Real-Time Maximum Torque per Ampere Control of Brushless DC Motor Drive with Minimum Torque Ripple

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Abstract - A nonlinear control scheme is proposed for real-time tracking of maximum torque per Ampere trajectory of the surface mounted brushless DC motor with non-sinusoidal back-EMF. The strategy employs the Lagrange’s Theorem to obtain the optimum stator current, at which both torque ripple and corresponding current magnitude become minimized. A new criterion is introduced in such a way that, when forced to zero, guarantees MTPA realization. The input-output feedback linearization (IOFL) control scheme is employed to improve tracking of the applied optimum currents. In the suggested approach, the magnitude of the stator current vector is minimized effectively in each instant. The performance of the control system is validated through experimental results.

I. INTRODUCTION

The brushless DC motor (BLDCM) have been increasingly used in various industrial and household applications, due to its high efficiency, high power density, and robust structure. In many applications, obtaining a ripple free torque control operation under minimum copper loss is the primary issue of BLDCM drives.

A wide variety of methodologies have been presented during the last two decades to minimize the torque ripple. The torque pulsation due to the commutation of the square-wave phase currents in the two-phase conduction mode has been addressed in [1, 2]. Through a comparative analysis, it has been demonstrated in [3] that the sinusoidal excited currents produce smaller torque ripple than square-wave currents. Several studies have recommended a non-sinusoidal injection scheme in which a complex Furrier series analysis is employed to determine harmonic coefficients of the reference current [4]. Both the torque ripple and copper loss minimization has been achieved by means of numerical optimization techniques [5]. In order to avoid complex exponential decomposition of these methods, Park, et al. [6] have proposed a straightforward solution to determine optimum current waveform using the dq-frame analysis, which has been previously employed for the sinusoidal permanent magnet synchronous machine (PMSM), extensively. Applying the dq-frame analysis to the sinusoidal PMSM, would result in constant state variables in the steady state without any position dependent terms. In the case of surface mounted PMSM, the generated torque would be a linear function of q-axis current. Accordingly, ripple-free torque control can be simply achieved by varying the q-axis current while keeping the d-axis current at zero. Due to non-sinusoidal back-EMF of the surface-mounted BLDCM, the d-axis current would appear in the torque relation which results in cross-coupling effect in torque relation. In addition, state variables would not be constant and vary with respect to the rotor angular position. In [6], in order to achieve a constant torque, the rotor position-dependent variation in the back-EMF is effectively reflected on the desired q-axis current, and d-axis current is forced to zero. Although this reference current will result in constant torque, its magnitude would not be minimum. There exist an opportunity to minimize the stator current magnitude by an MTPA strategy. It is well known that, among various control strategies, the MTPA ensures minimal copper loss [7].

In this study, a real-time MTPA strategy is proposed for BLDCM, which ensures torque ripple minimization under minimum vector current magnitude. The strategy employs Lagrange’s Theorem to obtain the optimum currents. In this method a non-sinusoidal reference current, which contain fundamental and harmonic components, are developed directly without requiring complex exponential decomposition. The output torque is improved due to contribution of both fundamental and harmonic components of stator currents and back-EMFs in the torque production. The effectiveness of the proposed MTPA strategy is validated experimentally on a 200W outer rotor BLDCM prototype.

II. PROPOSED MTPA APPROACH

Three-phase equivalent circuit of BLDCM drive is demonstrated in Fig. 1. The phase resistance and inductance are represented by $R_s$ and $L_s$, and the phase currents and back-EMFs are demonstrated by $i_j$ and $e_j$, respectively ($j=a, b, c$). No assumption is considered on the shape of the back-EMFs; therefore, the effect of non-ideal back-EMF is considered. The electromagnetic torque $T_e$ is given by

$$T_e = \lambda_d i_d + \lambda_q i_q + \lambda_c i_c$$

(1)

where $\lambda_j$ is the ratio of back-EMFs with respect to the rotational speed ($\omega_m$). The dq-axis variables in the rotor reference frame can be obtained by applying the power-invariant Park transformation (2) on the phase variables as

![Fig. 1 Equivalent circuit of BLDCM](image)
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where \( X \) is the variable and \( \theta_e \) is the rotor electrical position. The electromotive force and the square of the current vector magnitude can be obtained as

\[
T_e = \lambda_d i_d + \lambda_q i_q \quad \text{(3)}
\]

\[
i^2 = i_d^2 + i_q^2. \quad \text{(4)}
\]

The objective is to obtain a constant torque with minimum current magnitude. Without loss of generality, rather than the current magnitude, its square \((i^2)\) is aimed to be minimized. In the case PMSM with sinusoidal back-EMF, \(\lambda_d\) is zero, and \(\lambda_q\) is a constant value; therefore, a constant torque can be simply achieved by controlling the \(q\)-axis current and keeping the \(d\)-axis current at zero. However, due to the non-sinusoidal back-EMF of the BLDCM, both of \(\lambda_d\) and \(\lambda_q\) would not be constant anymore and vary with respect to the electrical position as depicted in Fig. 2 for the specified motor in Table I. This results in the cross-coupling effect between the \(d\)- and \(q\)-axis currents in the torque equation.

According to (3), in order to achieve a ripple free constant torque \(T_e^*\), following condition must be satisfied:

\[
i_q = \frac{T_e^* - \lambda_d i_d}{\lambda_q}. \quad \text{(5)}
\]

In this regard, in [6], the \(i_d = 0\) strategy is proposed in which the variation of back-EMF is reflected on the \(q\)-axis current as

\[
i_q^* = \frac{T_e^*}{\lambda_q}, \quad i_d = 0. \quad \text{(6)}
\]

Three-phase reference currents are then deduced, using the reversed-Park transformation. Although this method is able to achieve ripple-free constant torque straightly, the current magnitude, as will be proven, would not be minimized. According to (5), the constant torque curve can be plotted as a diagonal line with the slope of \(-\lambda_d/\lambda_q\) in the \(i_d - i_q\) plane, as indicated in Fig. 3(a). Depending on the electrical position, the slope could be negative or positive. On the same plane, according to (4), constant current magnitude takes the form of a circle. In a particular instant \((e.g. \theta_e = \theta_1)\), each point on the solid line produces constant torque \(T_e^{*1}\). Among them, the specified point \((i_d^{opt}, i_q^{opt})\), at which the current magnitude curve becomes tangent with the torque curve will result in minimum current magnitude and hence, MTPA will be obtained. It can be obviously observed that \(i_d = 0\) strategy \((6)\), requires greater current magnitude. In other words, in each rotor position such as \(\theta_e = \theta_1\), a right-angled triangle is formed. The magnitude of the hypotenuse \((I_0)\), is obviously greater than the magnitude of the side of

\[
\text{Fig. 2 Non-sinusoidal back-EMF ratio of BLDCM at rated speed}
\]

\[
\begin{bmatrix}
X_d \\
X_q
\end{bmatrix} = \begin{bmatrix}
\cos(\theta_e) & \cos(\theta_e - 2\pi/3) & \cos(\theta_e + 2\pi/3) \\
\frac{1}{3} & -\sin(\theta_e) & -\sin(\theta_e - 2\pi/3) & -\sin(\theta_e + 2\pi/3)
\end{bmatrix} \begin{bmatrix}
X_d \\
X_q
\end{bmatrix}
\]

\[
(2)
\]

\[
\text{Fig. 3 Constant torque and constant current magnitude curves on the \(i_d - i_q\) plane. (a): Comparison of the \(i_d = 0\) and the proposed MTPA strategies. (b): Variation of the optimum point with respect to \(\theta_e\)}
\]

\[
\text{Fig. 4 Locus of the optimum point in \(i_d = 0\) and the proposed MTPA strategies for the specifed motor in Table I}
\]
According to (9) and (10), the dq-axis current references are not constant. Moreover the plant is a highly nonlinear system. Therefore, simple proportional-integral controllers may result in nonzero steady-state error, especially at higher speeds where fundamental-to-sampling frequency ratio is high. In order to cope with the tracking problem, the nonlinear IOFL control scheme is employed. The dq-axis currents are selected as control outputs and dq-axis voltages are inputs. Derivative of outputs can be obtained from dynamic model of BLDCM in the rotor reference frame [8] as follows

\[
\begin{align*}
i_q &= -\frac{R_i}{L_s} i_q + \frac{1}{L_s} \omega i_q + \frac{1}{L_s} v_q \\
\end{align*}
\]

\[
\begin{align*}
i_d &= -\frac{R_i}{L_s} i_d + \omega i_d + \frac{1}{L_s} v_d \\
\end{align*}
\]

where \( \omega \) is electrical speed. According to IOFL, the nonlinear coupled BLDCM can be transformed to an exact linear system with poles at the origin by using following control laws:

\[
\begin{align*}
v_q &= e_d + R_i i_d - L_s \omega i_q + L_s v_i \\
v_d &= e_q + R_i i_q - L_s \omega i_d + L_s v_i. \\
\end{align*}
\]

The decoupled closed loop system can be controlled properly by means of the proportional-integral (PI) controller as

\[
\begin{align*}
i_q &= \left( K_p + \frac{1}{\omega} K_i \right) i_d + \frac{1}{\omega} K_i \left( i_d - i_q \right) - \frac{1}{\omega} \int (i_d - i_q) \, dt \\
i_d &= K_p (i_q - i_d) + \frac{1}{\omega} K_i \left( i_q - i_d \right) - \frac{1}{\omega} \int (i_q - i_d) \, dt. \\
\end{align*}
\]

The PI gains must be designed, such that the bandwidth set and corresponding phase margin could support fast changing variations of the reference currents in (8, 9). The block diagram of the current control loop is shown in Fig.5. According to (15), the linearized and decoupled plant is represented by an integrator. The delay introduced by sampling and algorithmic execution is denoted by \( \tau_d \), which is normally equating to one switching period \( (\tau_{SW}) \). Inverter assumed to have unit gain with a time constant \( (\tau_p) \) equal to half of \( \tau_{SW} \). In order to actually track up to \( 12^{th} \) harmonic, the required bandwidth should be set on \( 12\pi R_c \) \( \omega_{rated} \) where \( p \) is number of pole pairs. For the specified machine in Table I, selecting \( K_p = 3028 \) and \( \tau_d = 0.2 \), would result in the required bandwidth with a phase margin equal to 64.7\( ^\circ \)deg and a gain margin equal to 19.9 \( \text{dB} \), which provide a good robustness. The block diagram of whole control system is demonstrated in Fig. 6(a).

**III. Experimental Results**

The effectiveness of the suggested controller is experimentally validated for a 200 W outer rotor BLDCM, characterized in Table 1. As demonstrated in Fig. 6(b), the experimental setup consists of a 200 W outer rotor BLDCM, coupled with a 250 W DC generator by a timing belt. The rotor position is sensed by means of an incremental encoder with 1024 pulses per round. Load variation can be applied by an external rheostat connected to the DC generator. A DSP-based digital control board is employed for machine drive control including the following sections. IGBT based inverter with IGBT driver, HCPL 316J, which guarantees isolation between control and power system. The inverter switching frequency is 10 kHz with the dead-time equal to 1\( \mu \)s. A discrete signal processor (TMS320F28335), designed with Texas Instruments Co. for motor control application. Stator phase currents and voltages are measured by three Hall-effect currents (LEM LA-55P) and voltage (LEM LV-25-P) sensors. Analog second-order low-pass filters with 2.6 kHz cut-off frequency are employed for filtering sensed currents and voltages. Internal A/D channels of DSP are used to get all measured variables.

A sample test has been designed to compare the performance of the proposed MTPA strategy with two other techniques. The desired torque \( T^e \) is equal to 2.2 \( \text{Nm} \) (33% of rated) is experimentally applied to three control methodologies. In the first method, sinusoidal (constant dq-axis) currents are applied to the stator to provide the demanded torque. In the second method, the \( i_d = 0 \) strategy [6], and in the third one, the proposed MTPA strategy is applied to the control system. Results are presented in Fig. 7. In Figs. 7 (a), (b) the experimental results of the sinusoidal current injection scheme are demonstrated. The \( q \)-axis current is controlled at its desired constant value, while the \( d \)-axis current is kept constant at zero.

<table>
<thead>
<tr>
<th>TABLE I MACHINE PARAMETERS</th>
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<tbody>
<tr>
<td><strong>Rated speed</strong></td>
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<tr>
<td><strong>Rated torque</strong></td>
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<tr>
<td><strong>Number of pole pairs</strong></td>
</tr>
<tr>
<td>( R_c, L_s )</td>
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![Fig. 6 (a) Diagram of the proposed MTPA strategy (b) Experimental setup](image-url)
It can be seen that undesirable torque ripples are generated; however, these ripples are much less than the square-wave current injection scheme [3].

Torque ripple is reduced significantly in the second method (Figs. 7(c), (d)), in which the back-EMF variations are being reflected on the q-axis current while keeping the d-axis current at zero (6). However, as demonstrated earlier there exist an opportunity to reduce the required current magnitude to provide the same load torque by the proposed MTPA strategy. It is obviously observed in Figs. 7 (e), (f), that the variations of the back-EMF are reflected on both d- and q-axis current references to achieve both torque ripple reduction and current magnitude minimization, simultaneously. In addition, current controller has succeeded to track its desired value. The MTPA criterion is forced to zero as shown in Fig. 7 (h). All three phase currents are indicated in the Fig.7 (g). As demonstrated, in comparison with \(i_d = 0\) strategy, the realization of the proposed MTPA has effectively reduced the current magnitude at each instant under the same load conditions. Peak-to-peak value of torque ripple (\(T_{\text{ripple}}\)) in each method is also presented in Fig. 7 (c).

In order to analyze how the suggested method improves the torque production, exponential decomposition of back-EMFs and currents are employed as following:

\[
\lambda = \lambda_1 \sin \omega t + \lambda_3 \sin 3\omega t + \lambda_5 \sin 5\omega t + \lambda_7 \sin 7\omega t
\]

\[
i_d = I_d \sin \omega t + I_3 \sin 3\omega t + I_5 \sin 5\omega t + I_7 \sin 7\omega t
\]

Third harmonic of stator current would not be circulated due to Y connection of windings. Regarding (1), interaction of currents and back-EMFs produce following torque profile[9]:

\[
T = T_0 + T_6 \cos 6\omega t + T_{12} \cos 12\omega t
\]

\[
T_6 = [\lambda_3 I_x + \lambda_5 I_y + \lambda_7 I_z], \quad T_{12} = [\lambda_5 I_x - \lambda_3 I_y - \lambda_7 I_z]
\]

In the sinusoidal current injection scheme (\(i_d = 0\)), it is the fundamental components of the currents and back-EMFs that contribute in the torque production. In the non-sinusoidal harmonic injection scheme (NSHI), injected current harmonics are determined to minimize torque ripple by eliminating \(T_6\) and \(T_{12}\). Moreover, dc component \(T_0\) increases due to contribution of both fundamental and harmonic components in torque production. In comparison with (NSHI) [4], the suggested approach has two improvements:

- Rather than complex exponential decomposition, the optimum reference current in the proposed method would be obtained simply by forcing the MTPA criterion to zero.
- Unlike NSHI strategy, in which limited harmonic components are considered to reduce complexity, the proposed one considers all harmonic components since it considers general form of back-EMF.

The harmonic components of injected stator currents for each method of Fig. 7 is demonstrated in Fig. 8 (a). As demonstrated in Fig. 8 (b), 6th component and its coefficients are appeared in the produced torque. The harmonic components of produced torque is reduced in both \(i_d = 0\) and the proposed MTPA strategies by automatically injecting current harmonics. In comparison with \(i_d = 0\) strategy, the reference current harmonics in the proposed MTPA are determined such that RMS value of current is minimized. The ratio of average torque per Ampere (RMS) (\(T/I\)) is demonstrated for each method in Fig. 8 (c). Increment of this parameter in the proposed method implies that as per a constant current, the produced torque has been increased in this method. This occurs due to contribution of both fundamental and harmonic components of the currents and back-EMFs in the torque production.
The performance of the proposed MTPA strategy in various load conditions is demonstrated in Fig. 9. The RSMS value of the phase current $i_{ph}$ is compared for $I_d = 0$ and the proposed MTPA strategies in different load conditions. The trend of reduction in RSMS value of $i_{ph}$, is denoted by the dotted curve. The RSMS of phase current, required to produce same torque, is indicated to have 1.5% to 9% reduction depending on load torque, in such a way that, the rate of reduction is more dominant at light loading conditions.

In order to illustrate torque control performance, a step change in torque command from 3.5 Nm to 4.5 Nm is applied to the proposed control approach of Fig.6 (a) at 2.5s. The response of torque and corresponding dq-axis currents are demonstrated for rotational speed equal to 10 rad/s in Fig.10 (a), (b) and for speed of 30 rad/s in Fig.10 (c), (d). It is clearly observed that the back-EMF variations are reflected on both d- and q- axis currents to obtain the desired torque with minimum ripples as per the minimum current. Consequently, torque is properly tracked at its commanded value while the MTPA criterion is forced to zero to guarantee MTPA realization.

Fig.11 shows performance of the proposed MTPA in comparison with sinusoidal injection under different loading conditions equal to 2.2, 4.4 and 6.6 Nm. The produced torque and corresponding MTPA criterion in each load is presented in Fig.11 (a) and Fig.11 (b), respectively. When MTPA is applied, torque ripple is reduced effectively and MTPA criterion is forced to zero, that guarantees minimum copper loss.

IV. CONCLUSIONS

A new MTPA control strategy based on Lagrange’s Theorem has been developed and validated for BLDCM. The optimized dq-frame current waveforms of the proposed MTPA are applied as the reference values and the motor winding currents are forced to track them employing the IOFL method. Both analytical study and experimental results have revealed that applying the proposed MTPA strategy would result in both torque ripple and current magnitude minimizations simultaneously, especially under light loading conditions.

REFERENCES