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Effects of Different Parameters on The Crack Growth of Rotary Disks with Eccentricity Hole Using Emulator and Finite Element Techniques

M. Shariati and M.J. Maghrebi

The Faculty of Mechanical Engineering,
Shahrood University of Technology, Iran

Abstract

In this paper the crack growth in rotary disks with eccentricity hole are studied using both emulator and finite element techniques. It is assumed that a crack which is initiated from the inner surface of a disk is developed in radial direction and in longitudinal direction by length a. The effects of different parameters such as the disk radius, the distance between the hole and the disk center, the crack length and the radius of the hole on the behavior of the crack growth are investigated. The results of the FEM and the emulator techniques are used to establish a mathematical statement predicting the crack growth and the stress intensity factor. The effects of each parameter for any of the proposed model during the crack growth are studied separately. The mathematical model can be used to predict the crack growth and the stress intensity factor precisely and conveniently.

Key words: Crack growth; Rotary disks; Eccentricity hole; Emulators; FEM

Introduction

Machine elements such as turbines, turbo-compressors, generators and flywheels operate at very high speeds which produce high centrifugal forces. These forces can cause catastrophic failures as a result of flaws or crack-like defects which are initially found in the components or initiated by repeated stressing and grow finally to a critical size[l]. To assess the structural reliability of cracked structures it is necessary to know both the strength of the cracked component and the growth rate under in-service fatigue loads. Both the strength and crack growth rate depend on the stress intensity factor K. Therefore, an accurate determination of this parameter is of utmost importance to optimize the dimensions and ensure the in-service safety of rotating structures. Many research works have been conducted to analyze the strength and safety of rotary disks using the methods of fracture mechanics. Rook and Tweed [1] obtained an analytical solution for the stress intensity factor for a rotary disk containing a radial crack.

Chen and Lin [2] computed the stress intensity factor using the results of finite element simulation for a rotary disk with a crack initiated from the disk center. Smith [3] obtained the numerical solution of the stress intensity factor for a radial crack of a thin circular disk under a periodic loading. Sukere[4], Younis and Zacharys[5] obtained the stress intensity factor of a radial crack disks using an electro-optical approach known as Caustics method. Xu[6] introduced an analytical approach to study the stress intensity factor of a radial crack disk. Bert and Tapan[7] investigated the fatigue of rotary disks.

Ramesh et. al. [8] obtained the stress intensity factor for radial crack thin disks under the periodic loadings. Dhondt and Kohl [9] studied the effects of geometry and the loads rate on the dynamic fatigue of rotary disks using the concept of bending beam theory. In all of these investigations, the stress intensity factor of a solid rotary disk crack was studied when initiation started from the disk center. In the current work, the stress intensity factor of a crack for an eccentric hole is studied with emulators using the results of finite element simulation.

Emulator techniques

Emulators are used to both reduce the time cost and increase the accuracy in any optimization research work. Emulators are the fast shortcuts in optimization problems. In other words emulators are the statistical functions based on the data acquired from simulators and reflect the behavior of a system under investigation. They can be regarded as a proper substitution for predicting the behavior of a simulator. An overall comparison between an emulator and a simulator is indicated in table [1].

Table1: Comparison between an emulator and a simulator.

	SPEED	ACCURACY	OPTIMIZATION
Simulator	Slow	Very Good	Local
Emulator	Fast	Good	Global

The first step in creating an emulator is to acquire data properly. Based on the collected data, design points, an emulator can be constructed. Any design point contains the effective parameters in a problem and the corresponding outputs from the simulator inputs.

The accuracy of an emulator is important and must be taken into consideration. The accuracy is directly depending on both the selection of the effective parameters on a system and a number of design points on which an emulator is constructed.

The usage of an emulator can indicates an overall picture for an optimized design in a complicated system and reduce the time, cost and risk in different problems.

Problem definition

In the current investigation the growth rate in a rotating disk with an eccentric hole is studied. In the eccentric hole disk studied, cracks are initiated and growth in either radial or azimuthal direction (see Figs 1 and 2).

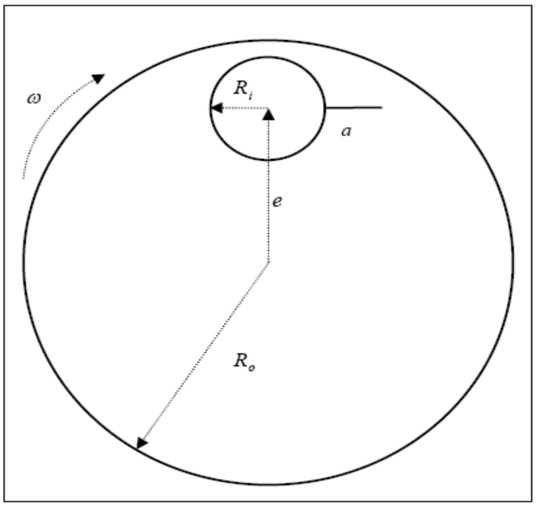

Figure 1: Eccentric hole disk with radial crack.

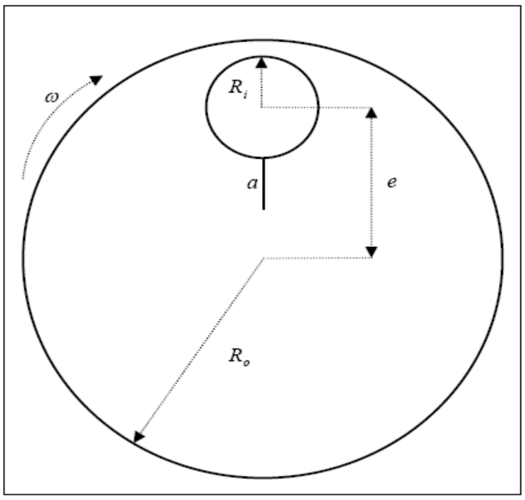

Figure 2: Eccentric hole disk with perpendicular crack.

Since the thickness of the disk is small in compare with the other geometries the state of the plane stress is dominated and the stress intensity factors for modes I and II are studied only. The data behavior from the simulation of a radial cracked disks indicate that the ratio of the stress intensity factor for mode II with respect to mode I exhibits the order of 10^{-4}.The other values for this geometry also agrees well with the assumption of the plane stress. Thus in the study of the radial cracked disks K_{II} is ignored without a loss of accuracy. Observations also indicate that the value of K_{II} is small for the case of small values of e/R_0 where R_0 is the disk radius and e is the distance between two centers. By increasing the value of e/R_0 the effects of the shear stresses are observed and K_{II} values become considerable. For both of the cases the value of K_I is considerable and must be taken into serious consideration. Therefore, in what follows K_I for the radial crack and K_I and K_{II} for the perpendicular crack's disks are studied.

Numerical simulations

Finite element method is one of the most applied numerical techniques used to obtain an approximate solution in engineering problems. This technique is used here for modeling and simulation of a rotating disk. First the disk is discretised into finite elements as shown in Figure [3].

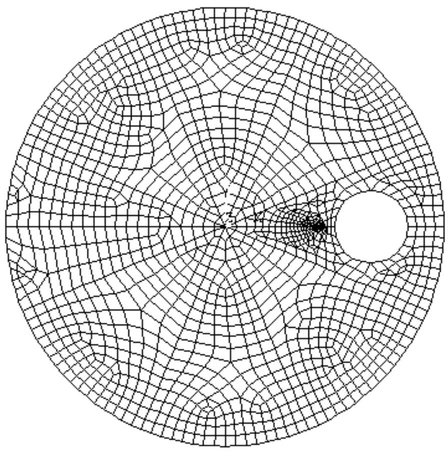

Figure 3: Discretization of a radial cracked rotating disk with an eccentric hole.

The stress intensity factors for a number of specific geometries are found in the text books of fracture mechanics. For complicated geometries there is no reference to address to address these factors. The finite element is the best technique for computing the crack nose parameters in complex geometries. In summary, the following steps are required for a finite element simulation of the crack nose parameters. First, the geometrical model must be constructed in the finite element software. Then, the model must be discretised into finite elements. Due to the presence of a crack in the model there are singular terms at the vicinity of the crack nose and the stress are so high in these regions.

$$\sigma = \frac{K_I}{\sqrt{2\pi r}} f(\theta) \qquad (1)$$

As the formula indicates, the stress is proportional to $\frac{1}{\sqrt{r}}$. Since r tends to zero at the crack nose, stress tends to infinity. In discretization of a cracked model conditions must be met so that the stress at the crack nose regions tends to infinity. In other words the discretization must be performed such that the stress profile at the crack nose region is a function of $\frac{1}{\sqrt{r}}$. In general the second order approximating function is used in finite element simulation of a cracked specimen. The shape of the finite elements is either triangular or rectangular. In triangular elements of the second order there is one node at each vertices and another node in the middle of each sides. In other words, each finite elements of the second order rectangular element contains eight nodes. To take into account the singular terms for the second order elements at the crack regions the middle nodes are considered at ¼ distance from the main vertices of a triangular elements. This is shown in Fig. 4.

Effects of parameters in models

To study the effects of each parameter separately, all parameters except the one under investigation are assumed constant. The effects of each parameter on stress intensity factors are then studied by varying the parameter.

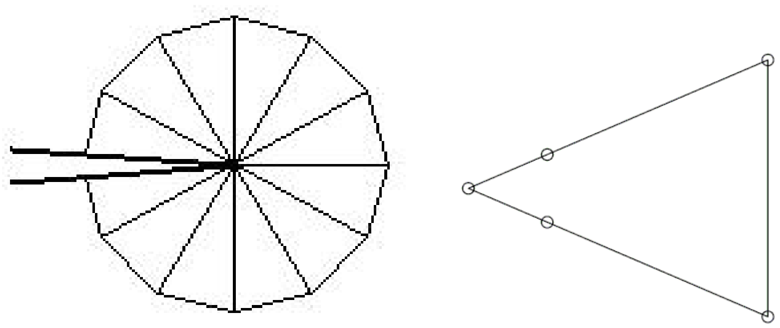

Figure 4: An illustration of a triangular element at crack nose region

Disk radius R_0

Stress in a rotating disk is created as a result of (inertia/centrifugal) forces related to the disk mass and as a consequence relates to the disk radius. The variation of K_I with respect to R_0 for radial cracked and perpendicular cracked disks are shown in Figures [5] and [6] respectively. The variation of K_{II} with any increase in the disk radius () is also shown in Figure [7]. The figures indicate that the values of K_I and K_{II} for the both disk geometries are increased as the radius increase.

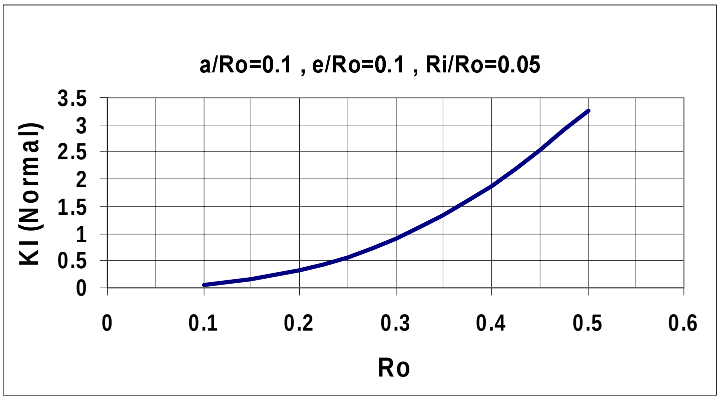

Figure 5: Variation of K_I with respect to R_0 for radial cracked disks

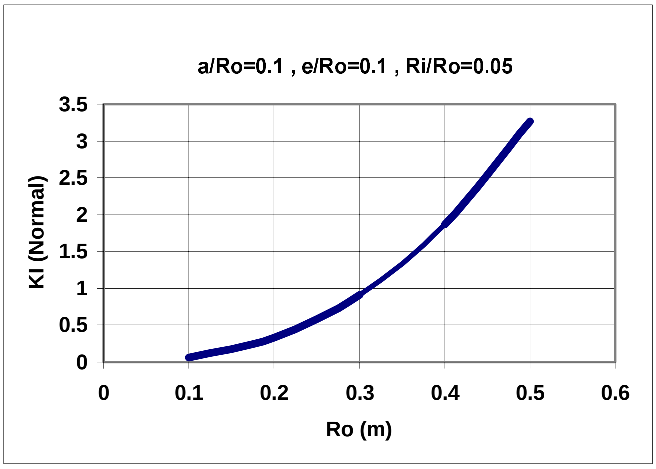

Figure 6: Variation of K_I with respect to R_0 for perpendicular cracked disks

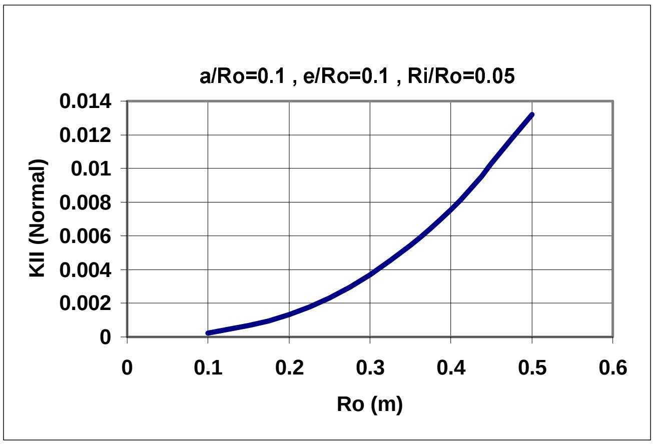

Figure 7: Variation of K_{II} with respect to R_0 for perpendicular cracked disks

Crack length (a / R_o)

Both $K_I = \beta\sigma\sqrt{\pi a}$ and experimental observations indicate that any increase in the crack length increase the stress intensity factors. This behavior is clearly observed in Figure [8].

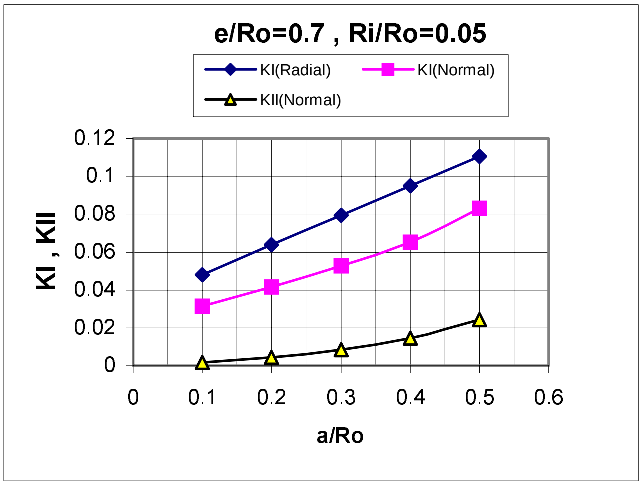

Figure 8: Stress intensity factors K_I and K_{II} for radial and perpendicular cracks as a function of a/R.

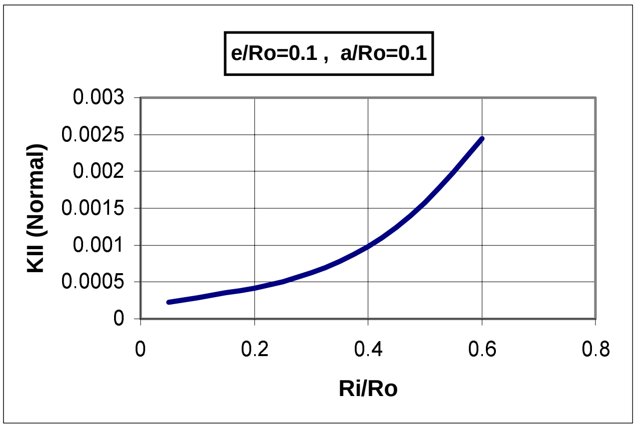

Figure 9: K_{II} as a function of R_i/R_o for a radial cracked disk.

Hole diameter dimension (Ri / Ro)

The effect of change in Ri/Ro ratio on stress intensity factors is studied. As indicated in Figure[9] the crack position is affected by the change in the hole dimension for the radial cracked disks such that any increase in the diameter increases K_{II} . Although the effect of increase in the disk hole radius decreases the stiffness, it increases the stress intensity factor for mode I. This behavior is observed in Figure [10],[11].

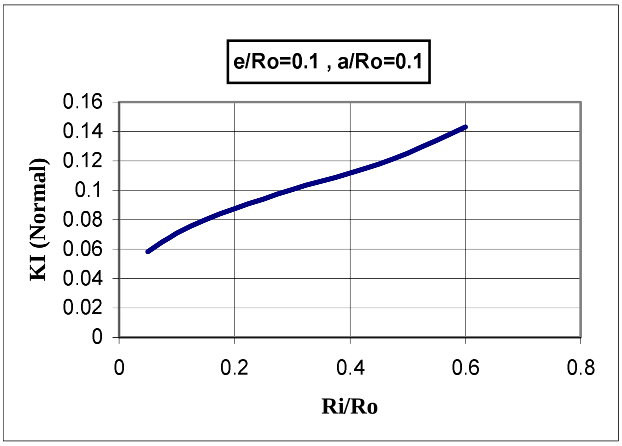

Figure 10: K_I as a function of R_i/R_o for a perpendicular cracked disk.

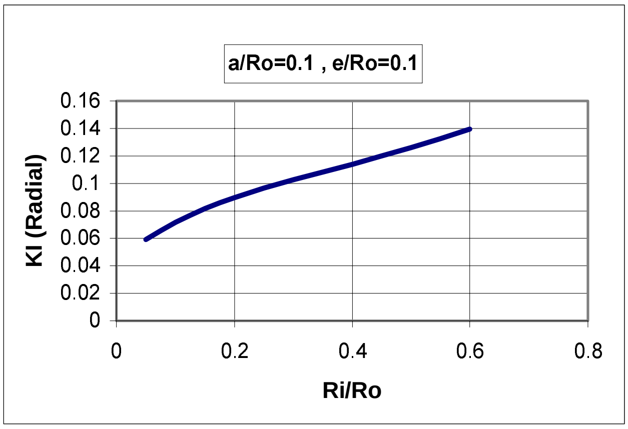

Figure 11: K_I as a function of R_i/R_o for a radial cracked disk.

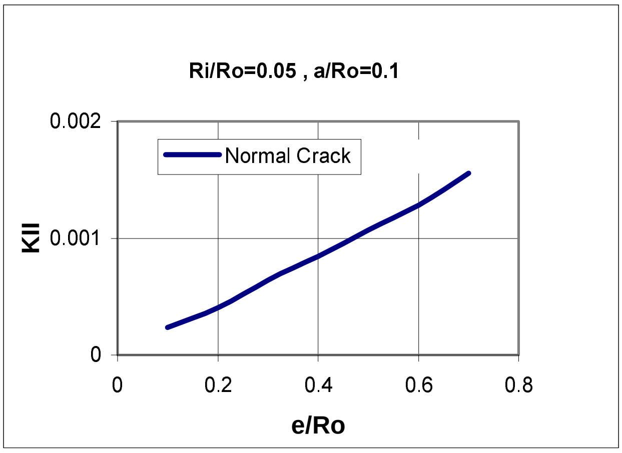

Figure 12: K_{II} as a function of e/R_o for a perpendicular cracked disk.

Hole position (e / Ro **)**

Variation in the hole position have dual effects for perpendicular cracked disks in which K_{II} is significant. Any increase in the ratio of e/Ro increases K_{II}. This is as a result of increase in the shear stress. See Figure [12]. However, by increasing the center distances K_I is decreased for both disks. These facts are shown in Figures[13],[14]. K_I is mostly affected by the shear stress which is decreased by the distance from the center.

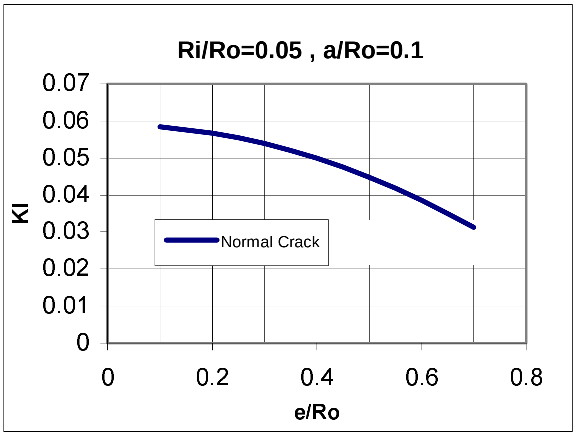

Figure 13: K_I as a function of e/R_0 for perpendicular cracked disks

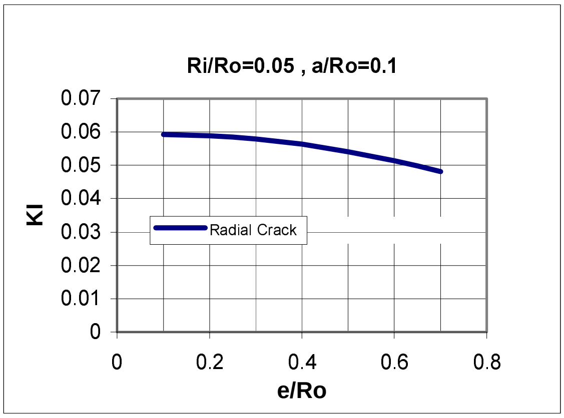

Figure 14: K_I as a function of R_i/R_0 for radial cracked disks

Poisson ratio

The mechanical properties of the disks can affect the fatigue crack growth. One of these properties is the Poisson ratio. The stress intensity factors K_I , K_{II} with respect to the change in the Poisson's ratio are shown in Figures 15,16 respectively. The figures show that any increase in the Poisson's ratio increases K_I and decreases K_{II}.

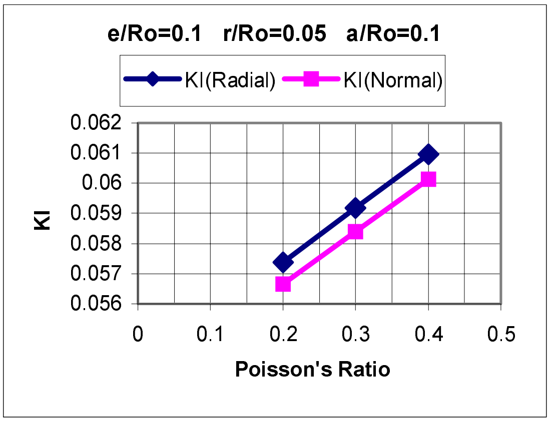

Figure 15: K_I as a function of Poisson's ratio.

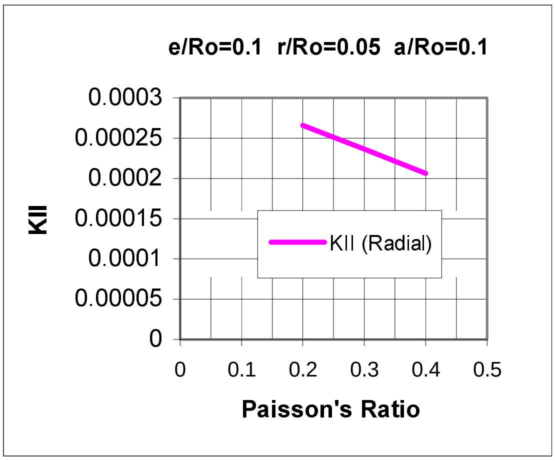

Figure 16: K_{II} as a function of Poisson's ratio.

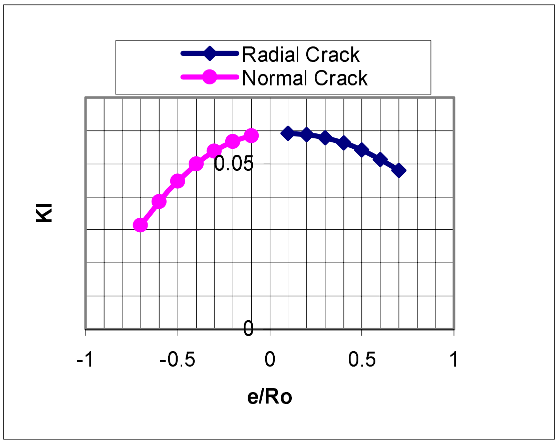

Figure 17: Results of two models with respect to e/R_o

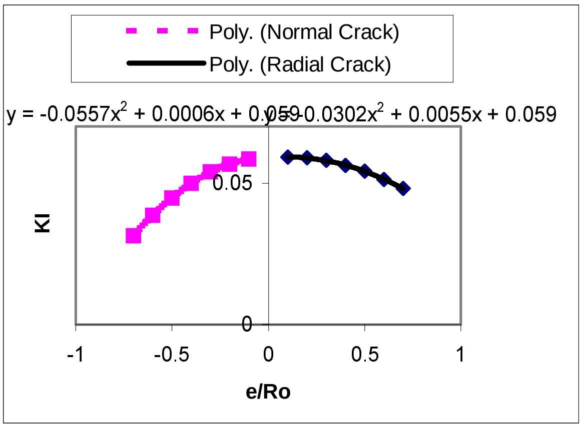

Figure 18: K_I values for both disks as e/R_o tends o zero.

Fatigue crack growth in disks as e tends to zero

It is expected that the stress intensity factors become the same for both disks as e tends to zero.

In other words, the extrapolation results for both models must be the same as e tends to zero. As indicated in Figures [17],[18] the values of K_I for the both cracked disks become the same as e tends to zero.

If K_I with respect to e/R_O , which are shown in Figures [17],[18], is extrapolated with a quadratic function for both disks it will be found that K_I =0.059 for both disks at X=0. This result can be regarded as a good test indicating the accuracy of the method.

Emulators design

The independent parameters and the stress intensity factors obtained by each running of the simulation containing inputs and outputs as design points are used to set up the relevant emulators. The desired model containing all independent parameters is selected and the result which is the emulator is used. Datafit 8.1 software which is the product of Oakdale engineering company in 2005 for regression analysis is used to obtain the emulators. For this reason the independent parameters and the value of stress intensity factor are entered in the related columns. The form of the desired formula with all independent parameters is entered in the software and the results of the regression analysis are used as the desired emulator. The input form containing the mathematical expression with unknown coefficients is recovered using a statistical analysis. A measure of accuracy for the formulas with the data of finite element simulation is found to be $R^2 > 0.98$ for all formulas obtained.

K_I for radial cracked disks

K_I for a radial crack is calculated using the form indicated by equation [2] in which the coefficient constants as in table [2].

Table 2: Coefficient values in the formula

a1	0.050
a10	1.511E-02
a11	0.567
a12	17.236
a13	5.449
a14	2.187
a2	2.359
a3	-0.103
a4	-4.839
a5	32.817
a6	0.154
a7	5.201
a8	-1.378E-02
a9	-0.0627

K_I for perpendicular cracked disks

K_I for a perpendicular crack with an acceptable accuracy is given as the preceding form with different coefficients as indicated by table[3].

Table 3: Coefficient values in the equation

a1	0.577

a10	-1.559E-02
a11	3.759E-03
a12	3.619
a13	2.492
a14	1.229
a2	5.869
a3	0.167
a4	-1.525
a5	-0.909
a6	-0.934
a7	1.772
a8	-6.599
a9	-5.146E-02

$$K_I = 4.0542(\rho\omega^2 R_o^{5/2})\left\{a_1 + (a_2 + a_3\nu)^{a_4}\left[a_5(\frac{R_i}{R_o}) + a_6(\frac{e}{R_o}) + a_7(\frac{a}{R_o}) + a_8\right]\right\}$$
$$\left\{(\frac{R_i}{R_o})^{a_9}(\frac{e}{R_o})^{a_{10}}(\frac{a}{R_o})^{a_{11}} + a_{12}(\frac{R_i}{R_o})^{a_{13}}(\frac{a}{R_o})^{a_{14}}\right\}$$

K_{II} for perpendicular cracked disks

K_{II} for a perpendicular crack with an acceptable accuracy is given as the preceding form with different coefficients as indicated by table[4].

Table 4: Coefficient values in the equation

a1	-2.121E-04
a10	1.991
a11	1.945
a12	1.722E-02
a13	0.345
a14	1.162
a2	0.318
a3	-0.255
a4	0.9243
a5	2.965
a6	4.898E-02
a7	0.364
a8	0.175
a9	-4.511E-02

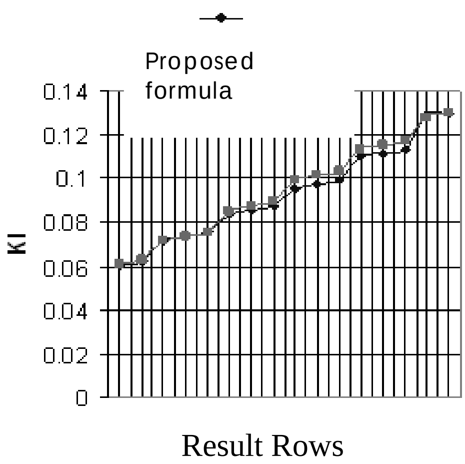

Figure 19: Comparison between K_I from the numerical analysis and the proposed formula for radial crack.

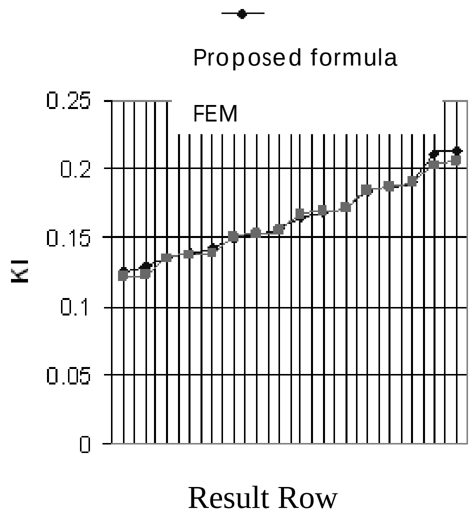

Figure 20: Comparison between K_I from the numerical analysis and the proposed formula for radial crack.

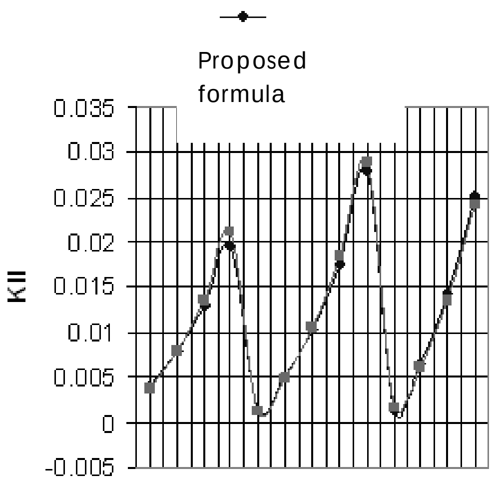

Figure 21: Comparison between K_{II} from the numerical analysis and the proposed formula for perpendicular crack.

Accuracy of emulator

To investigate the accuracy of emulator the results of finite element method and emulator for K_I and K_{II} are plotted and compared in Figures [19-21]. As indicated by the figures the results are very close together which reveals the emulators capability and the accuracy of the results and analysis .

Conclusion

The results of this investigation reveal that :

1) The stress intensity factors for both radial and perpendicular cracks are increased as the disk diameter increases.
2) K_I , K_{II} are increased as the crack length increases.
3) K_I , K_{II} for both types of cracks are increases as the internal hole radius increases.
4) For both types of cracks with any increase in the distance of the hole center from the disk center K_{II} increases , while K_I decreases
5) With any increase in the Poisson's ratio K_I increases but K_{II} decreases.
6) Emulators (mathematical expressions) are easy, proper and accurate methods for analysis of any engineering problem.

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