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Impact of loss on the light propagation in 1D optical waveguide array in the presence of Kerr-type nonlinearity

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A B S T R A C T

The interplay between radiation loss (diagonal and off-diagonal) and Kerr-type nonlinearity on the light propagation in 1D array of nonlinear dissipative optical waveguides are investigated numerically. Our results show that, at low nonlinear parameters, the diagonal loss only reduces the light intensity in the guides and does not affect the ballistic regime of light spreading. However, for nonlinear parameters above a critical value, the transition from the localized to the ballistic regime can be observed, after certain propagation distance. The study of the interplay between off-diagonal loss term and Kerr type nonlinearity, demonstrates that the results depend mainly to the nonlinear parameter strength. In this case, and for low strength of nonlinearity, the transition from ballistic to diffusive regime is observed after a critical propagation distance, while, spreading from localized to diffusive regime occurs at high nonlinear parameters (above the critical one). In addition, we have examined the impact of the both diagonal and off-diagonal losses in highly nonlinear optical lattices. In this case, by increasing the propagation distance, three different regimes of light spreading (from localized to the ballistic, and then, from ballistic to the diffusive) can be observed. Both critical propagation distances in which these transitions occur increase by the magnitude of the nonlinear parameter, while, decrease by the enhancement of the loss coefficients.

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1. Introduction

In recent years, by appropriate design of guides for light propagation, the optical waveguide arrays provide experimental tools to simulating and testing certain fundamental theories and phenomena in some branches of physics, such as condensed matter and quantum optics [1–4]. In design, there are some basic effects such as disorder, loss and gain, surface and nonlinear effects, which affects the light propagation in optical waveguide arrays [1–3,5–8]. The presence of loss leads to a non-Hermitian system with imaginary eigenvalues, and violates the energy conservation, because of the energy transfer from the guides to the environment. The co-existence of loss and gain in double lattices open a new research about the non-Hermitian parity-time reversal ($\mathcal{PT}$) symmetric lattices with real eigenvalues and conserved energy [9–16].

The $\mathcal{PT}$-symmetric lattices can be created by importing the gain and loss to the double-lattices and appropriate design of the coupling coefficients between guides [9–14]. Moreover, the symmetric treatment can be observed in the passive systems contain the diagonal loss term without the gain [9,14].

In the previous work [17], we investigated the impact of loss on the light propagation in linear optical waveguide arrays. As shown in [17], loss introduces an extra imaginary term to the coupling coefficients between neighbor guides, beside an imaginary term to the propagation constant of each guide, which are called off-diagonal and diagonal loss terms, respectively. In a linear system, the off-diagonal loss term results on the transition of the light spreading from the ballistic to the diffusive regime, after a critical propagation distance.

In propagation of high-power light in optical waveguide arrays, the nonlinear effects must be considered. The most important nonlinear effect on light propagation in the coupled waveguide array is the third-order Kerr-type nonlinearity. In the absence of loss, and for a nonlinear parameter above the critical one, the Kerr-type nonlinearity hindered the ballistic expansion of light, through the self-trapping mechanism [18–21].

In this paper, we investigate the interplay between the loss (diagonal and off-diagonal terms) and Kerr-type nonlinearity. Both effects exist in 1D optical waveguide array, and in the case of high-power input light and striking loss of guides, should be considered together.

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We have obtained different regimes of light spreading (from the transverse localization to diffusive and ballistic regimes) based on interplay between diagonal/off-diagonal loss term and Kerr-type non-linearity. The transition between different regimes occurs after some critical propagation distances that depend to the loss coefficients and nonlinear parameter.

We believe that our findings are significant for the study of discrete-optical solitons in waveguide arrays and optical fibers. Furthermore, these results can be useful for high-intensity light propagation in non-Hermitian and $\mathcal{PT}$-symmetric waveguide lattices [1,3,15,16].

This paper is organized in four sections. Section 2 is devoted to the theoretical model. Numerical results and discussion are presented in Section 3. Finally, we conclude and summarize our results in Section 4.

2. Theoretical model

There are two sources of loss in waveguide lattices: material absorption and geometrical loss. The first source play a role when the frequency of incident light is near the one of absorption frequencies of waveguide material [22]. The later loss is related to the geometry of the waveguide's boundary. At the boundary, tail of the electric field, in the nearby environment, move with different velocity respect to the electric field profile in the guide, and causes the transfer of energy from the middle of guide to its surrounding medium, to compensate the velocity mismatch. This type of loss is known as a radiation loss and can be controlled by the appropriate design of guide's boundaries [22]. The radiation loss introduces inherently in the light propagation along the 1D array of optical waveguides, while the material loss needs tuning of the incident light frequency.

In our previous work [17], the radiation loss is introduced by considering the electric permittivity of guides and surrounding medium as two different complex numbers. In the presence of radiation loss and by employing the slowly varying envelope approximation (SVEA), the light propagation in 1D array of optical waveguides (see Fig. 1) can be described by the following tight-binding (TB) equations [17]:

$$-i\frac{dE_n(z)}{dz} = (K_n + i\kappa_n)E_n(z) + (C_{n-1} + iC'_{n-1})E_{n-1} + (C_n + iC'_n)E_{n+1},$$
$$n = 1, 2, \ldots, N, \tag{1}$$

where, $E_n(z)$ is the electric field amplitude of light wave in the nth guide, which propagate along the z direction (see Fig. 1), K_n is the propagation constant of nth guide, C_n is the coupling coefficient between nth and $(n+1)$th guides, and N is the number of waveguides. Imaginary parts κ_n and C'_n indicate the radiation loss. Diagonal loss term κ_n depends on the imaginary parts of the dielectric constants of the system, while off-diagonal term C'_n is proportional to the mismatch between the imaginary parts of the dielectric constants of guides and their surrounding medium(absorption discrepancy). C'_n is also proportional to the coupling coefficient C_n between guides. This imaginary off-diagonal term strongly affects the dispersion relation of the system and, in the linear case, after a critical propagation distance, changes the transverse spreading of light from the ballistic to diffusive regime [17].

In the presence of third-order Kerr-type nonlinearity, the system of Eq. (1) are modified as follow:

$$-i\frac{dE_n(z)}{dz} = (K_n + i\kappa_n)E_n(z) + (C_{n-1} + iC'_{n-1})E_{n-1} + (C_n + iC'_n)E_{n+1}$$
$$+ \gamma \mid E_n(z)\mid^2 E_n(z), \tag{2}$$

where $\gamma = \frac{n_2\omega}{cA_{eff}}$ is the nonlinear parameter. Moreover, n_2, ω, c and A_{eff} are the nonlinear refractive index, frequency of incident light, speed of light in vacuum and the effective area of single-mode guide, respectively.

Here, we consider a periodic 1D array of identical guides surrounded by the same medium. Therefore, $K_n = K$, $\kappa_n = \kappa_0$, $C_n = C$, $C'_n = C' = \alpha C$, and we have:

$$-i\frac{dE_n(z)}{dz} = (K + i\kappa_0)E_n(z) + C(1 + i\alpha)(E_{n-1} + E_{n+1})$$
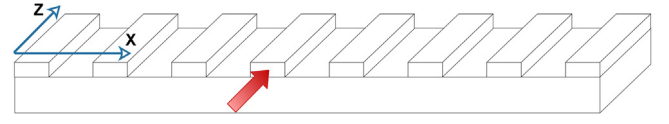$$+ \gamma \mid E_n(z)\mid^2 E_n(z). \tag{3}$$

Fig. 1. (Color online) Array of optical waveguides.

By applying $\varphi_n(z) = \frac{E_n(z)}{\sqrt{P}}e^{-i(K+i\kappa_0)z}$, $Z = Cz$, $\kappa = \frac{\kappa_0}{C}$ and $\chi = \frac{\gamma P}{C}$, we obtain the following set of the dimensionless nonlinear coupled equations:

$$-i\frac{d\varphi_n(Z)}{dZ} = (1 + i\alpha)(\varphi_{n-1}(Z) + \varphi_{n+1}(Z)) + \chi e^{-2\kappa Z} \mid \varphi_n(Z)\mid^2 \varphi_n(Z),$$
$$n = 1, 2, \ldots, N. \tag{4}$$

Here χ is normalized nonlinear parameter, $P = \sum_{n=1}^{N}\mid E_n(Z=0)\mid^2$ is the total power of light at the entrance plane, and κ and α are the dimensionless diagonal and off-diagonal loss terms, which are normalized to the coupling coefficient C between neighbor guides. It is important to note that, in these equations, the exponential decay of light intensity is factored out in $\varphi_n(Z)$, and instead of it, dimensionless nonlinear parameter $\chi e^{-2\kappa Z}$ decreases along the propagation distance. This clearly shows the reduction of nonlinear effects by loss, during propagation. We use the Runge–Kutta Fehlberg method to solve numerically these equations for $N = 200$ waveguides, with zero boundary conditions, when the middle guide ($n_0 = 100$) is excited at the entrance plane ($\varphi_n(Z=0) = \delta_{n,n_0}$).

3. Numerical results and discussion

We define the participation rate ($PR(Z)$) in $(1+1)$D optical waveguide arrays as a measure to study the different regimes of light spreading along the transverse direction:

$$PR(Z) = \frac{\left(\int_{-\infty}^{\infty}\mid \varphi(X,Z)\mid^2 dX\right)^2}{\int_{-\infty}^{\infty}\mid \varphi(X,Z)\mid^4 dX} = \frac{\left(\sum_{n=-\infty}^{\infty}\mid \varphi_n(Z)\mid^2\right)^2}{\sum_{n=-\infty}^{\infty}\mid \varphi_n(Z)\mid^4}. \tag{5}$$

The last term comes from the discretization of the middle term along the transverse direction. This measure counts the number of guides contain nonzero light amplitude. In completely extended finite system with $\varphi_n(Z) = \frac{1}{\sqrt{N}}$, the participation rate equals to the total number of guides, i.e. $PR(Z) = N$, while in exactly localized regime where $\varphi_n(Z) = \delta_{n,n_0}$, the participation rate equals one.

In $(1+1)$D optical system, the participation rate has the length dimension and can be interpreted as a beamwidth of light ($w(Z) = PR(Z)$), while in $(2+1)$D systems the participation rate has the length square dimension and the beam width can be defined as the square root of the participation rate ($w(Z) = \sqrt{PR(Z)}$). The participation rate (beamwidth) in 1D array of optical guides can change with the propagation distance as $PR(Z) \propto Z^{\beta}$, where $\beta = 1, 0.5, 0$ referring to the light spreading in ballistic, diffusive and localized regimes, respectively.

Fig. 2 shows the light intensity distribution ($I_n(Z) = \mid \varphi_n(Z)\mid^2$) and their corresponding beamwidth along the propagation distance in linear ($\chi = 0$) dissipative system for different off-diagonal loss terms (α) [17]. In this case, according to Eqs. (4), the value of diagonal loss κ does not affect the intensity distribution of the system. As shown in this figure, in the presence of off-diagonal imaginary term (α), the mechanism of light spreading in transverse direction changes from ballistic to diffusive regime after a critical propagation distance ($Z_c \simeq 10, 5$ in Fig. 2(b) and (c), respectively). The critical propagation distance decreases by enhancement of α. This result is in agreement with previous results in [17].

In Fig. 3, we investigated the impact of Kerr-type nonlinearity on light propagation in the absence of any loss ($\kappa = 0$, $\alpha = 0$). Kerr-type
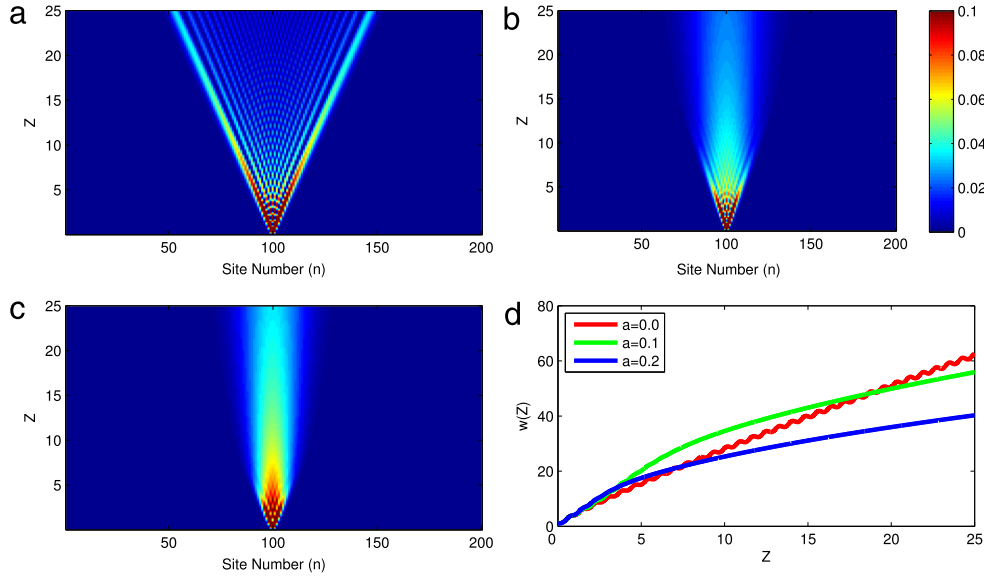
Fig. 2. (Color online) Light intensity profiles in linear ($\chi = 0$) dissipative system, with: (a) $\alpha = 0.0$, (b) $\alpha = 0.1$, (c) $\alpha = 0.2$ and (d) their corresponding beam widths.
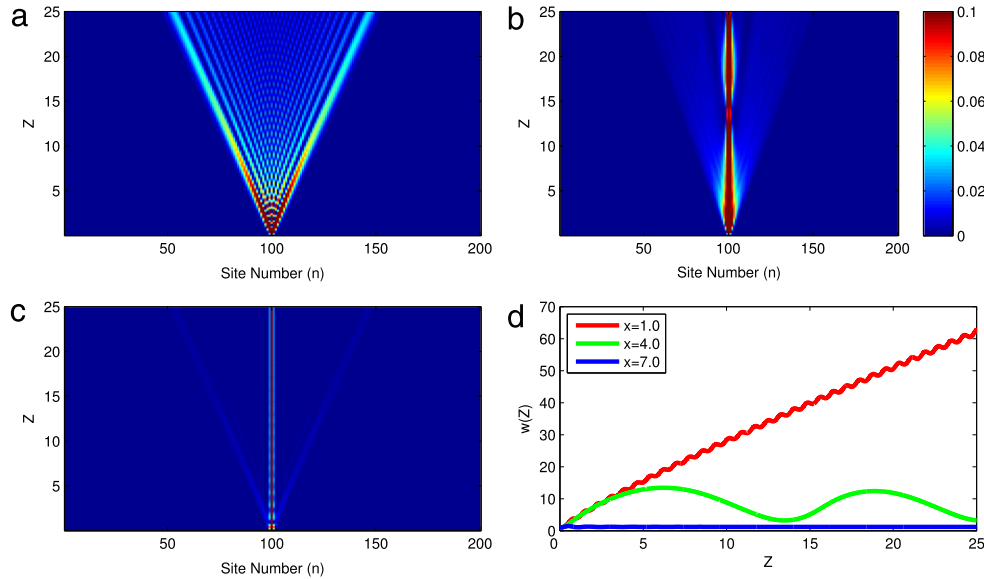

Fig. 3. (Color online) Light intensity profiles in nonlinear non dissipative system ($\kappa = \alpha = 0.0$), with: (a) $\chi = 1.0$, (b) $\chi = 4.0$, (c) $\chi = 7.0$ and (d) their corresponding beam widths.

nonlinearity can be appeared for the high light intensity at the entrance plane ($\chi \propto P$), within the waveguide lattices that fabricated from the materials with high nonlinear refractive index such as $GaAs$ or $CS3-68$ glasses [23]. For instance in $CS3-68$ glass lattices with $n_0 = 1.5$, $n_2 = 2.3 \times 10^{-14} \frac{m^2}{W}$ [23] and typical values of $A_{eff} = 50 \ \mu m^2$, $\lambda = 0.8 \ \mu m$, $C = 0.1 \ mm^{-1}$ [17], the input power equals $P \cong 18.5\chi$ (mW). As known the Kerr-type nonlinearity affects the light propagation throw the self-trapping mechanism [24,25]. For nonlinear parameters above a critical value ($\chi \geq \chi_c \simeq 4.0$), the self-trapping effect leads to the confinement of light in the incident waveguide. As shown in Fig. 3(a), light spreading is not affected by small nonlinear parameters [7]. However, by increasing the nonlinear parameter above the critical value, the self-trapping causes the transverse light localization. The figure for $\chi = 4.0$ shows the strong interplay between discrete diffraction and Kerr-type nonlinearity confinement, and marked as the critical nonlinear parameter.

In the next step, we investigate interplay between the diagonal loss ($\kappa \neq 0, \alpha = 0$) and Kerr-type nonlinearity. Figs. 4 and 5 shows the light intensity profiles and their corresponding beam widths versus the

propagation distance, at critical nonlinear parameter $\chi_c = 4.0$ and a higher value $\chi = 7.0$, respectively. As can be seen from Fig. 4, for $\chi = 4.0$, diagonal radiation loss quenches the nonlinear effect, and leads to the light spread in the ballistic regime. We find that, the behavior is the same for $\chi < \chi_c$. At higher value of nonlinear parameter $\chi = 7.0$, and at the initial propagation distances, one can observe the transverse localized regime, which demonstrates the self-trapping effect. However, after a critical propagation distance ($Z_c \simeq 5, 2.5$ in Fig. 5(b) and (c), respectively), the diagonal radiation loss suppresses the nonlinear effect, causes the scape of light from the initial injected guide, and the light spread in the ballistic regime. Also, we find that the critical propagation distance that the transition from the localized to the ballistic regime occurs, depends on the ratio between nonlinear and diagonal loss coefficients ($Z_c \propto \frac{\chi}{\kappa}$).

Figs. 6 and 7 indicate the interplay between off-diagonal radiation loss ($\kappa = 0, \alpha \neq 0$) and Kerr-type nonlinearity. These figures show the light intensity profiles and their corresponding beam widths versus the propagation distance, at two different nonlinear parameters, below
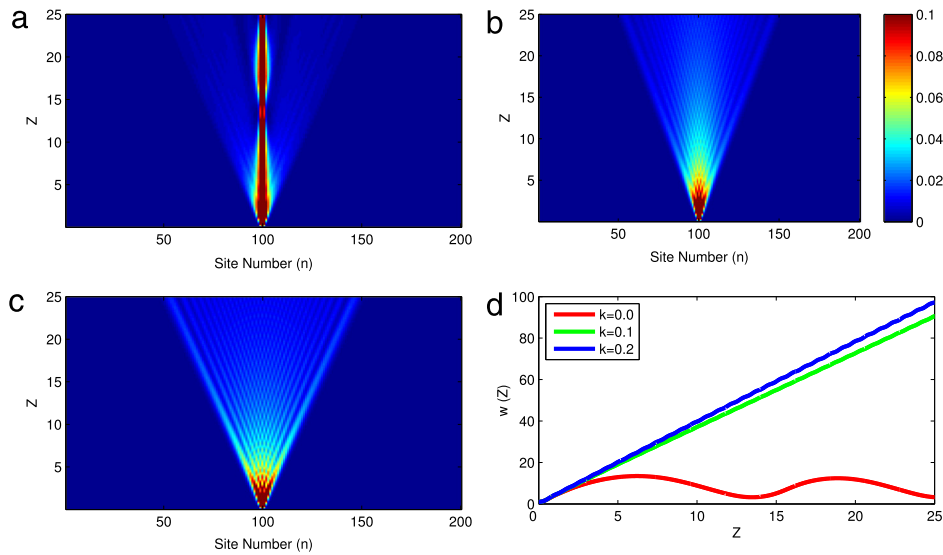
Fig. 4. (Color online) Light intensity profiles in nonlinear system ($\chi = 4.0$), in the absence of off-diagonal loss ($\alpha = 0.0$), with: (a) $\kappa = 0.0$, (b) $\kappa = 0.1$, (c) $\kappa = 0.2$ and (d) their corresponding beam widths.
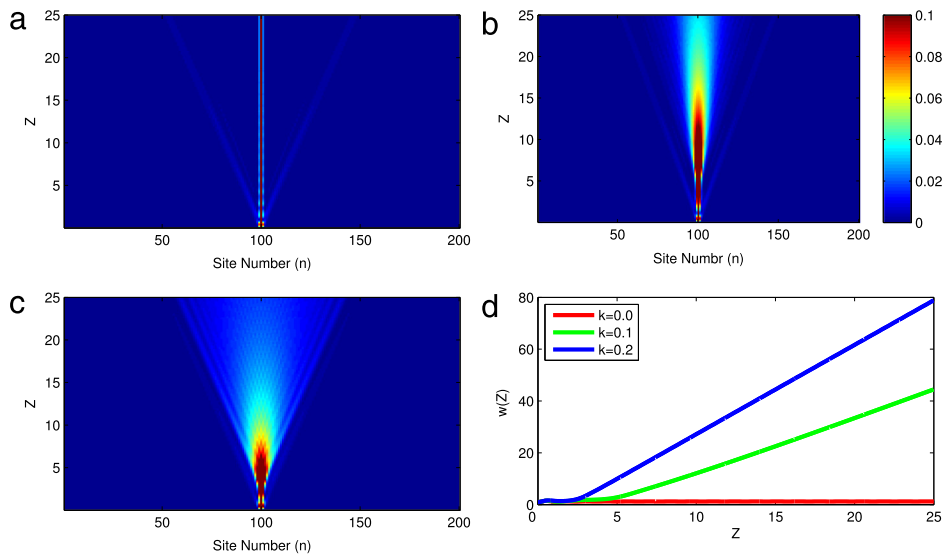

Fig. 5. (Color online) Light intensity profiles in nonlinear system ($\chi = 7.0$), in the absence of off-diagonal loss ($\alpha = 0.0$), with: (a) $\kappa = 0.0$, (b) $\kappa = 0.1$, (c) $\kappa = 0.2$ and (d) their corresponding beam widths.

the critical one $\chi < \chi_c$, and a $\chi > \chi_c$, respectively. At low nonlinear parameters $\chi \leq \chi_c$, where light spreading is not affected by nonlinear self-trapping mechanism, the off-diagonal loss term causes the transition from ballistic to diffusive regime after a critical propagation distance (see the inset of Fig. 6(d) in log–log scale). This behavior is similar to the linear dissipative system [17].

Our numerical results show that the off-diagonal loss term suppress the nonlinear self-trapping effect, and causes the light expansion in a diffusive regime. This result is reasonable; loss reduces the light power in the system, and leads to the reduction of nonlinear effects. In this case, as shown in Ref. [17] and also observed in Fig. 2, the off-diagonal loss causes the transverse light spreading from ballistic to diffusive regime, after a critical propagation distance. This critical propagation distance depends inversely to the off-diagonal loss coefficient α. Thus, by increasing of α, diffusive regime starts at lower propagation distance, and results in the smaller beam width. Therefore, the beam width is decreased by increasing of α (see Fig. 6(d)).

Fig. 7 is similar to Fig. 6, but for the higher nonlinear parameter $\chi = 7.0$. This figure confirms the strong interplay between the nonlinear

parameter and off-diagonal loss term. The loss term, by suppressing the nonlinear effect, leads to the scape of light from the initially injected guides. Therefore, the loss eliminates the light self-trapping effect after a certain propagation distance. This critical propagation distance decreased by increasing the off-diagonal loss coefficient. After this critical propagation distance, system behaves same as a linear one. Therefore, the loss term, by reduction of the energy and removing the self-trapping effect, causes the transition from nonlinear to linear system.

As known, in linear waveguide lattices, the discrete diffraction plays the role and causes the light wave spreading in the transverse direction [26]. There is a main difference between the types of diffraction in discrete and continuous mediums. In continuous medium, the light beam experiences normal diffraction and spreads gradually in the transverse direction during propagation. This changes dramatically in a discrete medium in which the light spread non-uniformly in the transverse direction, and the most of the optical energy is carried out along two major side lobes far from the center [1]. This behavior comes from the nature of different order Bessel functions as a Green function
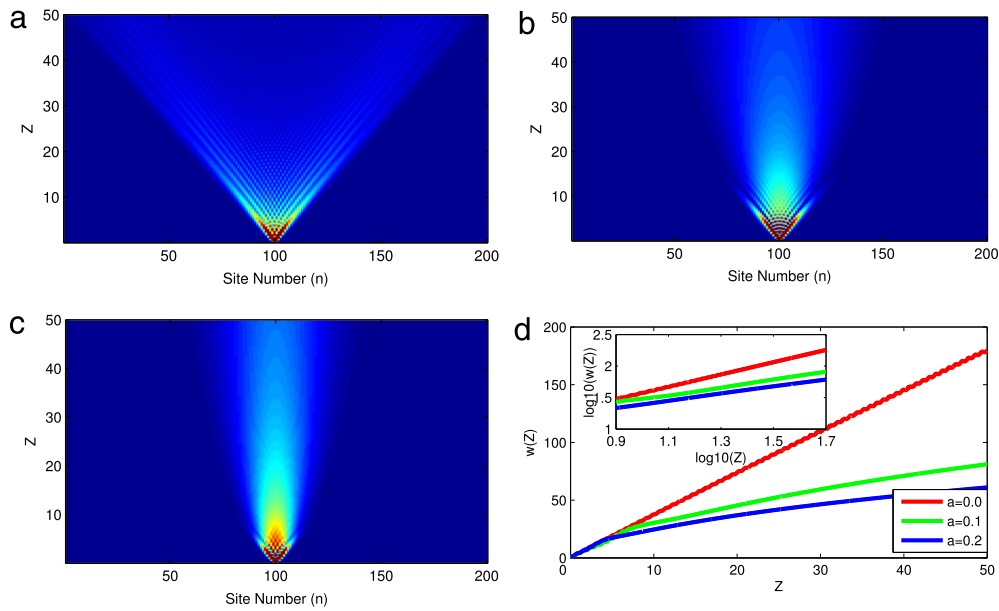
Fig. 6. (Color online) Light intensity profiles in nonlinear system ($\chi = 3.0$), in the absence of diagonal loss ($\kappa = 0.0$), with: (a) $\alpha = 0.0$, (b) $\alpha = 0.1$, (c) $\alpha = 0.2$ and (d) their corresponding beam widths.

Fig. 7. (Color online) Light intensity profiles in nonlinear system ($\chi = 7.0$), in the absence of diagonal loss ($\kappa = 0.0$), with: (a) $\alpha = 0.0$, (b) $\alpha = 0.1$, (c) $\alpha = 0.2$ and (d) their corresponding beam widths.
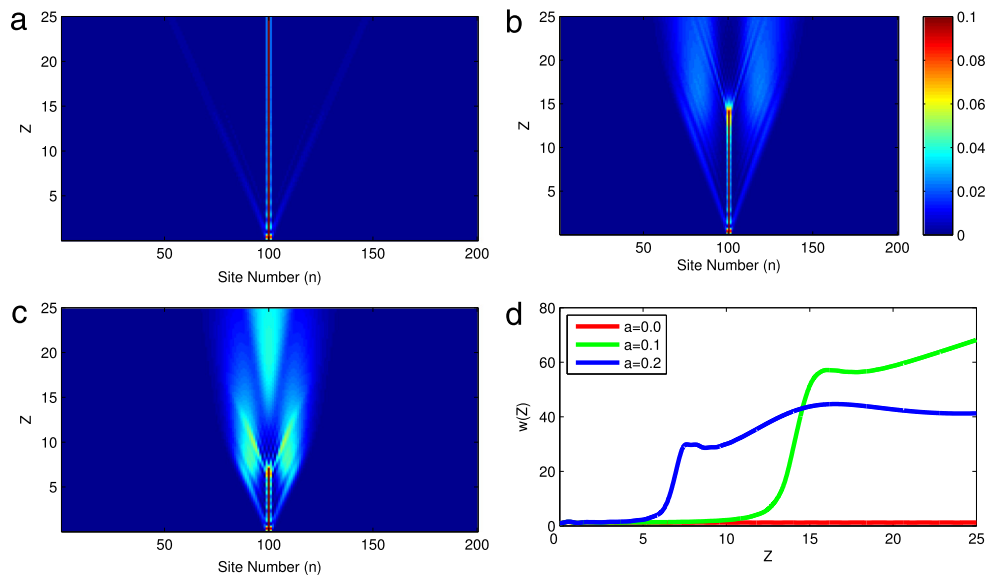
of the waveguide lattices [1]. In discrete nonlinear waveguide lattices there is a competition between nonlinear self-trapping mechanism and the discrete diffraction. For strongly nonlinear ($\chi > \chi_c = 4.0$) dissipative system ($\alpha \neq 0$), at low propagation distance, the nonlinear self-trapping mechanism is predominated and causes the transverse localization of light. However, at high propagation distance, due to the presence of off-diagonal loss α, the light intensity (and hence the nonlinear effect) decreases and the discrete diffraction dominates. Therefore, after a certain propagation distance (which decreases by increasing α), light wave spreads along two strong side lobes. This phenomenon appears as a light split into two branches (high intensity at side lobes and negligible intensity at the center). See Fig. 7(b) and (c) at $Z \simeq 15$ and $Z \simeq 8$, respectively. Furthermore, at higher propagation distance, the off-diagonal loss, by reduction of the interference effects, causes the light expansion in the diffusive regime [17]. In this case, the light intensity follows the diffusive regime profile, in which the light wave intensity is concentrated mostly in the central guides (see Fig. 7(c) for $Z > 15$).

In the last step, we have investigated the interplay between co-existence of diagonal and off-diagonal loss terms, and nonlinear parameter. In experiment, the off-diagonal loss coefficient is lower than the diagonal one [17]. Fig. 8(c) and (d) show the results for high nonlinear parameter ($\chi = 7.0 > \chi_c$) (Fig. 8(a) and (b) are depicted for comparison). These figures indicate three different regimes of light expansion. At the low propagation distances, where the power does not attenuate gravely, the nonlinear mechanism is dominated and causes the transverse light localization ($Z \leq Z_1 \simeq 2.5$, black dashed line in Fig. 8). At larger propagation distances, the diagonal loss term causes the transition to the ballistic regime ($Z_1 \leq Z \leq Z_2 \simeq 7.5$, yellow dashed line in Fig. 8). Finally, at higher propagation distances, the off-diagonal loss term affects the diffraction pattern and causes the transition to the diffusive regime ($Z \geq Z_2$). Inset in Fig. 8(d) shows the beam width ($w(Z)$) versus the propagation direction, in log–log scale, at high propagation distances ($Z > Z_2$), correspond to Fig. 8(a), (b)
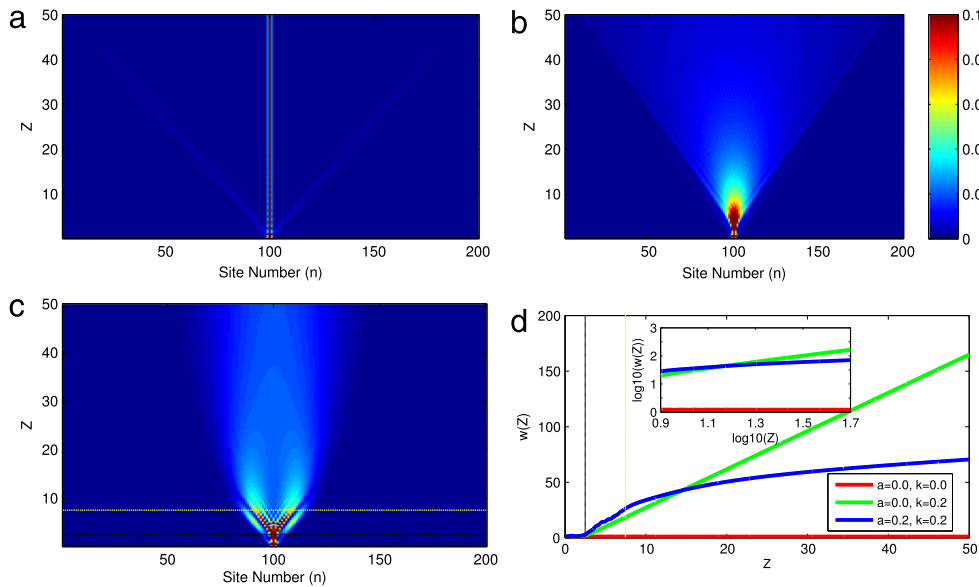
Fig. 8. (Color online) Light intensity profiles in nonlinear system ($\chi = 7.0$), with: (a) $\kappa = 0.0, \alpha = 0.0$, (b) $\kappa = 0.2, \alpha = 0.0$, (c) $\kappa = 0.2, \alpha = 0.2$ and (d) their corresponding beam widths.

and (c). This figure confirms three different regimes of light spreading: transverse localization with zero slope curve in nonlinear lossless system (Fig. 8(a)), ballistic regime with the slope equals one in the presence of diagonal loss (Fig. 8(b)) and diffusive regime with half slope when off-diagonal loss effect dominates (Fig. 8(c) at high propagation distances).

At the end, we explored the experimental conditions to see the radiation loss effects. In the experiment, the waveguide arrays can be written with a high power femtosecond laser in (2 + 1)D bulk silica glass or other materials [1,9]. To enhance the radiation loss, each guide is sinusoidal bended along the one of the transverse directions (Y-direction). All of the bended guides are located periodically in the other transverse direction (X-direction). The light wave is injected in one guide at the entrance plane, and the light intensity is monitored along the Z-direction. The loss coefficient can be tuned with the period and the amplitude of bended guides. The nonlinear parameter can be altered with the initial light intensity.

We expect these results help us to much understand the physics behind the non-Hermitian (for example $\mathcal{PT}$-symmetric) photonic structures. Also, these results may be useful in the study of optical solitons in the waveguide arrays and fibers.

4. Conclusion

In conclusion, we have investigated the interplay between diagonal and off-diagonal loss terms, and Kerr-type nonlinearity on the light propagation in 1D optical waveguide lattices. Our numerical results show that, in the presence of diagonal loss, for low nonlinear parameters, the light spreads ballistically in the transverse direction. However, by enhancement of the nonlinear parameter above a critical one, the situation changes dramatically. In this case, at low propagation distance, the light localized transversely in the injected guide because of the self-trapping mechanism. Meanwhile, by increasing the propagation distance, the diagonal loss term, by reduction the power, decreases the nonlinear effect, and causes the scape of light from the excited waveguide. In this position, light spreads in the ballistic regime similar to the linear system. The critical propagation distance, where the transition from nonlinear behavior to linear one occurs, depends to the ratio between the nonlinear parameter and diagonal loss coefficient ($Z_c \propto \frac{\chi}{\kappa}$).

In addition, the interplay between off-diagonal loss and Kerr type nonlinearity, at low nonlinear parameter causes the light spreading transition from ballistic to the diffusive regime. In this manner, the system

behaves similar to the linear ones and our result is in agreement with previous work [17]. Meanwhile, by increasing the nonlinear parameter, the self-trapping mechanism is predominated at low propagation distance and the light localized in the injected guide. Although, at high propagation distance the off-diagonal loss decreases the self-trapping chance by exchange the light intensity between injected guide and environment. Therefore, above the critical propagation distance the system behaves similar to the linear ones and light spreads in diffusive regime. Again, the critical propagation distance, depends on the ratio between the nonlinear parameter and off-diagonal loss coefficient ($Z_c \propto \frac{\chi}{\alpha}$).

Furthermore, for simultaneous interplay between diagonal loss, off-diagonal loss, and Kerr type nonlinearity (at high nonlinear parameters), one can observe three different regimes of light spreading. At low propagation distance ($Z < Z_1$), the self-trapping mechanism is predominated and the light wave is trapped in the injected guide. For larger propagation distance ($Z_1 < Z < Z_2$), the diagonal loss plays the role and causes the light spreading in the ballistic regime. Nevertheless, at higher propagation distance ($Z > Z_2$), the off-diagonal loss causes the light spreading transition from ballistic to diffusive regime. Both critical propagation distances Z_1 and Z_2 increase by the nonlinear parameter, while, diagonal and off-diagonal loss terms reduce them, respectively.

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