Simulation of droplet impact on a curved solid surface by Lattice Boltzmann Method

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Abstract
In this paper, a two-phase multi-relaxation time lattice Boltzmann model is implemented to investigate droplet impact on a curved solid surface. The improved model is capable of simulating the phenomenon at low viscosities, different contact angles, and relatively high density ratios. Effects of varying the liquid viscosity and surface wettability are shown and discussed. For the impact on a hydrophilic surface, it was found that increasing the liquid viscosity slows down its spreading motion and the liquid shows more resistance to be stretched by the gravity force below the solid. In the case of droplet impact on a hydrophobic surface, secondary drops are separated from the spreading liquid before it even reaches the bottom of the surface.

Keywords: *LBM, Droplet, Impact, Curved Surface.*

Introduction

Droplet impact on solid surfaces has a wide range of applications in different fields. Ink-jet printing, spray cooling, and painting, hot-spot cooling, thermal spray coating, and soil erosion by rain drops are some examples where droplet impact on a solid surface is involved [1].

The Weber and Reynolds number as well as the contact angle are the most important variables in studying droplet impact on a solid surface. Effects of gravity force can be also considered by using the Bond number. The phenomenon of droplet impact on a flat solid surface has been studied thoroughly by performing different experiments, theoretical analysis, and numerical simulations [2]; however, number of papers investigating droplet impact on a curved surface is much lesser because of more complexity. The lattice Boltzmann method (LBM) is a powerful kinetic-based numerical model capable of simulating different problems in fluid mechanics especially multi-phase flows. In contrast with the conventional methods in which the interface is tracked and constructed, the position of the interface is automatically given using the pseudo-potential multi-phase LBM implemented in this paper. There are only a few papers simulating droplet impact on a curved solid surface by using the LBM [3-5]. Furthermore, the numerical model used in these few papers is limited to low Reynolds numbers or low density ratios. Therefore, in this study, an improved two-phase model based on the pseudo-potential LB model is used to study the droplet impact on a solid circular cylinder. The model has certain capabilities which make it applicable to cases with relatively high density ratios, low viscosities, and tunable values of surface tension independent of the density ratio without losing the simplicity and computational efficiency of the original model.

Numerical model

Details of the numerical model used in this paper as well as its validation test cases are given elsewhere [6]. The main description of the model is presented here; however, the validation part is not repeated.

The multi-relaxation time lattice Boltzmann equation for multi-phase flows is expressed as follows:

$$f_\alpha(\boldsymbol{x}+\boldsymbol{e}_\alpha\delta t,\ t+\delta t)-f_\alpha(\boldsymbol{x},t) = -\sum_i \Lambda_{\alpha i}\left(f_i-f_i^{eq}\right)+\delta t\left(s_\alpha-\frac{1}{2}\sum_i \Lambda_{\alpha i}s_i\right) \quad (1)$$

where $\boldsymbol{x}$ is the spatial position, $\boldsymbol{e}_\alpha$ the discrete velocity in the αth direction, δt the time step, Λ the relaxation matrix, s the forcing term, and f_α and f_α^{eq} represent the particle distribution and the equilibrium distribution functions in the αth direction, respectively. In low Mach limit, the equilibrium distribution function is given as follows:

$$f_\alpha^{eq} = \omega_\alpha\rho\left[1+\frac{\boldsymbol{e}_\alpha\cdot\boldsymbol{V}}{c_S^2}+\frac{(\boldsymbol{e}_\alpha\cdot\boldsymbol{V})^2}{2c_S^4}-\frac{\boldsymbol{V}\cdot\boldsymbol{V}}{2c_S^2}\right] \quad (2)$$

where ω_α is the weight factor in the αth direction and c_s is the speed of sound in lattice which is equal to $c_S = 1/\sqrt{3}$. The macroscopic density and velocity are given by:

$$\rho = \sum_\alpha f_\alpha \quad , \quad \rho\boldsymbol{V} = \sum_\alpha f_\alpha \boldsymbol{e}_\alpha + \frac{\delta t}{2}\boldsymbol{F} \quad (3)$$

where $\boldsymbol{F}$ is total force exerted on each fluid particle equal to $\boldsymbol{F}=\boldsymbol{F}_1+\boldsymbol{F}_2+\boldsymbol{F}_3$ in this paper. It is more convenient to do the collision step in the momentum space; therefore, the right-hand side (RHS) of Eq. (1) is transformed into the momentum space by multiplying it by the transformation matrix $\mathbf{M}$ as follows:

$$\mathbf{M}(\text{RHS Eq. 1}) = \mathbf{m} = -\sum_i \hat{\Lambda}_{\alpha i}\left(\hat{f}_i-\hat{f}_i^{eq}\right) + \delta t\sum_i\left(I_{\alpha i}-\frac{1}{2}\hat{\Lambda}_{\alpha i}\right)\hat{s}_i \quad (4)$$

where $\hat{f} = \mathbf{M}f$, I is the identity matrix and $\hat{\Lambda}$ is the diagonal relaxation matrix in the momentum space. The macroscopic kinematic viscosity is given by $\nu = (\tau_\upsilon - 0.5)c_S^2$. According to Li et al. [7], the surface tension can be tuned independent of the density ratio by adding a source term, $\delta t C_\alpha$, to Eq. (1) and is implemented in this paper. The streaming step is then performed in the velocity space:

$$f_\alpha\left(\boldsymbol{x}+\boldsymbol{e}_\alpha\delta t,\ t+\delta t\right) = \mathbf{M}^{-1}(\hat{f}_\alpha+\mathbf{m}) \quad (5)$$

The fluid-fluid interaction force is given by:

$$\boldsymbol{F}_1(\boldsymbol{x},t) = -G\psi(\boldsymbol{x},t)\left[\sum_\alpha w\left(|\boldsymbol{e}_\alpha|^2\right)\psi(\boldsymbol{x}+\boldsymbol{e}_\alpha,t)\boldsymbol{e}_\alpha\right] \quad (6)$$

where G = -1 and $w(1)=1/3$, $w(2)=1/12$ are the weights in the D2Q9 lattice model. The interaction potential follows:

$$\psi = \sqrt{\frac{2\left(P_{\text{EOS}}-\rho c_s^2\right)}{Gc^2}} \quad (7)$$

where c = 1, c_s^2 = 1/3 and P_{EOS} is the equation of state which is the Carnahan-Starling in this paper.The fluid-solid interaction force is similarly given by:

$$\boldsymbol{F}_2(\boldsymbol{x},t) = -G\psi(\boldsymbol{x},t)\left[\sum_\alpha w\left(|\boldsymbol{e}_\alpha|^2\right)\psi(\rho_w)S(\boldsymbol{x}+\boldsymbol{e}_\alpha)\boldsymbol{e}_\alpha\right] \quad (8)$$

where ρ_w is a fictitious wall density varied to achieve different wettability and S is equal to one for solid nodes and zero elsewhere. The gravity force is imposed by defining $\boldsymbol{F}_3(\boldsymbol{x},t) = -\rho(\boldsymbol{x},t)g$, where g is the gravity acceleration.

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Results and Discussion

As a validation for the model, a comparison between results of current LBM simulation and the finite volume method available in ref. [8] at $We = \frac{\rho_l V^2 D}{\sigma} = 30$, $Re = \frac{V_0 D}{\nu_l} = 66$, static contact angle of $\theta = 90^\circ$, and $D/D_s = 0.13$ is shown in Fig. 1, where a good agreement is observed. In above definitions, ρ_l and υ_l are the liquid density and kinematic

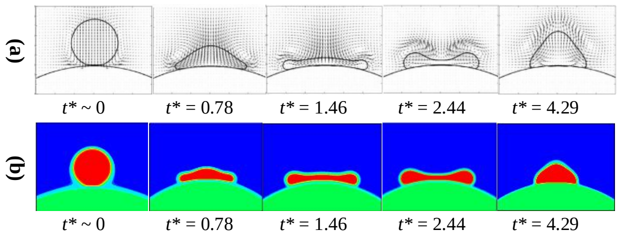

Fig. 1: Simulation images of a drop impact on a curved surface at *We* = 30, *Re* = 66; (a) finite volume method [8] (b) LBM

viscosity, V_0 is the droplet impact velocity, D its diameter, σ the surface tension, and D_s is the solid diameter. Effect of gravity is neglected here. The dimensionless time is $t^* = tV_0/D$ which is zero when the droplet touches the surface tip. The bounce back no-slip boundary condition is imposed on the solid circular surface in the LBM.

Figure 2 shows results of droplet impact at We = 110, Re =1200, $Bo = \frac{\rho_l g D^2}{\sigma} = 4.4$, ρ_l/ρ_g = 140 (T/T_c = 0.6), $\theta = 60^\circ$, and D/D_s = 0.5. where ρ_g is the gas density and g is the gravity acceleration. The domain size is 400×600 and the periodic boundary condition is applied on all sides.

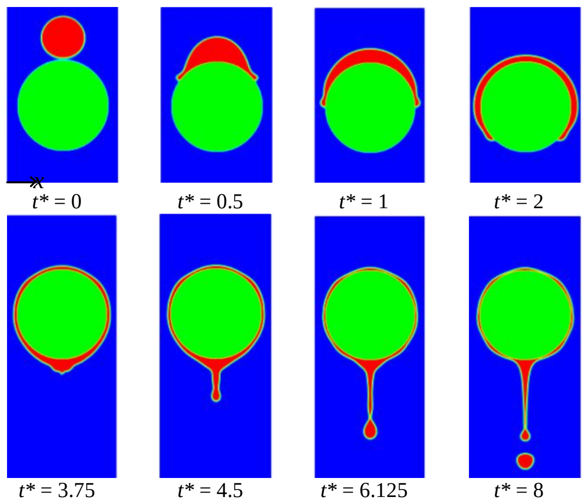

Fig. 2: Dynamics of droplet impact on a hydrophilic circular surface at $Re = 1200$.

As it can be observed from Fig. 2, the droplet spreads on the surface and forms a liquid film. Because the surface is hydrophilic, the liquid film sticks firmly to the surface and moves toward the lowest point. When the liquid film reaches the bottom of the cylinder, the amount of liquid mass is accumulated. After a while, the gravity force effect becomes considerable and the liquid is stretched downward until smaller drops drip from its bottom due to the Rayleigh instability. The effects of varying only the Reynolds number by changing the liquid viscosity is shown in Fig. 3, indicating that increasing the liquid viscosity slows down its motion. Results of a drop impact on a hydrophobic surface at $\theta = 130^\circ$ is observed in Fig. 4. In this case, secondary drops are separated from the film before reaching the bottom point of the cylinder. As the contact angle is higher in this case, the liquid curvature on the solid surface is increased leading to a higher surface tension force that causes formation of the secondary drops.

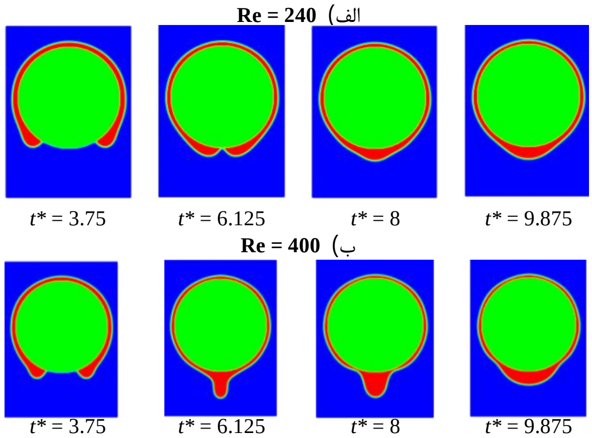

Fig. 3: Effect of different Reynolds numbers.

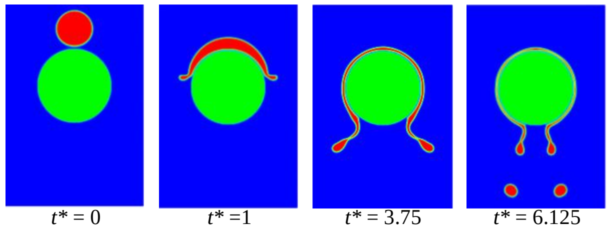

Fig. 4: Dynamics of droplet impact on a hydrophobic circular surface at $Re = 1200$.

Conclusion

In this paper, the complex phenomenon of droplet impact on a curved solid surface is simulated using the LBM. For the impact on a hydrophilic surface, dripping of smaller drops are observed at relatively high Reynolds numbers (Re = 1200) but not at lower values such as Re = 240. For the impact on a hydrophobic surface, however, separation of secondary drops from the liquid film is observed before it even reaches the bottom of the cylinder.

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