Net present value maximization in project scheduling with an external resource

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1 Introduction

We consider a project with a number of activities that are related to each other via precedence constraints. Each activity is characterized by a known duration (a natural number) and a (positive or negative) cash flow that is incurred when the activity is completed. We adopt a discrete time horizon, so that the *net present value* (in short, NPV) of each activity's cash flow with discount rate α when it is completed in any given time period, can be computed in a preprocessing step. We also assume that sufficient internal resources are available to complete the activities (and thus, their availability is not modelled), but some of the activities require an external (and relatively expensive) resource (such as a crane, loader, concrete mixer, bulldozer, excavator, etc.) that can only be used by at most one activity at each moment in time. The external resource should be rented for one uninterrupted time interval, and a cost is incurred per time unit that it is rented (this rent is paid per time unit, at the end of the unit). The objective is to schedule the activities such that the total net NPV of the activity cash flows and the renting costs (which represent negative cash flows) is maximized.

Let P denote the above-described problem, and let $1|prec, l_{ij}|C_{max}$ denote the single machine scheduling problem with minimal-time-lag precedence constraints for makespan minimization. The following two results imply that P is NP-hard in the strong sense.

Theorem 1 *When* $\alpha = 0$*, P is equivalent to* $1|prec, l_{ij}|C_{max}$*.*

Theorem 2 $1|prec, l_{ij}|C_{max}$ *is strongly NP-hard.*

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If none of the activities requires the external resource then the problem is equivalent to the so-called "project scheduling problem with irregular starting time costs" (in short, PSPwIC) introduced by Maniezzo and Mingozzi [1]. For PSPwIC, there is a time-indexed formulation with totally unimodular coefficient matrix, which implies a pseudo-polynomial-time algorithm. Another pseudo-polynomial-time method for PSPwIC is based on a transformation to the min-cut problem (following [2]).

2 Solution methods

We develop two different methods for solving P: one based on Benders decomposition, and one branch-and-bound procedure. Both methods follow the same two-step logic. In the first step, we choose the sequence in which the activities that require the external resource are to be executed. Given such a sequence, the original instance can be converted to an instance of PSPwIC and thus, in the second step, we schedule the activities using the pseudo-polynomial algorithm of Möhring et al. [3]. The fact that the proposed time-indexed formulation for PSPwIC is totally unimodular guarantees that the resulting Benders cuts will be sufficiently tight.

In the branch-and-bound algorithm, we branch over disjunctive arcs that determine the sequence of the activities on the external resource. In order to achieve better pruning in the exploration of the search tree, we compute Lagrangian-relaxation-based bounds by relaxing those constraints associated with the external resource (following Möhring et al. [3]) and penalizing infeasibilities. The relaxed problem is equivalent to PSPwIC, which (as mentioned supra) can be solved in pseudo-polynomial time [2].

References

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