation is likewise. (The negative sign is to make w_j^i bigger than zero as balance directions between objectives are against each other.)

$$w_j^i = -\frac{\Delta f_1}{\Delta f_j} \qquad (8)$$

The suggested method for the second step: to determine W in this step like the previous method, there is a need to interact with decision maker. In order to enter decision maker comments into gradient algorithm, fuzzy logic was used. To obtain W in the previous method, increase in objectives versus decrease in the function of decision maker comments was done with definite numerical values, to the extent that indifference point of the decision maker was obtained. It usually is time consuming to get to difference point in this way.

In this suggested method for calculation of W from the level of objectives importance compared to reference objective is a question. In this method, the decision maker can express the level of objectives importance in proportion to reference objective in a step with expression like very much, much, and etc.. Through this method, plays of time over reaching to indifference point are avoided. Here, fuzzy approach is as the mediator between the decision maker and analyst. Therefore, W is defined in this way:

1. Selecting a reference function
2. Decision maker is questioned about the importance of other functions compared to reference function.
3. Assigning fuzzy numbers to verbal expressions
4. The process of defuzzification
5. Normalizing obtained weights.

defuzzification operation: defuzzification is a crucial step in fuzzy systems. In fuzzy systems, results of an approximate reasoning often is obtained in single or multiple fuzzy sets. In this case, it is essential to convert output fuzzy system into a typical number (non-fuzzy). There are various methods that includes gravity center, area center, maximum center, total center, and average weight center method (Radfar et al., 2010).

Development of the third step: optimum $\tilde{\lambda}^i$ that was discussed before should be obtained from the following formula:

$$\begin{aligned} &\max : U\left\{F\left(X^i + \lambda^i d_i\right)\right\} \\ &s.t : 0 \le \lambda^i \le 1 \end{aligned} \tag{9}$$

In order to use decision maker help to estimate $\tilde{\lambda}^i$, $F\left(X^i + \lambda^i d_i\right)$ changes must be provided in a function of λ variable (in a range of 0 to 1). This can be through a table that shows changes of the function for every $0 \le \lambda^i \le 1$.

Termination criterion:

Theoretically, end of movements is the time that two consecutive solutions $\{X^{i+1}, X^i\}$ become similar, of course, ideal situation rarely happens. Thus, to present a practical criterion for termination of an algorithm, it should be noticed that as a utility function $U(F)$ is presumably concave, there is a F^i for each point:

$$\begin{aligned} &\forall F_i ; F_i \in H \\ &U(F) \le U\left(F^i\right) + \nabla_F U\left(F^i\right).\left(F - F^i\right) \\ &\rightarrow^{or}\ U(F) - U\left(F^i\right) \le \nabla_F U\left(F^i\right).\left(F - F^i\right) \\ &\rightarrow^{or}\ U\left(F^{i+1}\right) - U\left(F^i\right) \le \nabla_F U\left(F^i\right).\left(F^{i+1} - F^i\right) \end{aligned} \tag{10}$$

It means that the upper bound of optimum in utility function from F^i to $\{U(F^{i+1}) - U(F)\} F^{i+1}$ equals to $\nabla_F U\left(F^i\right).\left(F - F^i\right)$. Although $\nabla_F U\left(F^i\right)$ is not known, meaningful comparisons of $w^i = \{1, w_2^i, \ldots w_k^i\}$ values in different points can be conducted. Considering the available relationship for w_j^i, utility function gradient for each $F_i \in H$ point is:

$$\nabla_F U\left(F^i\right) = \frac{\partial U}{\partial f_1}.\left(1, w_2^i, \ldots, w_k^i\right) \tag{11}$$

As for algorithm hypothesis, $\frac{\partial U}{\partial f_1} = C > 0$ is positive for each practical points (which means it has positive marginal utility and its value for each practical point is almost fixed):
It is:

$$\nabla_F U\left(F^i\right) = C.\left(1, w_2^i, \ldots, w_k^i\right) \tag{12}$$

And finally:

$$\nabla_F U\left(F^i\right).\left(F^{i+1} - F^i\right) = C.\left(1, w_2^i, \ldots, w_k^i\right).\left(F^{i+1} - F^i\right) \tag{13}$$

Therefore, optimization proportion of i^th movement for, for example, one can be calculated in this way:

$$\frac{\Delta^i}{\Delta^1} = \frac{C.\left(1, w_2^i, \ldots, w_k^i\right).\left(F^i - F^{i+1}\right)}{C.\left(1, w_2^1, \ldots, w_k^1\right).\left(F^1 - F^0\right)} \tag{14}$$

Hence, this optimization proportion, by deleting the proximate zero C, is calculable and termination algorithm is chosen in the way that if $(i > 1)\frac{\Delta^i}{\Delta^1} \le \varepsilon$ is met, terminates and the final solution for the problem be obtained; $\frac{\Delta^i}{\Delta^1} \le \varepsilon$ means that no more optimization is possible for the problem.

The reason for using fuzzy logic in the second step:

In case fuzzy sets theory is to be explained, it must be mentioned that it is a theory for acting in insecure conditions. This theory can give mathematic face to many concepts, variables, and systems that are not accurate; it can also provide the condition to reason, deduce, control, and make decision under uncertainty. It is clear that many of our decisions and practices are in uncertainty conditions and clear and unambiguous modes are rare (Tehrani, 1996).

Fuzzy logic can enter parameters like knowledge, experience, judgment, and decision making in the pattern and along with creating flexibility provides a gray picture of the gray world.

Definition of fuzzy number

If X is the discussing world and its members x, then fuzzy set of A from X with ordered pair is defined by the following equation: μA(x) is membership function, X in A.

$$A = \{(x, \mu_A) \mid x \in X\} \tag{15}$$

- fuzzy set, is called real fuzzy number whenever $\mu_{\tilde{A}}(x)$ (membership function of $\tilde{A}$) has the following traits: $\mu_{\tilde{A}}(x)$ has the upper semi continuous
- Out of a range like $\mu_{\tilde{A}}(x) = 0, [a, b]$
- b, c, real numbers be in a way that $a \le b \le c \le d$ and for each ascending $\mu_{\tilde{A}}(x), x \in (a, b)$ within the range of $[c, d]$ be descending and for every $\mu_{\tilde{A}}(x) = 1, x \in (a, b)$.

Since $\mu_{\tilde{A}}(x)$ is upper semi continuous for $\tilde{A}$, then $\{x : \mu_{\tilde{A}}(x) \geq \alpha\}$ set for all α in A is closed. Triangular fuzzy number is a special kind of fuzzy number that is defined by three a, b, c numbers.
Triangular fuzzy number: $\tilde{A}$ is called a triangular fuzzy number whenever a, b, c real numbers be available in a way that a<b<c

- $Y = \mu_{\tilde{A}}(x)$ diagram becomes a triangle in a way that its base is on $[a,c]$ and its head is on.
- Triangular fuzzy numbers are shown in this way $\tilde{A} = (a,\ b,\ c)$ (Soleimani, 2012).

EXAMPLE

Consider the following multi-objective problem.

$$\max f_1(x) = -x_1 - 2x_2 - 3x_3$$
$$\max f_2(x) = -x_1 - 2x_2 - x_3$$
$$\max f_3(x) = -2x_1 - x_2 - x_3$$
$$s.t:$$
$$2x_1 + 5x_2 + 3x_3 \geq 30$$
$$x_1 + x_2 + x_3 \geq 20$$
$$10x_1 + 8x_2 + 4x_3 \geq 40$$
$$3x_1 + 2x_2 \geq 20$$
$$x_1, x_2, x_3 \geq 0$$

The first movement
The first step: practical point of $X \in S$

$$x = (5,4,12) \qquad (16)$$
$$i = 1, \varepsilon = 0.15$$

The second step: determining the optimum solution y1 and consequently accessing to efficient movement.

$$F^0 = \{F_1(X^1), F_2(X^1), F_3(X^1)\} = \{-49,-25,-26\} \qquad (17)$$

Calculating w_j^1 weights: the following adjustments are conducted by the decision maker.

(18)

$\rightarrow$ for f_2, f_3 versus f_1:

$$\{-49,-25,-26\} \square \{(-49+0.5),(-25-4),(-26)\}$$
$$\{-49,-25,-26\} \square \{(-49+0.75),(-25),(-26-10)\}$$
$$w_1 = -\frac{0.5}{-4} = 0.125$$
$$w_2 = -\frac{0.75}{-10} = 0.075 \rightarrow w^1 = \{1, 0.25.0.075\}$$

to calculate y^1 and d_1, we have:

$$\begin{aligned}
&\max f(y^1): f_1(y^1)+0.125 f_2(y^1)+0.075 f_3(y^1)\\
&s.t\\
&y^1 \in s\\
&\max f: -1.275 y_1 - 2.325 y_2 - 3.2 y_3\\
&s.t:\\
&y^1 \in s
\end{aligned} \tag{19}$$

$$\begin{aligned}
&y^1 = \{20,0,0\},\ f = -25.5\\
&d_1 = y^1 - x^1 \rightarrow d_1 = \{15,-4,-12\}
\end{aligned}$$

The third step: specify $\tilde{\lambda}^1$ optimum, $F^1(X^1+\lambda^1 d_1)$ values for $0 \le \lambda^1 \le 1$ is set in the table(1):

Table 1. Calculation of values in F[1] function for different LAMBDA

F^1 \ λ	0	0.4	0.8	1
f_1	-49	-37.4	-25.8	-20
f_2	-25	-23	-21	-20
f_3	-26	-31.7	-37.2	-40

The above table will be shown to the decision maker to select the following solution.

$$\begin{aligned}
&F^1 = \{-37.4,-23,-31.7\},\ \lambda = 0.4\\
&X^2 = \{11, 2.4, 7.2\}
\end{aligned} \tag{20}$$

Set i+1=2 and shift to the second movement.

The second movement:
The second step: navigation for d_2 .
change calculation of W_j^2 weights of reference objective from F_1 to F_3 because changes of F_3 with F_1 and F_2 are in contrast. Now decision maker states the following adjustments considering F_3 as a reference.

→ For F_1 and F_2 versus F_3, we have:

$$\begin{aligned}
&\{-37.4,-23,-31.7\} \ \square\ \{(-37.4-10),(-23),(-31.7+0.5)\}\\
&\{-37.4,-23,-31.7\} \ \square\ \{(-37.4),(-23-5),(-31.7+0.5)\}\\
&W^2 = \{0.05, 0.1, 1\}
\end{aligned} \tag{21}$$

to calculate Y^2 and d_2, it is:

$$\begin{aligned}
&\max f: -2.15 y_1 - 1.3 y_2 - 1.25 y_3\\
&s.t: y \in s\\
&y^2 = \{0,10,10\},\ f = -25.5\\
&d_2 = \{-11, 7.6, 2.8\}
\end{aligned} \tag{22}$$

The third step: calculation of $\tilde{\lambda}^2$ for different values shown in the table(2).

Table 2. Calculation of F^2 function for different values of LAMBDA

F^2 \ λ	0	0.4	0.8	1
f_1	-37.4	-42.44	-47.48	-50
f_2	-23	-25.8	-28.6	-30
f_3	-31.7	-26.96	-22.32	-20

The decision maker prefers the following solution.

$$F^2 = \{-42.44, -25.8, -26.96\}, \lambda^2 = 0.4 \tag{23}$$

Termination criterion: the proportion of optimization of the second movement to the first is:

$$\frac{\Delta^2}{\Delta^1} : \frac{(0.05, 0.1, 1)(-42.44+37.4, -25.8+23, -26.96+31.7)^t}{(1, 0.25, 0.075)(-37.4+49, -23+25, -31.7+26)^t} = 0.3 \tag{24}$$

$$\frac{\Delta^2}{\Delta^1} > \varepsilon$$

Termination criterion is not provided, continue the algorithm to the point that termination control becomes provided.

SOLVING BY THE SUGGESTED METHOD

$$\max f_1(x) = -x_1 - 2x_2 - 3x_3$$
$$\max f_2(x) = -x_1 - 2x_2 - x_3$$
$$\max f_3(x) = -2x_1 - x_2 - x_3$$
$$s.t:$$
$$2x_1 + 5x_2 + 3x_3 \geq 30$$
$$x_1 + x_2 + x_3 \geq 20$$
$$10x_1 + 8x_2 + 4x_3 \geq 40$$
$$3x_1 + 2x_2 \geq 20$$
$$x_1, x_2, x_3 \geq 0$$

The first movement

The first step: $X \in S$ practical point

(25)

$$x = (5, 4, 12)$$
$$i = 1, \varepsilon = 0.15$$

The second step: specifying the optimum solution for y^1 and consequently accessing to the efficient movement.

(26)

$$F^0 = \{F_1(X^1), F_2(X^1), F_3(X^1)\} = \{-49, -25, -26\}$$

Calculation of w_j^1 weights:

In this step after specifying reference function, the importance level of other functions in proportion to that function is questioned from the decision maker. Table (3) shows Fuzzy numbers to specify preferences. In this step, $f_1(x)$ function is selected as a reference function. The importance level of $f_2(x)$ and $f_3(x)$ functions versus $f_1(x)$ is questioned from the decision maker. The decision maker will reply with expressions like *very important, important*, and etc. as an example, the decision maker estimates the importance of the second function rather than the first higher; and the importance of the third function compared to the first as more important. Table (3) shows Fuzzy numbers to specify preferences

Therefore:

Level of importance = {similar, much more important, more important)

Table 3. Fuzzy numbers to specify preferences (kazemi et., 2012)

Triangular fuzzy numbers	Preferences
(1, 1, 1)	Similar importance
(0.5, 1, 1.5)	Almost similar importance
(1, 1.5, 2)	A little more important
(1.5, 2, 2.5)	More important
(2, 2.5, 3)	Much more important
(2.5, 3, 3.5)	Thoroughly important

The level of importance is shown by ***imp***. According to the table, the level of importance is defined in this way:

$$\text{imp} = \{(1,1,1),\ (2,\ 2.5,\ 3),\ (1.5,\ 2,\ 2.5)\}$$

A.1. defuzzification in this research is conducted by the fuzzy average method.

(28)

$$w = \frac{m_l + 2m_m + m_k}{4}$$

$$w^1{}_1 = \frac{1+2+1}{4} = 1$$

$$w^1{}_2 = \frac{1.5+5+3}{4} = 2.5$$

$$w^1{}_3 = \frac{1.5+4+2.5}{4} = 2$$

$$w^1 = \{1, 2.5, 2\}$$

After normalization:

(29)

$w^1 = \{0.18, 0.45, 0.36\}$

to calculate y^1 and d_1:

(30)

$$\max\ f(y^1): 0.18 f_1(y^1) + 0.45 f_2(y^1) + 0.36 f_3(y^1)$$
$$s.t:$$
$$y^1 \in s$$
$$\max\ f: -1.35 y_1 - 1.62 y_2 - 1.35 y_3$$
$$s.t:$$
$$y^1 \in s$$

$$y^1 = \{6.66, 0, 13.33\},\ f = -27$$
$$d_1 = y^1 - x^1 \rightarrow d_1 = \{1.66, -4, 1.33\}$$

The third step: specifying the optimum $\tilde{\lambda}^1$, $F^1(X^1 + \lambda^1 d_1)$ values for $0 \le \lambda^1 \le 1$ is set in the table (4):

Table 4. Specifying F1 function for every different LAMBDA, by solving according to the suggested solution

F^1 \ λ	0	0.4	0.8	1
f_1	-49	-48.06	-47.12	-46.65
f_2	-25	-22.99	-20.99	-19.99
f_3	-26	-26.52	-26.52	-26.65

The above table is shown to decision maker that the following solution is chosen:
(31)

$$F^1 = \{-48.06, -22.996, -26.26\},\ \lambda = 0.4$$
$$X^2 = \{5.66, 2.4, 12.53\}$$

Set i+1=2 and go the second movement
The second movement:
The second step: navigation for d_2

calculation of w_j^2 weights changes objective reference from F_1 to F_3 because changes of F_3 with F_1 and F_2 is in contrast. Now, decision maker specifies the importance of two other functions compared to F_3. The level of importance= {a little more important, more important, similar}

(32)

$$imp = \{(1, 1.5, 2), (1.5, 2, 2.5), (1, 1, 1)\}$$

A.1. defuzzification:

(33)

$$w_1^2 = \frac{1+3+2}{4} = 1.5$$
$$w_2^2 = \frac{1.5+4+2.5}{4} = 2$$
$$w_3^2 = \frac{1+2+1}{4} = 1$$
$$w^2 = \{1.5, 2, 1\}$$

w^2 after normalization is:

$$w^2 = \{0.33, 0.44, 0.22\} \tag{34}$$

to calculate Y^2 and d_2:

$$\begin{aligned} &\max\ f : 0.33 f_1(x) + 0.44 f_2(x) + 0.22 f_3(x) \\ &\max\ f : -1.21 y_1 - 1.76 y_2 - 1.65 y_3 \\ &s.t : y \in s \\ &y^2 = \{20, 0, 0\},\ f = -24.2 \\ &d_2 = \{14.34, -2.4, -12.53\} \end{aligned} \tag{35}$$

The third step: Calculation of $\tilde{\lambda}^2$ for different values is shown in the table (5):
The decision maker prefers the following solution:

$$F^2 = \{-36.77, -21.73, -31.63\}, \lambda^2 = 0.4 \tag{36}$$

Termination criterion: the proportion of the second movement to the first is:

$$\frac{\Delta^2}{\Delta^1} : \frac{(0.33, 0.44, 0.22)(-36.77 + 48.06, -21.73 + 22.99, -31.63 + 26.26)^t}{(0.18, 0.45, 0.36)(-48.06 + 49, -22.99 + 25, -26.26 + 26)^t} = 3.16$$

$$\frac{\Delta^2}{\Delta^1} < \varepsilon$$

The termination criterion is not provided. Continue the algorithm to the point that termination criterion is provided.

Conclusion

In this article, gradient algorithm of Geoffrion which is a solution for multi objective problems, is presented. This method needs interaction with the decision maker during the problem solving. Of course, there are some criticism like: 1) the selection of objective function for the time the number of objectives is more than three, 2) the selection of $\tilde{\lambda}^i$ by the decision maker. However, its advantages are high convergence speed and solving non-linear problems.

Then, by using fuzzy logic, a development is achieved in the method of gradient of Geoffrion. Fuzzy logic has hegemony over traditional methods at the time of measuring human mental concepts. The reason is that the classic management methods are based on two or multiple value mathematics, requiring accurate quantitative and qualitative data. Fuzzy logic can easily enter preferences and judgments into mathematics patterns.

Finally, by presenting a problem, the solution was done by two methods: gradient of Geoffrion and its combination with fuzzy logic. In both methods, all steps of algorithm, selection of reference function and $\tilde{\lambda}^i$ were similarly done to have all the steps similar and comparable. However, in the second step of algorithm, using fuzzy logic causes a faster speed. By using fuzzy logic, estimation of w proportion is easier than the previous one.

The method of gradient of Geoffrion is presented by its combination with fuzzy logic. Selection of each of them for multi objective problems is on the elite and decision makers. It is suggested that in the future research: 1) apply combination of gradient method and fuzzy logic in different case studies and compare its result with gradient of Geoffrion method, 2) in the second step without defuzzification, enter obtained fuzzy numbers
for w in the continuation of algorithm and compare its results with the results of suggested method in this article.

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