Int J Adv Manuf Technol
DOI 10.1007/s00170-015-7290-0

ORIGINAL ARTICLE

A bi-objective MIP model for facility layout problem in uncertain environment

Mohammad Hassan Salmani [1] · **Kourosh Eshghi** [1] · **Hossein Neghabi** [1]

Received: 14 May 2012 / Accepted: 8 May 2015

Abstract Facility layout problem (FLP) is one of the classical and important problems in real-world problems in the field of industrial engineering where efficiency and effectiveness are very important factors. To have an effective and practical layout, the deterministic assumptions of data should be changed. In this study, it is assumed that we have dynamic and uncertain values for departments' dimensions. Accordingly, each dimension changes in a predetermined interval. Due to this assumption, two new parameters are introduced which are called length and width deviation coefficients. According to these parameters, a definition for layout in uncertain environment is presented and a mixed integer programming (MIP) model is developed. Moreover, two new objective functions are presented and their lower and upper bounds are calculated with four different approaches. It is worth noting that one of the objective functions is used to minimize the total areas, which is an appropriate criterion to appraise layouts in uncertain conditions. Finally, we solve some benchmarks in the literature to test the proposed model and, based on their results, present a sensitivity analysis.

Keywords Facility layout problem · Mixed integer programming · Robust optimization · Uncertainty modeling

✉ Mohammad Hassan Salmani
mhsalmani@gmail.com

Kourosh Eshghi
eshghi@sharif.edu

Hossein Neghabi
h_neghabi@ie.sharif.edu

[1] Department of Industrial Engineering, Sharif University of Technology, Tehran 14588-89694, Iran

1 Introduction

Facility layout problem (FLP) is the problem of arranging the location of predetermined facilities such as machines, employee workstations, utilities, customer service areas, restrooms, material storage areas, lunchrooms, drinking fountains, offices, and internal walls [1]. For manufacturing facilities, material handling cost turns out to be the most significant factor in determining the efficiency of a layout and, accordingly, attracts a lot of attention. It represents 20–50% of the total operating cost and 15–70% of the total cost of manufacturing a product [2].

In the literature, FLP falls into several categories, such as single-row facility layout problem (SRFLP) and multi-row facility layout problem (MRFLP). In MRFLP, the main object is to plan the placement of facilities on a discrete or continuous plant floor. Therefore, facilities should not have any overlapping in either the horizontal axis or the vertical one. Moreover, distances between each pair of facilities are usually calculated based on a rectilinear form. In this study, this problem has four major inputs: $L=[l_i]$ and $W=[w_i]$ which represent the length and width of departments, respectively, and $\alpha=[\alpha_{ij}]$ and $\beta=[\beta_{ij}]$ which represent the length and width deviation coefficients, respectively. These two coefficients are utilized to relate the area of each facility to its adjacencies. Generally, these coefficients help us to model dynamic values for facilities' dimensions, where we multiply facilities' length and width by length and width deviation coefficients, respectively. Therefore, if adjacencies of a special facility change, its deviation coefficients change too, which can alter the respective facility's dimensions.

A large number of articles, found in the literature, have presented different models and algorithms for FLP. However, only a few of them have considered developing an appropriate and efficient model or layout for industry needs. Robust

Published online: 26 May 2015

optimization is an approach which can be considered to bridge the gap between theory and practice. Generally, the most important problem with the proposed models for FLP in the past decades has to do with the assumption of deterministic values for parameters. It is almost impossible in practice to determine an exact value for a special parameter accurately. Therefore, in this paper, unlike most models presented in the literature, dynamic (nondeterministic or uncertain) values are considered for facilities' dimensions. This assumption seems more practical and invaluable when real departments are considered. Based on this premise, facilities may take up different areas in different layouts and their areas will change according to their adjacencies.

It should also be noted that our model completely differs from dynamic facility layout problem (DFLP). The DFLP is the problem of finding the position of departments on the plant floor for multiple periods (stochastic material flows between departments change during the planning horizon) such that the sum of the material handling and rearrangement costs is minimized [3]. But, in this study, proceeding with our definitions in the paper, we generate a robust layout for the whole planning horizon and the uncertainty may happen for departments' areas as a result of their placements in the layout and their adjacencies.

Another important aspect of this model is its flexibility in shaping the departments for a special facility. In other words, the rectangular departments can be suitably shaped for a facility; even in special cases with a highly complex design of facilities, the generated layout will require only a little smoothing to be implemented in real life and be fit for real applications. This smoothing can be achieved in accordance with the desires of the designers and implementers, and, of course, their knowledge and experience.

The importance of FLP has attracted many researchers' attention in the past few decades, such that various models and algorithms have been presented, resulting in appropriate layouts. However, it has been proved that the facility layout problem is an NP-complete problem, which cannot be solved in a reasonable time [4]. Therefore, a large number of studies have been done in this area during the past few decades. In the 1980s, the first stages were taken to achieve an appropriate model for MRFLP [5–7]. They focused on implementing cutting plane approach to generating a solution for MRFLP. A mathematical programming model was presented by Heragun and Kusiak [8], which is known as ABSMODEL. In this model, four constraints were considered for each pair of departments to prevent them from overlapping and then a rectilinear method was applied to calculate distances among departments. In 1998, a study applied simulated annealing algorithm to MRFLP [9]. The most important contribution of this paper was drawing attention to orientations, shapes, and areas of facilities. Afterward, Matsuzaki, Irohara, and Yoshimoto made a major contribution to MRFLP [10]. They assumed that a layout could have more than one floor; therefore, facilities would be located on different floors. Moreover, they restricted the capacity of material handling equipment such as forklift. Among these studies, a decision-making system was presented to design and generate a multi-row layout for a multi-objective model [11]. This system had a sequence of stages which were input, design, assessment, selection, and output.

In today's dynamic market, it is rather hard and impossible to generate a layout by considering exact values for parameters. Therefore, we should modify our approach to solving MRFLP in an uncertain environment. Generally, stochastic optimization and robust optimization are the most important approaches which can be implemented to solve MRFLP. However, the problem with stochastic optimization is that one must determine parameters' distribution function exactly. However, in some studies, the mean and standard deviation of a time series are substituted with their distribution function which results in a static model. This approach may lead to an appropriate model in special cases, but it is not accurate enough to bridge the existing gap between theory and practice. Therefore, robust optimization is the best choice to solve MRLP. History of robust optimization dates back as far as 1950, when some tools like worst case analysis and Wald's MaxMin model were used [12]. In the 1970s, robust optimization was classified in a separate field along with operation research, control theory, and statistics. Generally, three major methodologies have been utilized in robust optimization:

1. Scenario approaches, proposed by Mulvey and Vanderbei [13]. They characterize the desirable properties of a solution to a model, where the problem data are described by a set of scenarios for their values instead of using point estimations. A solution to an optimization model is defined as robust solution if it remains close to optimal for all scenarios of the input data.
2. Elliptic model of uncertainty, presented by Ben-Tal et al. [14–16]. In this method, elliptic data will be replaced with exact data. In other words, a new exact model will be extended to cover all data uncertainties.
3. Interval model of uncertainty, presented in different studies [17–19]. This method is different from the elliptic model in that the former uses intervals for parameters. Also, in this model, interval data are substituted with exact point data by applying an accurate method.

In the literature, two categories of flexible layout and robust layout have attracted more attention from the robust optimization point of view for MRFLP. It would be no stretch to claim that Shore and Tompkins [20] were pioneers in the field of flexibility in FLP. They presented a model to generate a flexible layout for uncertain environments with a penalty function criterion. Afterward, Kulturel-Konak, Smith, and

Norman [21] defined flexibility of FLP as the ability to manufacture various products. They used uncertainty in block layout design. Based on their definition, a robust layout has the most proportion of optimality in simulations. Rosenblatt and Lee [22] proceeded with the assumption of having stochastic product demands for a single period and presented a QAP model for the robust layout. Kouvelis, Kurawarwala, and Gutierrez [23] developed a new model to generate a robust layout under demand uncertainty in a multiple-period layout problem. Their proposed QAP model was solved by applying branch and bound approach. Along with these studies, a ranking method, based on fuzzy approach, was developed by Aiello and Enea [24]. They considered the level of decision makers' pessimism to find a robust layout. Azadivar and Wang [25] implemented stochastic characteristics such as inter-arrival time and diverse part routes. Cheng, Gen, and Tozawa [26] presented a model with a view to the uncertainty flow between departments as fuzzy numbers. This problem was solved by utilizing GA.

Moreover, a robust approach was presented to solve dynamic facility layout problem [27]. This approach to DPLP assumes that a layout accommodates changes from time to time with high rearrangement and production interruption costs, and that the machines cannot be easily relocated.

Recently, two different studies have been done in the field of robust layout. The first one, developed by [28], presented a new MIP model based on ABSMODEL to generate a robust model. The proposed model is solved using an algorithm where both qualitative and quantitative approaches are considered. The second focused on developing a robust model for multi-floored plants where facilities are located on different floors and storages are considered to be in the cellar [29]. In this problem, an elevator is embedded in the plant to convey materials among departments with different floor indexes.

The general structure of this paper is as follows. To begin with, five important definitions are presented for departments' adjacencies and robust layout in "Definition of facilities' adjacency and uncertainty" section. Next, the robust model is proposed and afterward its linear model is expanded. This is followed by the presentation of four different lower and upper bounds for objective functions of the proposed model. In the next two sections, computational results are presented and, according to these results, a comparison is made between ABSMODEL and robust models. Also, a sensitivity analysis is done on generated solutions in "Sensitivity analysis" section. Finally, it sums up the main points of the study.

2 Definition of facilities' adjacency and uncertainty

In this section, four important definitions are presented for different types of adjacencies of facilities in a layout where the robust layout is defined to develop a robust model. In other words, according to the definitions of departments' adjacencies, the robust mathematical model is developed.

Definition 1 (adjacency definition) Two facilities are adjacent to each other, if

- At least, they have the minimum predetermined value of joint boundaries, and
- The maximum distance between their joint boundaries does not exceed the maximum predetermined value.

Definition 2 (adjacency along the horizontal axis) Two facilities i and j are adjacent along the horizontal axis if they have joint boundaries along the horizontal axis (Fig. 1).

Definition 3 (adjacency along the vertical axis) Two facilities i and j are adjacent along the vertical axis if they have joint boundaries along the vertical axis (Fig. 2).

Definition 4 (adjacency of facilities) Two facilities i and j are adjacent if they become adjacent along either the horizontal axis or the vertical one, where the distance between their adjacent boundaries lies in an acceptable interval.

In definitions 2 and 3, how far the facilities are located from each other does not matter—what counts is only their joint distance. Therefore, if two facilities become adjacent along the horizontal axis, where their vertical distance is considerable, they would not be considered adjacent. Therefore, definition 4 is presented to rule out such kinds of unacceptable adjacencies.

Admittedly, we need to present a model to generate a layout which is more appropriate in comparison to typical models presented in the literature. As a matter of fact, robust optimization can be implemented as a very decent approach. Therefore, in this section, a new method is presented to replace dynamic values with exact values, which can result in robust layouts in uncertain environment that are more effective than other layouts generated in the past few decades.

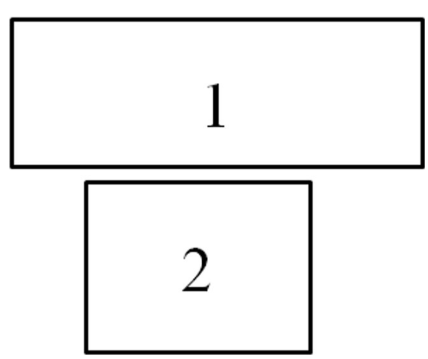


Fig. 1 Adjacency along the horizontal axis

Fig. 2 Adjacency along the vertical axis

As mentioned earlier, in this study an interval model of uncertainty is used to develop a mathematical model for MRFLP. On the other hand, the most important parameters in MRFLP are facilities' areas. Therefore, in this paper, we have attempted to consider dynamic and uncertain values for facilities' dimensions. These dimensions may change in different layouts. In other words, as it can be seen in Fig. 3, width and length of a special facility change if its adjacents change.

It is noticeable that if we change the orientation of facilities, the same layout will be generated more or less. This means that the orientation of facilities can be considered in this model by implementing the length and width deviation coefficients. The reason is that the model will control the shape of each facility by its center coefficients, length and width deviation coefficients.

Now, suppose that $l_i^{'}$ and $w_i^{'}$ represent the minimum values for length and width of facility i, respectively, and $l_i^{''}$ and $w_i^{''}$ are the maximum values for the length and width of facility i, respectively. Therefore, we will have two intervals of $l_i \in [l_i^{'}, l_i^{''}]$ and $w_i \in [w_i^{'}, w_i^{''}]$ as it is shown in Fig. 4.

Generally, the main idea to develop a robust model is to use both length and width deviation coefficients as variables which are implemented to multiply the length and width of departments, respectively. By using these coefficients, we will be able to assign the minimum and maximum values of lengths and widths to departments, and it would be possible to determine the exact values of the length and width of each department. Therefore, by utilizing this method, we will be able to propose a model in certain circumstances which can be solved by typical approaches for linear mathematical models.

We assume that each of $l_i^{'}$, $l_i^{''}$, $w_i^{'}$, and $w_i^{''}$ are the minimum and maximum values that facility i will take up in adjacency to special facilities. If we define l_i and w_i as basis values for length and width of facility i, the following equations will be obtained:

$$l_i^{'} = \text{Min}_j\{\alpha_{ij}\} \times l_i\,; \quad \forall i = 1, 2, \ldots, n \qquad (1)$$

$$l_i^{''} = \text{Max}_j\{\alpha_{ij}\} \times l_i\,; \quad \forall i = 1, 2, \ldots, n \qquad (2)$$

$$w_i^{'} = \text{Min}_j\{\beta_{ij}\} \times w_i\,; \quad \forall i = 1, 2, \ldots, n \qquad (3)$$

$$w_i^{''} = \text{Max}_j\{\beta_{ij}\} \times w_i\,; \quad \forall i = 1, 2, \ldots, n \qquad (4)$$

where n is the number of facilities, j is alias index with index i (both are departments' indexes), and α_{ij} and β_{ij} represent length and width deviation coefficients, respectively. These deviation coefficients change facilities' dimensions.

In fact, we plan to generate all possible values for the length of a special department i based on its basis value l_i. Therefore, we must relate $l_i^{'}$ and $l_i^{''}$ with l_i using the minimum and maximum length deviation coefficient values, respectively (relations (1)–(2)). Also, the same implication can be done for department width (w_i) to clarify the relations (3)–(4).

Definition 5 (robust layout in uncertain environment): A layout is robust in an uncertain environment if each facility can take up the maximum needed area in adjacency to other facilities, where no overlapping among facilities is allowed to occur.

As mentioned in this definition, no overlapping can occur in a robust layout. In other words, in generated layouts, none of the departments can have common areas where there are some points with the same coordinates.

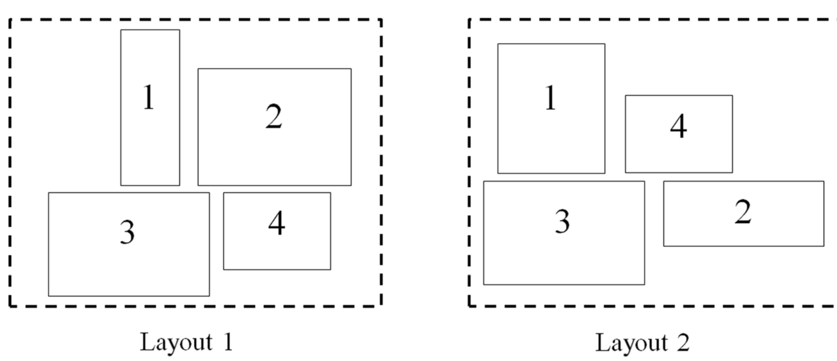
3 Robust MRFLP and the linear model

In this section, a mathematical model is presented, which is consistent with definition 5. In other words, output layout of this mathematical model is a robust layout according to our definition.

The major variables for this model are as follows:

x_i and y_i are positive variables to indicate facilities' center coordinates.

Fig. 3 For two different layouts, four existing facilities acquired different dimensions according to their adjacents



α_i and β_i represent the maximum needed value of length and width deviation coefficients for facility i according to its adjacents, respectively.

P_{ij} and Q_{ij} are binary variables which are used to linearize the mathematical model.

$$NX_{ij} = \begin{cases} 1 & When\ for\ two\ adjacent\ facilities\ i\ and\ j,\ definition\ 2\ is\ satisfied \\ 0 & Otherwise \end{cases}$$

$$NY_{ij} = \begin{cases} 1 & When\ for\ two\ adjacent\ facilities\ i\ and\ j,\ definition\ 3\ is\ satisfied \\ 0 & Otherwise \end{cases}$$

$$N_{ij} = \begin{cases} 1 & When\ for\ two\ adjacent\ facilities\ i\ and\ j,\ definition\ 4\ is\ satisfied \\ 0 & Otherwise \end{cases}$$

And M is a large number.

Due to the dynamic values for facilities' dimensions, we assume that the total layout area is unlimited; therefore, we present an objective function to minimize this total layout area. According to Eqs. (1)–(4), each facility has basis dimensions values which are multiplied by length and width deviation coefficients. Therefore, if we want to minimize the total area, it would be sufficient to minimize the sum of the maximum needed values for length and width deviation coefficients of each facility as relation (5).

$$\text{Max} \sum_{i=1}^{n} (\alpha_i + \beta_i) \tag{5}$$

On the other hand, another objective function is presented to maximize the sum of the adjacencies among facilities. If we disregard this objective function, the constraints of the model would not work properly. Therefore, it is mandatory to consider relation (6) as the second objective function.

$$\text{Max} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} N_{ij} \tag{6}$$

To prevent facilities from overlapping, relations (7) and (8) are presented. Because at least one of these inequalities should be satisfied, we use binary variable P_{ij}. As it can be seen in these constraints, the maximum needed values for length and width of each facility are considered by multiplying their basis length and width values by α_i and β_i, respectively.

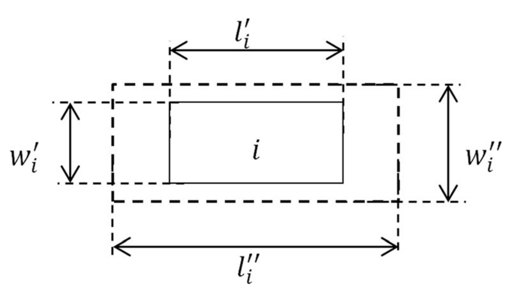


Fig. 4 Length and width interval for facility

$$|x_i - x_j| + MP_{ij} \geq \frac{1}{2}(\alpha_i l_i + \alpha_j l_j) \ ; \quad \forall i, j = 1,\ 2,\ \cdots,\ n \quad \& \quad i \neq j \tag{7}$$

$$|y_i - y_j| + M(1 - P_{ij}) \geq \frac{1}{2}(\beta_i w_i + \beta_j w_j) \ ; \quad \forall i, j = 1,\ 2,\ \cdots,\ n \quad \& \quad i \neq j \tag{8}$$

In this model, it is important to determine which facilities are adjacent to each other. Therefore, constraints (9) and (10) are presented to determine the values of NX_{ij} and NY_{ij}.

$$\left| \left(x_i + \frac{1}{2}\alpha_i l_i\right) - \left(x_j - \frac{1}{2}\alpha_j l_j\right) \right| \leq d_1 + (1 - NY_{ij})M + (\alpha_i l_i + \alpha_j l_j) \ ; \quad \forall i, j = 1,\ 2,\ \cdots,\ n \ \ \& \ i \neq j \tag{9}$$

$$\left| \left(x_i + \frac{1}{2}\beta_i w_i\right) - \left(x_j - \frac{1}{2}\beta_j w_j\right) \right| \leq s_1 + (1 - NX_{ij})M + (\beta_i w_i + \beta_j w_j) \ ; \forall i, j = 1,\ 2,\ \cdots,\ n \ \ \& \ i \neq j \tag{10}$$

As it can be seen in Fig. 5, s_1 and d_1 are respectively the maximum permissible vertical and horizontal distances between nearest boundaries of two adjacent facilities. Moreover, in this figure, d_2 and s_2 are respectively the minimum horizontal and vertical distances which two adjacent facilities should have as joint boundaries. These two parameters are used in constraints (11) and (12).

In inequalities (9) and (10), the maximum distance between nearest boundaries of two adjacent facilities is determined. According to these two constraints, for two facilities such as those in Fig. 4, the maximum horizontal distance between boundaries of two adjacent facilities should not exceed d_1. If this distance exceeds d_1, NY_{ij} will become zero. The same inference can be drawn for inequality (10) and variable NX_{ij} according to the value of s_1.

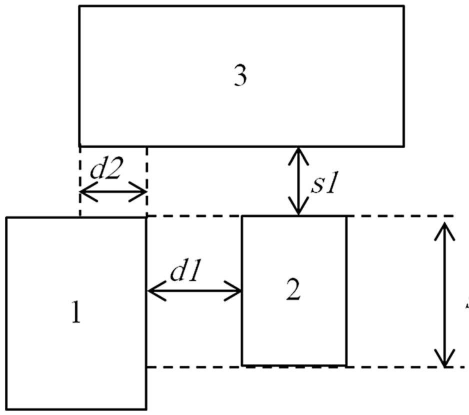


Fig. 5 Maximum and minimum distances between two adjacent facilities

To determine the minimum distances as joint boundaries between two adjacent facilities, inequalities (11) and (12) are utilized. To explain these constraints, consider two facilities 1 and 3 in Fig. 5. In inequality (11), we calculate the distance between the right boundary of facility 3 and the left boundary of facility 1. According to this constraint, this distance should not exceed the sum of the lengths of facilities 1 and 3 minus d_2 $(l_1+l_3-d_2)$; otherwise, NX_{13} will be zero. We can extend this inference to each pair of facilities i and j. The same conclusion can be drawn for constraint (12).

$$\left(x_i+\frac{1}{2}\alpha_i l_i\right)-\left(x_j-\frac{1}{2}\alpha_j l_j\right)\le(\alpha_i l_i+\alpha_j l_j)-d_2+\left(1-NX_{ij}\right)M \;; \quad \forall i,j=1,2,\cdots,n \;\&\; i\ne j \tag{11}$$

$$\left(y_i+\frac{1}{2}\beta_i w_i\right)-\left(x_j-\frac{1}{2}\beta_j w_j\right)\le(\beta_i w_i+\beta_j w_j)-s_2+\left(1-NY_{ij}\right)M \;; \quad \forall i,j=1,2,\cdots,n \;\&\; i\ne j \tag{12}$$

On the other hand, if the left sides of constraints (11) and (12) become negative, wrong solutions will ensue. In other words, inequalities (11) and (12) will be satisfied while definitions (2)–(4) are violated. Therefore, to rule out such wrong solutions, inequalities (13) and (14) are presented.

$$\left(x_i+\frac{1}{2}\alpha_i l_i\right)-\left(x_j-\frac{1}{2}\alpha_j l_j\right)\ge d_2-\left(1-NX_{ij}\right)M \;; \quad \forall i,j=1,2,\cdots,n \;\&\; i\ne j \tag{13}$$

$$\left(y_i+\frac{1}{2}\beta_i w_i\right)-\left(x_j-\frac{1}{2}\beta_j w_j\right)\ge s_2-\left(1-NY_{ij}\right)M \;; \quad \forall i,j=1,2,\cdots,n \;\&\; i\ne j \tag{14}$$

According to the definitions 2–4, we can relate NX_{ij}, NY_{ij}, and N_{ij} by using Eq. (15).

$$N_{ij}=NX_{ij}NY_{ij} \;; \quad \forall i,j=1,2,\cdots,n \;\&\; i<j \tag{15}$$

On the other hand, to cover the notifications in definition 5, we should determine the maximum needed length and width deviation coefficients for each facility. Therefore, constraints (16) and (17) can be used to determine the values of α_i and β_i.

$$\alpha_i\ge N_{ij}\alpha_{ij} \;; \quad \forall i,j=1,2,\cdots,n \;\&\; i\ne j \tag{16}$$

$$\beta_i\ge N_{ij}\beta_{ij} \;; \quad \forall i,j=1,2,\cdots,n \;\&\; i\ne j \tag{17}$$

On the other hand, to restrict the number of constraints in our model, we enforce the relation of $i<j$ to constraint (15). We also present Eq. (18) to determine the values of N_{ij} for all pairs of facilities. Therefore, we substitute some nonlinear constraints with linear constraints.

$$N_{ij}=N_{ji} \;; \quad \forall i,j=1,2,\cdots,n \;\&\; i<j \tag{18}$$

The nonlinearity of model (5)–(18) is clear. Therefore, to solve this model by using MIP exact techniques, we need to linearize it.

Firstly, by adding variable Q_{ij} to the proposed model, it is possible to linearize inequalities (7) and (8) as follows [30]:

$$x_i-x_j+P_{ij}M+Q_{ij}M\ge\frac{1}{2}(\alpha_i l_i+\alpha_j l_j) \;; \quad \forall i,j=1,2,\cdots,n \;\&\; i\ne j \tag{19}$$

$$-x_i+x_j+P_{ij}M+\left(1-Q_{ij}\right)M\ge\frac{1}{2}(\alpha_i l_i+\alpha_j l_j) \;; \quad \forall i,j=1,2,\cdots,n \;\&\; i\ne j \tag{20}$$

$$y_i-y_j+\left(1-P_{ij}\right)M+Q_{ij}M\ge\frac{1}{2}(\beta_i w_i+\beta_j w_j) \;; \quad \forall i,j=1,2,\cdots,n \;\&\; i\ne j \tag{21}$$

$$-y_i+y_j+\left(1-P_{ij}\right)M+\left(1-Q_{ij}\right)M\ge\frac{1}{2}(\beta_i w_i+\beta_j w_j) \;; \quad \forall i,j=1,2,\cdots,n \;\&\; i\ne j \tag{22}$$

On the other hand, we can easily linearize constrains (9) and (10) as follows:

$$\left(x_i+\frac{1}{2}\alpha_i l_i\right)-\left(x_j-\frac{1}{2}\alpha_j l_j\right)\le d_1+\left(1-NY_{ij}\right)M+\left(\alpha_i l_i+\alpha_j l_j\right) \;; \quad \forall i,j=1,2,\cdots,n \;\&\; i\ne j \tag{23}$$

$$\left(x_i+\frac{1}{2}\alpha_i l_i\right)-\left(x_j-\frac{1}{2}\alpha_j l_j\right)\ge-\left(d_1+\left(1-NY_{ij}\right)M+\left(\alpha_i l_i+\alpha_j l_j\right)\right) \;; \quad \forall i,j=1,2,\cdots,n \;\&\; i\ne j \tag{24}$$

$$\left(y_i+\frac{1}{2}\beta_i w_i\right)-\left(x_j-\frac{1}{2}\beta_j w_j\right)\le s_1+\left(1-NX_{ij}\right)M+\left(\beta_i w_i+\beta_j w_j\right) \;; \quad \forall i,j=1,2,\cdots,n \;\&\; i\ne j \tag{25}$$

$$\left(y_i+\frac{1}{2}\beta_i w_i\right)-\left(x_j-\frac{1}{2}\beta_j w_j\right)\ge-\left(s_1+\left(1-NX_{ij}\right)M+\left(\beta_i w_i+\beta_j w_j\right)\right) \;; \quad \forall i,j=1,2,\cdots,n \;\&\; i\ne j \tag{26}$$

Table 1 Output variables of solving different problems

Number of departments	Variable	Department number (i)							Objective function	Total time (s)
		1	2	3	4	5	6	7		
4	x_i	0.00	4.09	2.18	7.45	–	–	–	0.53	0.27
	y_i	0.00	0.87	7.61	1.84	–	–	–		
	α_i	1.28	1.08	1.36	1.20	–	–	–		
	β_i	0.82	1.20	1.45	1.22	–	–	–		
5	x_i	0.01	0.00	5.16	9.58	6.91	–	–	0.46	2.28
	y_i	0.67	6.74	0.00	2.48	6.31	–	–		
	α_i	0.67	0.97	1.36	1.02	1.42	–	–		
	β_i	1.26	1.20	1.40	0.70	1.31	–	–		
6	x_i	3.70	7.57	0.00	4.43	10.60	6.47	–	0.46	203.24
	y_i	0.00	2.64	9.38	6.89	6.89	11.10	–		
	α_i	1.28	0.97	1.07	1.20	1.42	1.30	–		
	β_i	1.26	1.20	1.45	0.93	1.17	1.35	–		
7	x_i	0.00	4.09	5.29	7.46	3.69	12.16	10.82	0.39	27,090.68
	y_i	5.53	6.79	0.00	6.77	10.96	6.61	9.58		
	α_i	1.28	1.08	1.07	1.20	1.42	1.40	1.08		
	β_i	0.92	1.20	1.45	1.22	1.17	1.35	1.37		

To linearize Eq. (15), Naslund's approximations can be used [31]. Therefore, it would be sufficient to substitute Eq. (15) with inequalities (27) and (28).

$$N_{ij}-NX_{ij}-NY_{ij}\leq 1.5 \quad ; \qquad \forall i,j=1,2,\cdots,n \quad \& \quad i<j \quad (27)$$

$$1.5N_{ij}-NX_{ij}-NY_{ij}\leq 0 \quad ; \qquad \forall i,j=1,2,\cdots,n \quad \& \quad i<j \quad (28)$$

As mentioned earlier, we enforce the restriction of $i<j$ to Eq. (15) and consequently to inequalities (27) and (28). Therefore, it can be said that we add Eq. (18) instead of using the relation of $i\neq j$to decrease the number of linear and nonlinear constraints in the proposed model. This will become obvious by comparing constraints (15), (18), (27), and (28).

4 Lower and upper bounds

In this section, two theorems are proved to present appropriate upper bounds for objective functions in relations (5) and (6). Moreover, two lower bounds for the objective functions are presented as two results. These bounds are narrow and according to these theorems, we can determine two intervals for objective functions to analyze the generated layouts.

Theorem 1 (upper bound of relation (5)) If we replace α_i and β_i with their maximum values for each facility:

(a) The value of the first objective function for the generated layout will be an upper bound for relation (5), and

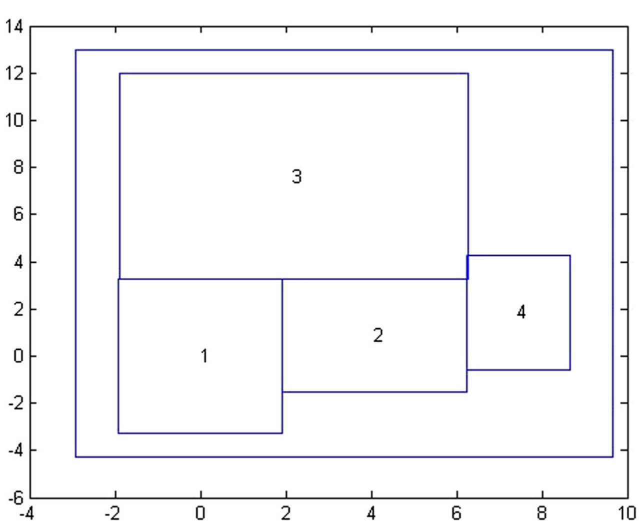


Fig. 6 Generated robust layout for the problem with 4 departments

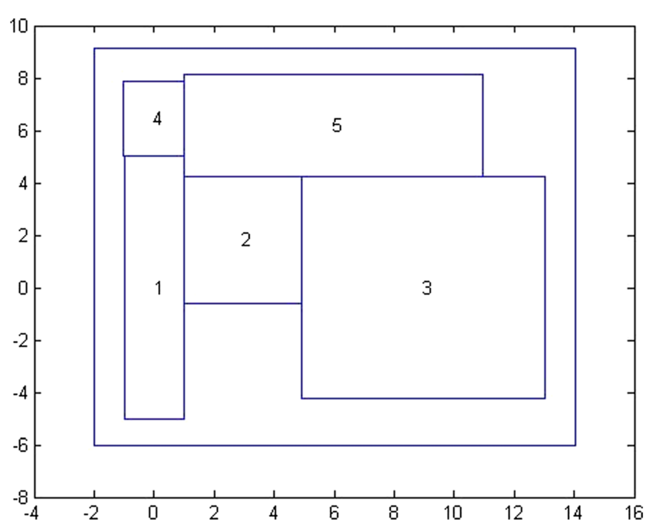


Fig. 7 Generated robust layout for the problem with 5 departments

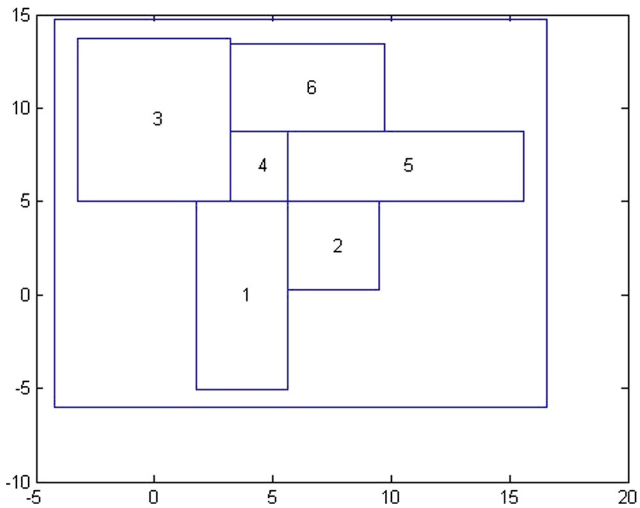


Fig. 8 Generated robust layout for the problem with 6 departments

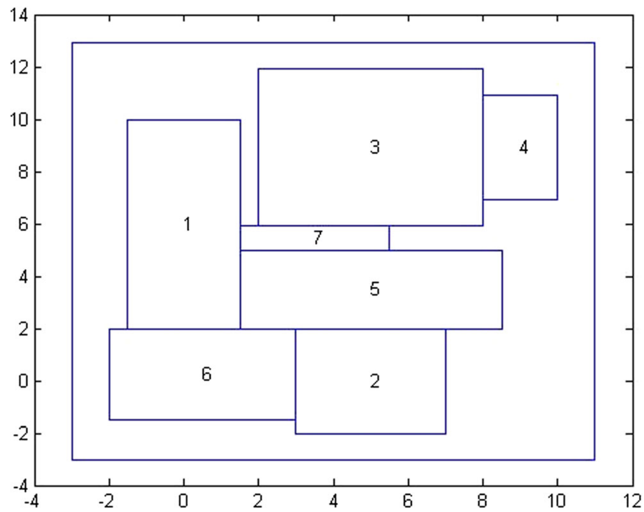


Fig. 10 Generated ABSMODEL layout for the problem with 7 departments

(b) The generated layout will be a robust layout, according to definition 5.**Proof.** (a) According to constraints (16) and (17), it can be inferred that α_{ij} and β_{ij} are lower bounds for α_i and β_i, respectively. Therefore, it is possible to replace α_i and β_i with their maximum possible values for each facility. On the other hand, if we replace these variables with their maximum possible values, relation (5) will change to relation (29).

$$\text{Min} \sum_{i=1}^{n} \left(\text{Max}_j(\alpha_{ij}) + \text{Max}_j(\beta_{ij}) \right) \tag{29}$$

It is obvious that an upper bound will be obtained for relation (5), according to relation (29).

(b) According to definition 5, it must be possible for each facility in a robust layout to take up its maximum needed area value. Now, we replace length and width deviation coefficients with their maximum possible values. Therefore, all of the facilities will be in their maximum possible areas. This means that the output layout of this new model, which is a feasible model, is a robust layout.

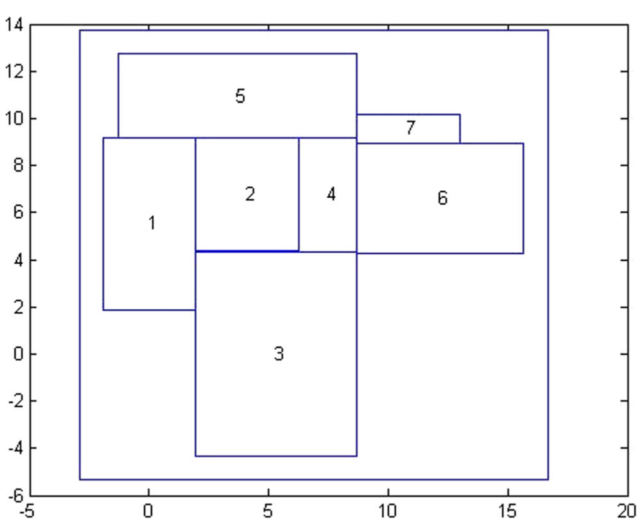


Fig. 9 Generated robust layout for the problem with 7 departments

As a result of Theorem 1, it can be said that if we replace α_i and β_i with their minimum values for each facility, then the value of the first objective function for the generated layout will be a lower bound for relation (5). However, the generated layout may be a robust layout according to definition 5. The reason is that according to relations (16) and (17), it can be inferred that α_{ij} and β_{ij} are lower bounds for α_i and β_i, respectively. Moreover, it is mentioned in definition 5 that it must be possible for each facility in a robust layout to take up its maximum needed area value. Nevertheless, we enforce all facilities to be in their minimum areas; however, it may be a feasible solution for model (5)–(18), which will be a consistent layout with definition 5. Therefore, the generated layout may be a robust layout.**Theorem 2 (upper bound of relation (6))** The upper bound of the second objective function in relation (6) is:

(a) $(3n-6)$, if s_1 and d_1 have zero values.

(b) $\binom{n}{2} = \frac{n(n-1)}{2}$, if s_1 and d_1 have positive values.

Proof. (a) If d_1 and s_1 have zero values, the best layout can be founded by maximal planar graph (MPG). Moreover, it has been proved [32] that the maximum number of edges which corresponds to the maximum number of adjacencies in a layout which is related to a MPG is $(3n-6)$.

(b) If d_1 and s_1 have positive values, it would be possible to have adjacency between two facilities by adding distance between them. This way, a layout can be generated where the number of adjacencies exceeds the maximum possible number for MPG. On the other hand, if we have adjacency

Table 2 Comparison results between robust and ABSMODEL models

Number of departments	Model	Total layout areas	Total departments' areas	Total wasted areas		Number of adjacencies		Total time (s)
				Value	Percentage	Number	Percentage	
4	Robust	161.09	128.63	32.46	20.15%	5	83.33%	0.27
	ABSMODEL	120.00	84.00	36.00	30.00%	3	50.00%	0.09
5	Robust	182.42	152.21	30.21	16.56%	7	70.00%	2.28
	ABSMODEL	143.00	105.00	38.00	26.57%	6	60.00%	0.61

between each pair of facilities, the total number of facilities will be restricted by $\binom{n}{2} = \frac{n(n-1)}{2}$. Therefore, the upper bound will be $\binom{n}{2}$.

Moreover, it is obvious that the lower bound of second objective function is zero.

the other hand, we assume that both objective functions have the same weights and neither is preferable over the other. So, we use relation (30) as the single objective function.

$$\text{Max}\left(\frac{\sum_{i=1}^{n}\sum_{j=i+1}^{n}N_{ij}}{\binom{n}{2}} - \frac{\sum_{i=1}^{n}(\alpha_i+\beta_i)}{\sum_{i=1}^{n}\sum_{j=1}^{n}(\alpha_{ij}+\beta_{ij})}\right) \quad (30)$$

5 Computational results

This section gives computational results on the performance of the proposed model. The proposed model was coded in GAMS 23.7 and ran on a Core i5 480M 2.4 GHz PC. We studied a set of test problems that include 30 instances given by Kar and Shih [33]. These test problems consist of 30 departments' dimensions. The results of the 4, 5, 6, and 7 departments test problems are given. To generate length and width deviation coefficients, it is assumed that these parameters have a uniform distribution in [0.5, 1.5]. All these data are presented in the Appendix section in Tables 8, 9, and 10.

In model (5)–(18), two objective functions are presented. Therefore, it would not be possible to solve this model by using exact techniques for ordinary MIP model with just one objective function. To solve this mode, we normalize two objective functions and then we combine them together. On

According to relation (30), it can be said that we maximize the total number of adjacencies while we try to minimize the total layout area.

The generated layouts for four different problems are shown. Furthermore, the total outputs for these problems are summarized in Table 1. The values of N_{ij} are shown in Tables 4, 5, 6, and 7 in the Appendix section. One important issue is that the values of objective functions for different problems do not change drastically. Therefore, it can be said that a narrow interval can be calculated which contains lower and upper bounds of relation (30).

The generated layouts for the proposed problems are shown in Figs. 6, 7, 8, and 9. As can be seen, no area is wasted and if we compare these layouts with Tables 4, 5, 6, and 7 in the Appendix, we can see that N_{ij} for all problems has the correct values. In other words, for every two adjacent facilities i and j, N_{ij} is one; otherwise, it has a zero value. For instance, N_{47} is equal to one, where in Fig. 9, facilities 4 and 7 are adjacent to each other; consequently, N_{74} has a value of one.

As it is obvious in these figures, areas of departments vary while their adjacencies change. For example, for the problem

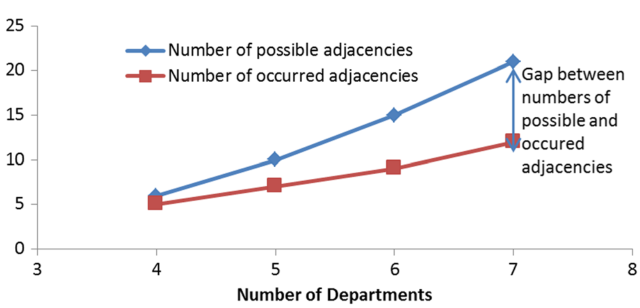


Fig. 11 Comparison between the number of possible and occurring adjacencies

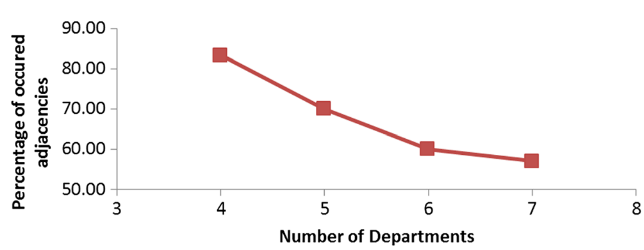


Fig. 12 Percentage of occurring adjacencies

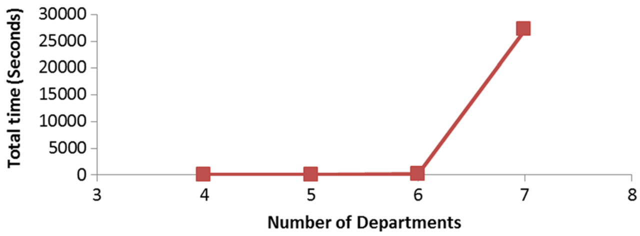


Fig. 13 Total time of solving different problems

with four departments, facility 1 has a value of 1.28 and 0.82 for α_1 and β_1, respectively, while for the problem with five departments, these values change to 0.67 and 1.26, respectively. In other words, adjacent facilities play a key role in determining facilities' areas.

6 Comparison between robust model and ABSMODEL

It is important to have a comparison among layouts which are generated based on robust and ABSMODEL models. Therefore, in this section, we point out the difference between these two models, where the problem with seven departments is used. The generated layout for the problem with seven departments which is based on ABSMODEL model is shown in Fig. 10. As can be seen in this layout, there are some wasted spaces which decrease the efficiency of the layout. Conversely, we do not have such spaces in the robust layout.

On the other hand, we can see that the total areas for different layouts have many differences (Table 2). Due to length and width deviation confidents, we consider more areas in the robust layout than in the ABSMODEL layout. Because additional areas are needed at the operational stage, these additional areas help us stave off crises.

Therefore, according to Table 2, the robust model is preferred to the ABSMODEL model because:

- The number of adjacencies in the robust layout will be more than that in the ABSMODEL layout.
- Compared to the ABSMODEL layout, the robust layout is expected to have fewer wasted spaces.
- Given the additional areas in the robust layout, we can stave off any likely crisis in the operational stage.
- The robust layout is more compatible with practical problems in comparison to the ABSMODEL layout.

7 Discussion

Two important factors to solve a mathematical model are the objective function and the total time of solving. For model (5)–(18), the value of second objective function in relation (6) can be predicted. For FLP, it is obvious that as the number of facilities increases, the maximum value of relation (6) increases, too. Nevertheless, as the number of facilities increases, the number of possible adjacencies, which can occur, decreases. By possible adjacency, we mean that it is possible that two departments *i* and *j* become adjacent with each other in the final layout, but by occurring adjacency, we mean two departments *i* and *j* are adjacent with each other in the final generated layout. This notification, which is expected for the proposed model, is proved according to Figs. 11 and 12. As shown in Fig. 12, the existing gap between the number of

Table 3 Result of sensitivity analysis

Iteration	α_{12}	Objective function	Total time (s)	α_1	Iteration	α_{12}	Objective function	Total time (s)	α_1
1	0.50	0.4691	1.98	0.67	11	1.00	0.4647	2.56	1.00
Original value	0.53	0.4692	2.73	0.67	12	1.05	0.4639	3.35	1.28
2	0.55	0.4604	3.59	1.28	13	1.10	0.463	1.92	1.28
3	0.60	0.4696	2.42	0.67	14	1.15	0.4633	2.45	1.28
4	0.65	0.4698	4.76	0.67	15	1.20	0.4635	3.31	1.28
5	0.70	0.4611	2.31	0.67	16	1.25	0.4637	2.31	1.28
6	0.75	0.4686	1.94	0.75	17	1.30	0.4637	4.91	1.30
7	0.80	0.4678	3.43	0.80	18	1.35	0.4629	3.78	1.35
8	0.85	0.4670	2.00	0.85	19	1.40	0.4621	2.45	1.40
9	0.90	0.4662	1.9	0.90	20	1.45	0.4578	3.51	1.45
10	0.95	0.4623	1.97	1.28	21	1.50	0.4606	1.59	1.50
Minimum value	0.50	0.4578	1.59	0.67					
Maximum value	1.50	0.4698	4.91	1.50					

possible adjacencies (PA) and occurring adjacencies (OA) increases by enlarging the problem size. Consequently, the percentage of occurring adjacencies (P) in Eq. (31) will be descending relative to the number of facilities (Fig. 12).

$$P = \frac{PA}{OA} \times 100 \tag{31}$$

As mentioned earlier, the proposed model is NP-complete. Therefore, it would be reasonable to expect an exponential treatment for the total time of solving model (5)–(18). As shown in Fig. 13, as the size of problem increases, the total time increases exponentially.

8 Sensitivity analysis

Sensitivity analysis (SA) is the study of how the variation (uncertainty) in the output of a statistical model can be attributed to different variations in the inputs of the model. Now we define the sensitivity of the model.

Generally, when a new layout follows as a result of a change in some input parameters, it can be said that the model is sensitive to those parameters. Admittedly, in any layout, the location of each facility and objective function are the most important issues which may be affected by changing the input parameters. Therefore, in sensitivity analysis of a layout, we analyze the effect of some important parameters on the generated layout, facilities areas, and the value of objective function.

In model (5)–(18), the length and width deviation coefficients are the most important input parameters which can affect the output layout drastically. Therefore, we restrict our analysis on the changing of a special length deviation coefficient such as α_{12} where the results can be readily extended to other length and width deviation coefficients. To do this analysis, we change the value of α_{12} by step of 0.10 from 0.50 to 1.50, which are the minimum and maximum permissible values for length and width deviation coefficients, respectively.

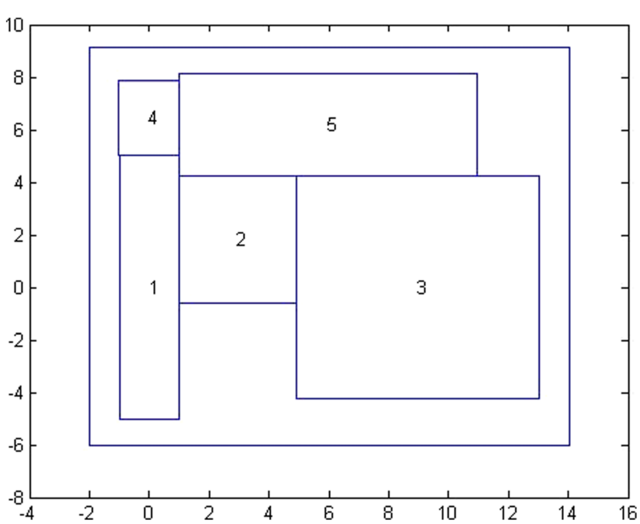


Fig. 14 Generated layout for the problem with five facilities and α_{12}=0.5

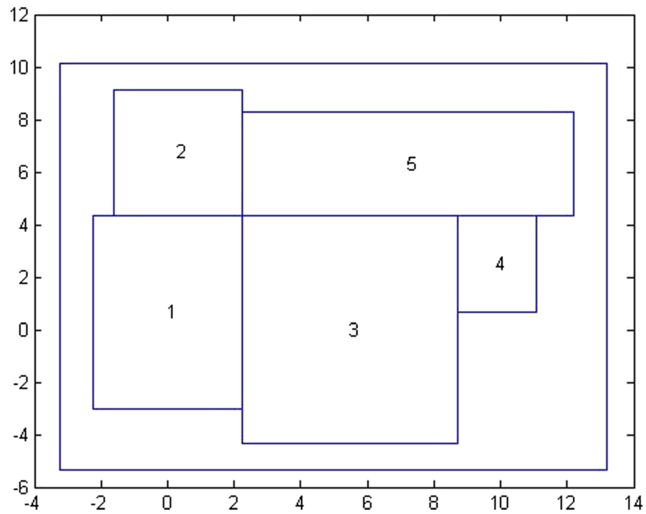


Fig. 15 Generated layout for the problem with five facilities and α_{12}=1.5

The results of sensitivity analysis are shown in Table 3. As shown, the value of objective function changes in the small range [0.4578, 0.4698]. This indicates that changing the value of length and width deviation coefficients does not affect objective function significantly. However, α_1 acquires various values, which means that adjacents of facility 1 changes significantly. In other words, changing the value of length and width deviation coefficients can affect the layout drastically. This is shown in Figs. 14 and 15, where α_1 has values of 0.50 and 1.50, respectively. On the other hand, the value of α_{12} can affect the total solving time of the model.

Therefore, it can be said that length and width deviation coefficients are the most important factors which can change the layout completely.

9 Conclusion

In this study, a bi-objective MIP model was developed for facility layout problem (FLP) under uncertain conditions. In this model, we assumed that the length and width of each department were not exactly determined, and both of them could change according to the deviation coefficients. Moreover, in this paper, we proposed two new objective functions. One of these objective functions had never been addressed in the literature while it is an effective criterion to assess the layouts in uncertain environment. Furthermore, we proved two theorems to calculate the upper bounds for two proposed objective functions, then we presented two results for lower bounds of objective functions.

The proposed model has some important advantages. First of all, instead of QAP models discussed in the literature, we present an MIP model. Secondly, based on two new parameters, length and width deviation coefficients, we are able to generate more flexible layouts. The third advantage is that as a new approach, we are able to determine which facilities are adjacent with each other by an MIP model. Finally, there are no predetermined areas needed for layout and departments and their areas will be determined according to a mathematical model.

Future research areas regarding the model (5)–(16) can be as follows:

- Applying bi-objective optimization approaches to MRFLP.
- Using a graph theory approach to solving the proposed model.
- Defining a new model based on areas instead of the dimensions of departments.
- Developing heuristic and meta-heuristic algorithms to solve larger size problems.

Appendix

Table 4 Value of N_{ij} for the problem with 4 departments

$j\downarrow i\rightarrow$	1	2	3	4
1	–	1	1	0
2	1	–	1	1
3	1	1	–	1
4	0	1	1	–

Table 5 Value of N_{ij} for the problem with 5 departments

$j\downarrow i\rightarrow$	1	2	3	4	5
1	–	1	1	0	0
2	1	–	0	0	1
3	1	0	–	1	1
4	0	0	1	–	1
5	0	1	1	1	–

Table 6 Value of N_{ij} for the problem with 6 departments

$j\downarrow i\rightarrow$	1	2	3	4	5	6
1	–	1	1	1	0	0
2	1	–	0	0	1	0
3	1	0	–	1	0	1
4	1	0	1	–	1	1
5	0	1	0	1	–	1
6	0	0	1	1	1	–

Table 7 Value of N_{ij} for the problem with 7 departments

$j\downarrow i\rightarrow$	1	2	3	4	5	6	7
1	0	1	1	0	1	0	0
2	1	0	0	1	1	0	0
3	1	0	0	1	0	1	0
4	0	1	1	0	1	1	1
5	1	1	0	1	0	0	1
6	0	0	1	1	0	0	1
7	0	0	0	1	1	1	0

Table 8 Facilities dimensions

Facility number	Area	Length	Width
1	24	8	3
2	16	4	4
3	36	6	6
4	8	4	2
5	21	3	7
6	17.5	3.5	5
7	3.6	0.9	4

Table 9 Length deviation coefficients

α_{ij}	1	2	3	4	5	6	7
1	1.00	0.53	1.28	0.66	0.67	0.96	1.46
2	0.97	1.00	0.79	1.08	0.63	1.36	0.86
3	1.07	1.36	1.00	1.07	0.73	0.83	0.59
4	1.02	1.02	1.20	1.00	0.77	1.20	0.80
5	0.80	0.83	1.20	1.42	1.00	0.62	0.50
6	1.13	0.51	0.66	1.30	0.73	1.00	1.40
7	1.10	1.09	0.75	0.64	0.82	1.08	1.00

Table 10 Width deviation coefficients

β_{ij}	1	2	3	4	5	6	7
1	1.00	0.57	0.82	1.26	0.92	1.16	1.36
2	1.20	1.00	0.74	1.07	0.71	0.99	1.44
3	0.70	1.21	1.00	1.45	1.40	1.26	1.10
4	0.70	1.22	0.93	1.00	0.51	0.65	1.21
5	1.03	1.01	1.31	1.17	1.00	1.03	1.03
6	1.05	0.86	0.99	1.35	0.87	1.00	1.27
7	1.32	0.70	1.45	0.61	1.37	0.67	1.00

References

1. Francis RL, JA White, F. McGinnis (1992) Facility layout and location: an analytical approach. Prentice-Hall
2. Tompkins JA (2010) Facilities planning, 4th edn. John Wiley, New York
3. McKendall ARJ, Hakobyan A (2010) Heuristics for the dynamic facility layout problem with unequal-area departments. Eur J Oper Res 201:171–182
4. Suresh G, Sahu S (1983) Multiobjective facility layout using simulated annealing. Int J Prod Econ 32(2):239–254
5. Bazzara MS, Sherali MD (1980) Benders' partitioning scheme applied to a new formulation of quadratic assignment problem. Nav Res Logist Q 27(1):29–41
6. Bukard RE, Bonniger T (1983) A heuristic for quadratic Boolean program with application to quadratic assignment problems. Eur J Oper Res 13:347–386
7. Kaku BK, Thompson GL (1986) An exact algorithm for the general quadratic. Eur J Oper Res 23(3):382–390
8. Kusiak, Heragu S (1987) The facility layout problem. Eur J Oper Res 29:229–251
9. Chwif L, Barretto MRP, Moscato LA (1998) A solution to the facility layout problem using simulated annealing. Comput Ind 36:125–132
10. Matsuzaki K, Irohara T, Yoshimoto K (1999) Heuristic algorithm to solve the multi-floor layout problem with the consideration of elevator utilization. Comput Ind Eng 36:487–502
11. Ning X, KC Lam, MCK Lam (2010) A decision-making system for construction site layout planning. Automation in Construction
12. Wald A (1950) Statistical decision functions which minimize the maximum risk. Ann Math 46(2):265–280
13. Mulvey JM, Vanderbei RJ (1995) Robust optimization of large scale systems. Oper Res 43(2):264–281
14. Ben-Tal A, LE Ghaoui, A Nemirovski (2009) Robust optimization. Princeton Series in Applied Mathematics. Princeton Series in Applied Mathematics
15. Ben-Tal A et al (2004) Adjustable robust solutions of uncertain linear programs. Math Program 99:351–376
16. Ben-Tal A, Nemirovski A (2002) Robust optimization—methodology and applications. Math Program 92:453–480
17. Bertsimas D, Sim M (2006) Tractable approximations to robust conic optimization problems. Math Program 107(1):5–36
18. Bertsimas D, Sim M (2003) Robust discrete optimization and network flows. Math Program 98:49–71
19. Soyster A (1973) Convex programming with set-inclusive constraints and application to inexact linear programming. Oper Res 21(5):1154–1157
20. Shore RH, JA Tompkins (1980) Flexible facilities design. Am Inst Ind Eng Trans: p. 200–205
21. Kulturel-Konak S, Smith AE, Norman BA (2004) Layout optimization considering production uncertainty and routing flexibility. Int J Prod Res 42(21):4475–4493
22. Rosenblatt MJ, Lee HL (1987) A robustness approach to facilities design. Int J Prod Res 25(4):479–486
23. Kouvelis P, Kurawarwala AA, Gutierrez GJ (1992) Algorithms for robust single and multiple period layout planning for manufacturing systems. Eur J Oper Res 63(2):287–303
24. Aiello G, Enea M (2001) Fuzzy approach to the robust facility layout in uncertain production environments. Int J Prod Res 39(18):4089–4101
25. Azadivar F, Wang J (2000) Facility layout optimization using simulation and genetic algorithms. Int J Prod Res 38(17):4369–4383
26. Cheng R, Gen M, Tozawa T (1996) Genetic search for facility layout design under interflows uncertainty. Jpn J Fuzzy Theory Syst 8(2):267–281
27. Pillai VM, IB Hunagund, KK Krishnan (2011) Design of robust layout for dynamic plant layout problems. Comput Ind Eng. 61
28. Neghabi H, Eshghi K, Salmani MH (2014) A new model for robust facility layout problem. Inf Sci 278:498–509
29. Izadinia N, Eshghi K, Salmani MH (2014) A robust model for multi-floor layout problem. Comput Ind Eng 78:127–134
30. Heragu SS, Kusiak A (1991) Efficient models for the facility layout problem. Operations 53(1):1–13
31. Olson DL, Swenseth SR (1987) A linear approximation for chance-constrained programming. J Oper Res Soc 38(3):261–267
32. Legendre A (1794) Élements de géometrie. Paris: Firmin Didot
33. Tam KY, Li SG (1991) A hierarchical approach to the facility layout problem. Int J Prod Res 29(1):165–184