Travel time computations using a compact eikonal equation for vertical transverse isotropic media

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ABSTRACT

Eikonal solvers often have stability problems if the velocity model is mildly heterogeneous. We derive a stable and compact form of the eikonal equation for P-wave propagation in vertical transverse isotropic media. The obtained formulation is more compact than other formulations and therefore computationally attractive. We implemented ray shooting for this new equation through a Hamiltonian formalism. Ray tracing based on this new equation is tested on both simple as well as more realistic mildly heterogeneous velocity models. We show through examples that the new equation gives travel times that coincide with the travel time picks from wave equation modelling for anisotropic wave propagation.

Key words: Anisotropy, Numerical modelling, Ray tracing.

INTRODUCTION

Over the past few decades, the growing need for fast and accurate prediction of waveform attributes (especially the travel time) in heterogeneous 2D and 3D media has spawned a prolific number of grid- and ray-based solvers. It is important to have an efficient travel time solver as they form the backbone of pre-stack depth migration methods such as pre-stack Kirchoff depth migration (Alkhalifah 2011) and pre-stack Gaussian beam depth migration (Hill 2001).

Traditionally, the method of choice has been ray tracing (Julian and Gubbins 1977; Cerveny 1987; Virieux and Farra 1991; Cerveny 2001), in which the trajectory of paths corresponding to wavefront normals are computed between two points. This approach is often highly accurate and efficient, and naturally lends itself to the prediction of various seismic wave properties. However, it is not always robust, and may fail to converge to a true two-point path even in mildly heterogeneous media.

Grid-based schemes, which usually involve the calculation of travel times to all points of a regular grid that spans the velocity medium, have become increasingly popular in recent times. They are often based on finite difference solution of the eikonal equation (Kim and Cook 1999; Popovici and Sethian 2002; Rawlinson and Sambridge 2004; Fomel, Luo and Zhao 2009; Luo and Qian 2012; Waheed, Alkhalifah and Wang 2015) or shortest path (network) methods (Nakanishi and Yamaguchi 1986; Moser 1991; Cheng and House 1996), both of which tend to be computationally efficient and highly robust, which makes them viable alternatives to ray tracing. Wavefronts and rays can be obtained a posteriori if required by either contouring the travel time field or following the travel time gradient from receiver to source.

In the ray-shooting method, which was popular in the 1990s and early 2000s, in the presence of smooth velocity variations, the kinematic ray tracing equations provide the desirable solution, but as soon as complex velocity heterogeneity is introduced, ray bending distorts the calculation to become complicated or in some cases virtually impossible to solve (Cerveny 1987; Sambridge and Kennett 1990; Virieux and Farra 1991; Velis and Ulrych 1996, 2001; Rawlinson and Sambridge 2004; Waheed et al. 2013). Given the potential pitfalls of using an iterative non-linear solver in two point shooting methods, it would appear that fully non-linear solvers would be at least worthy of investigation. However, there are relatively few examples in the recent literature,
Figure 1 Geometrical interpretation of the angles $\alpha_h$ and $\alpha_z$ appeared in the proposed eikonal equation.

perhaps due to the recent proliferation of grid-based and wavefront construction type schemes that are designed to overcome these limitations. In this paper, we observe that forming a Hamiltonian system (ray-shooting method) for our newly modified form of the eikonal equation would prevent the rays to bend severely, while it preserves the travel time values as accurate as the full PDE finite difference solver for the wave propagation. This can potentially revive the field of ray-shooting method that has practically abandoned for many years.

All ray- and grid-based methods are subject to the so-called high frequency approximation, that is the wavelength of the propagating wave is substantially shorter than scale of the seismic heterogeneities that characterize the medium through which the waves pass. Under this assumption, Alkhalifah (1998, 2000) proposed an approximate P-wave eikonal equation for VTI media. The eikonal equation is derived from a fourth-order anisotropic P-wave propagation equation that combines the computational efficiency of acoustic wave propagation with the accuracy of anisotropic P-wave propagation (Alkhalifah 2000). Further, Zhou, Zhang and Bloor (2006) proposed two coupled second-order wave equations for the same propagation behaviour.

This paper is organized as follows: we first derive the eikonal and transport equations from Zhou et al. (2006) for VTI wave propagation equations and obtain an algebraically similar form to the eikonal equation of Alkhalifah (2000). We then represent a compact form of this equation using some trigonometric functions. We use ray shooting to solve our

```plaintext
1: Assign initial slowness: $p_x \leftarrow \cos(\psi) / v_h$
   $p_z \leftarrow \cos(\psi) / v_z$

2: for $i \leftarrow 1$ : $nsteps$ do
3:    $\alpha_h \leftarrow \text{sign}(v_h p_h) \cos^{-1}(v_h p_h)$
    $\alpha_z \leftarrow \text{sign}(v_z p_z) \cos^{-1}(v_z p_z)$
4:    if $(\psi \leq 45^\circ)$ then
5:        Assign base step: $\Delta s_b$
6:        Normalize $\Delta s$: $\Delta s \leftarrow \Delta s_b \cos^2 \alpha_h \sin \alpha_h / v_h^2$
7:        Update $(p_x, p_z, x, z, \text{traveltime})$ according to Eq.16
8:    if $(\psi > 45^\circ)$ then
9:        Assign base step: $\Delta s_b$
10:       Normalize $\Delta s$: $\Delta s \leftarrow \Delta s_b \cos^2 \alpha_z \sin \alpha_z / v_z^2$
11:       Update $(p_x, p_z, x, z, \text{traveltime})$ according to Eq.17
12: end for
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compact form of the eikonal equation and compare the travel time results with travel times obtained by finite difference solver for PDEs for VTI wave propagation (Zhou et al. 2006). Our results show that with the ray shooting solution for our new compact form of eikonal equation, we not only avoid the ray bending for heterogeneous velocity models, but also obtain reasonably accurate travel time results. At the end, we discuss some limitations that we face in the implementation of the solver for the compact form of the eikonal equation and possible solutions for these problems.

THEORY

In the formulation of Zhou et al. (2006), the propagation of P-waves through a VTI medium is described by

\[
\frac{\partial^2 \hat{p}}{\partial t^2} = v_p^2 (1 + 2\delta) \nabla^2 \hat{q} (\hat{p} + \hat{q}) + v_p^2 \frac{\partial^2 \hat{p}}{\partial z^2},
\]

\[
\frac{\partial^2 \hat{q}}{\partial t^2} = 2v_p^2 (\epsilon - \delta) \nabla^2 \hat{q} (\hat{p} + \hat{q}),
\]  

(1)

Here, \( \hat{p} = \hat{p}(x, t) \) denotes the wavefield, \( \epsilon \) and \( \delta \) are the Thomsen parameters describing the anisotropic medium, \( v_p \) is the velocity in the vertical direction and \( \nabla^2_b = \hat{\alpha}_x^2 + \hat{\alpha}_y^2 \). The auxiliary function \( \hat{q} = \hat{q}(x, t) \) was used to reduce the fourth-order VTI equation given by Alkhalifah (2000), to the above couple of equations, it should be easier to implement and solve.

A compact form of eikonal equation

In this subsection, we derive an eikonal equation for equation (1). For this purpose, we apply a change of variables \( (p = \hat{p}, q = \hat{p} + \hat{q}) \). This gives

\[
\frac{\partial^2 p}{\partial t^2} = v_n^2 \nabla^2 q + v_p^2 \frac{\partial^2 p}{\partial z^2},
\]

\[
\frac{\partial^2 q}{\partial t^2} = 2v_p^2 (\epsilon - \delta) \nabla^2 q + v_p^2 \frac{\partial^2 p}{\partial z^2},
\]  

(2)

with \( v_n^2 = v_p^2 (1 + 2\delta) \) and \( v_p^2 = v_p^2 (1 + 2\epsilon) \). We now insert the high frequency approximations

\[
p(x) = A_p(x) e^{-i\omega T_p(x)},
\]

\[
q(x) = A_q(x) e^{-i\omega T_q(x)},
\]  

(3)

in equations (2). Here, the amplitude functions \( A_p,q \) and travel time functions \( T_p,q \) are smooth scalar functions of \( x \) and ...
Because $p$ and $q$ in the wave equation (2) are the components of the same wavefield, in this case, $p$ and $q$ in equation (2) have the same phase. As a result, $T_p$ and $T_q$ are identical to each other ($T_p = T_q = T$).

Inserting equation (3) into (2), after going to the frequency domain and using some algebra gives the following homogeneous system of equations:

$$
\begin{pmatrix}
\omega / 2
& 0 & 0 & 0 \\
0 & \omega / 2
& 0 & 0 \\
0 & 0 & \omega / 2
& 0 \\
0 & 0 & 0 & \omega / 2
\end{pmatrix}
\begin{pmatrix}
A_1 - i \omega B_1 - \omega^2 C_1 \\
A_2 - i \omega B_2 - \omega^2 C_2 \\
A_3 - i \omega B_3 - \omega^2 C_3 \\
A_4 - i \omega B_4 - \omega^2 C_4
\end{pmatrix}
\begin{pmatrix}
e^{-i \omega T(x)} \\
e^{-i \omega T(x)}
\end{pmatrix}
= 0
$$

with

$$
A_1 = v_p^2 \frac{\partial}{\partial z} A_p, \quad B_1 = v_p^2 \left( A_p \frac{\partial T}{\partial z} + 2 \frac{\partial A_p}{\partial z} \frac{\partial T}{\partial z} \right),
$$

$$
C_1 = v_p^2 A_p \left( v_p^{-2} - \left( \frac{\partial T}{\partial z} \right)^2 \right),
$$

$$
A_2 = v_n \nabla^2_h A_q,
$$

$$
B_2 = v_n^2 \left( A_q \nabla^2_h T + 2 \left( \frac{\partial A_q}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial A_q}{\partial y} \frac{\partial T}{\partial y} \right) \right),
$$

$$
C_2 = v_n^2 A_q \left( \left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2 \right),
$$

$$
A_3 = A_1, \quad B_3 = B_1, \quad C_3 = v_p^2 A_p \left( \frac{\partial T}{\partial z} \right)^2,
$$

$$
A_4 = v_n^2 A_p \nabla^2_h A_q, \quad B_4 = v_n^2 \left( A_q \nabla^2_h T + 2 \left( \frac{\partial A_q}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial A_q}{\partial y} \frac{\partial T}{\partial y} \right) \right),
$$

$$
C_4 = v_n^2 A_q \left( v_n^{-2} - \left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2 \right).
$$

Non-trivial solutions for equation (4) exist if

$$
\begin{vmatrix}
A_1 - i \omega B_1 - \omega^2 C_1 \\
A_2 - i \omega B_2 - \omega^2 C_2 \\
A_3 - i \omega B_3 - \omega^2 C_3 \\
A_4 - i \omega B_4 - \omega^2 C_4
\end{vmatrix} = 0.
$$
or, equivalently, if

$$Q_0 + Q_1 \omega + Q_2 \omega^2 + Q_3 \omega^3 + Q_4 \omega^4 = 0. \quad (7)$$

Here, the $Q_j$ (for $j = 1 \ldots 4$) are functions of $A_i$, $B_i$, and $C_i$ (with $i = 1 \ldots 4$). Since this equation should be satisfied for any frequency $\omega$, the expressions $Q_j$ must vanish. For high frequencies the most important term is that of $\omega^4$: $Q_4$. Setting

Figure 6 Anisotropic constant gradient velocity model (a), computed ray paths (b), travel time (c).

Figure 7 Isotropic random velocity model (a), computed ray paths (b), travel time (c).
\[ Q_4 = 0 \] gives the eikonal equation for the VTI medium

\[

v_b^2 \left( \frac{\partial T}{\partial x} \right)^2 - \left( \frac{\partial T}{\partial y} \right)^2 \left( \frac{\partial T}{\partial z} \right)^2 \right) \left( v_p^2 - \left( \frac{\partial T}{\partial z} \right)^2 \right) = \eta \left( \left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2 \right) \left( \frac{\partial T}{\partial z} \right)^2 .
\]

(9)

Using \( p = \nabla T \) this can be written as

\[
(v_b^2 - p_x^2 - p_y^2) (v_p^2 - p_z^2) = \eta \left( p_x^2 + p_y^2 \right) p_z^2,
\]

(10)

where

\[
\eta = \frac{1 + 2\delta}{1 + 2\epsilon}.
\]

(11)

It should be noted that this equation can also be obtained directly from the eikonal equation of Alkhalifah (2000), Stovas and Alkhalifah (2013) and Hao and Stovas (2015). The transport equation is obtained by setting \( Q_3 = 0 \). This gives

\[
2 \frac{\partial A}{\partial x} \frac{\partial T}{\partial x} + 2 \frac{\partial A}{\partial y} \frac{\partial T}{\partial y} + A \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \times \left( v_p^2 - (1 + \eta) \frac{\partial T}{\partial z} \right) + \left( 2 \frac{\partial A}{\partial z} \frac{\partial T}{\partial z} + A \frac{\partial^2 T}{\partial z^2} \right) \times \left( v_b^2 - (1 + \eta) \left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2 \right) = 0.
\]

(12)

Using the definition of \( \nabla T \), and by defining \( \frac{\partial f}{\partial x} = f_x \), with \( f = T \) or \( f = A \), and similarly for the other partial derivatives, this can be rewritten as

\[
(2 A_x p_x + 2 A_y p_y + A \nabla^2_z T) \left( v_p^2 - (1 + \eta) p_z^2 \right) + (2 A_z p_z + A \nabla^2_x T) \left( v_b^2 - (1 + \eta) \left( p_x^2 + p_y^2 \right) \right) = 0.
\]

(13)

Ray tracing

The eikonal equation (10) is equivalent to that of equation (A2) in Alkhalifah (2000). We can represent this formulation in a more compact form with an interesting geometrical interpretation. For this, we divide both sides of equation (10) by \( p_b^2 p_z^2 \) (with \( p_b^2 = p_x^2 + p_y^2 \)) and define angle \( \alpha_b \) by

\[
\cos \alpha_b = v_b p_b.
\]

(14)

Similarly, angle \( \alpha_z \) is defined by

\[
\sin \alpha_z = v_p p_z.
\]

(15)
The eikonal equation, equation (10), now takes the simple form:

\[ \tan \alpha_h = \eta \tan \alpha_z \]

or, equivalently,

\[ \cot \alpha_z = \eta \cot \alpha_h \]

with

\[ \alpha_h = \text{sign}(v_h, p_h) \cos^{-1}(v_h, p_h) \]

and

\[ \alpha_z = \text{sign}(v_p, p_z) \sin^{-1} v_p, p_z \]

where \( \text{sign}(.) \) stands for the sign function and the angles \( \alpha_h \) and \( \alpha_z \) are defined on \([0, 2\pi]\). A geometrical interpretation of equations (16) and (17) is shown in Fig. 1. \( \alpha_h \) and \( \alpha_z \) can be interpreted as the angles between the vectors \( p_{iso_v}^{h} \) and \( p_{iso_v}^{p} \), respectively (see Fig. 1). These two slowness vectors are the slowness vectors if the media is assumed to be isotropic with velocity \( v_h \) and \( v_p \), respectively. The projection of \( p_{iso_v}^{h} \) on the \( x - y \) plane and the \( z \)-component of \( p_{iso_v}^{p} \) are the components of the anisotropic slowness vector \( p \) (see Fig. 1). In the case of isotropy, we have \( \alpha_h = \alpha_z \) and the slowness vectors \( p_{iso_v}^{h} \), \( p_{iso_v}^{p} \) and \( p \) coincide.

Because of the singularities that occur at \( \pi/2 \) and \( 3\pi/2 \) in the case of equation (16) and at \( 0 \) and \( 2\pi \) in the case of equation (17), it is recommended that equation (16) be used in the intervals \([0, \pi/4]\) and \([3\pi/4, 7\pi/4]\) and equation (17) be used for the intervals \([\pi/4, 3\pi/4]\) and \([7\pi/4, 2\pi]\). The nonlinear partial differential equations (10) (or (16) and (17)) can be converted to a set of ordinary differential equations using the method of characteristics (Courant and Hilbert 1962; Kravtsov and Orlov 1980).

Ray equations

In the previous subsection two eikonal equations were derived. We use the method of characteristics, using the Hamiltonian formulation, to derive the corresponding ray equations. For the eikonal equation (10), we define the Hamiltonian \( H \) as
Figure 11 Synthetic seismograms for the source–receiver configuration shown in Fig. (10) with the Ricker wavelet as source. The solid black line indicates the travel time calculated by solving the eikonal equation presented in this paper.

\[ H \equiv Q_t. \] We insert this in the general Hamiltonian equations

\[
\frac{dx}{ds} = \frac{\partial H}{\partial p}, \quad \frac{dp}{ds} = -\frac{\partial H}{\partial x},
\]

where \( s \) is the parametric value in Hamiltonian system. Depending on the Hamiltonian system \( s \) can be unitless or with unit (such as when it represent an arclength). In our Hamiltonian derivation we found that \( s \) has units of \( s^3/m^2 \).

The initial conditions are \( x(0) = x_s \), with \( x_s \) the source location and \( p(0) = p_s \), with \( p_s \) the initial slowness vector (typically parametrized by two take-off angles). The travel time can be computed using

\[
\frac{d\tau}{ds} = \frac{1}{\cos \alpha_h \sin \alpha_h v^2},
\]

with

\[
\hat{p} = (v^2 p_x, v^2 p_y, v^2 p_z),
\]

\[
g_1 = \frac{1}{\cos^3 \alpha_b \sin^2 \alpha_b v^2},
\]

\[
g_2 = -\frac{1}{\cos \alpha_b \sin \alpha_b v^{-1}},
\]

\[
g_3 = -\eta \sin \alpha_z v^{-1}
\]

and

\[
g_4 = -\tan \alpha_z.
\]

The initial conditions are the same as in the previous case. The travel time can be found using

\[
\frac{d\tau}{ds} = \frac{1}{\cos \alpha_h \sin \alpha_h + \eta \sin \alpha_z \cos \alpha_z}.
\]

If we use \( H = \cot \alpha_z - \eta \cot^3 \alpha_b \), then the ray tracing equations become

\[
\frac{dx}{ds} = g_1 \hat{p}
\]

and

\[
\frac{dp}{ds} = g_2 \nabla v_h - g_3 \nabla p - g_4 \nabla \eta
\]

with

\[
\hat{p} = (v^2 p_x, v^2 p_y, v^2 p_z),
\]

\[
g_1 = \eta \frac{1}{\sin^3 \alpha_b \cos \alpha_b v^2},
\]

\[
g_2 = \frac{1}{\sin^3 \alpha_b \sin \alpha_b v^2}
\]

\[
g_3 = -\eta \sin \alpha_z \cos \alpha_z v^{-1}
\]

and

\[
g_4 = -\cot \alpha_b.
\]

It is clear that the first set of ray equations is considerably more complicated than the latter two sets. Therefore, the latter two sets of ray equations should improve the computational efficiency.
NUMERICAL MODELLING

In this section, we implement the ray tracing using equations for the first quadrant \([0, \pi/2]\). As mentioned in the previous section, there are singularities at \(\pi/2\), in case the Hamiltonian is expressed in terms of the tangent function, and 0 in case the Hamiltonian is expressed in terms of the cotangent function. We therefore use both formulations: for angles between 0° up to 45°, we use the tangent formulation (equation (16)) and for angles between 45° and 90°, we use the cotangent formulation (equation (17)). Figure 2 shows the algorithm for hybrid ray tracing.

Ray tracing in VTI media

We show the ray tracing results for five different 2D models. In all five cases, we also show the travel time calculated using an eighth order in space and second order in time finite difference solver of equation (1) for comparison. Travel times are calculated by picking the time of peak value for the receiver waveforms in synthetic seismograms. The finite difference solver is eighth order in space and second order in time. The model is defined on \(\Omega\) with

\[
\Omega = \{(x, z) \in \mathbb{R}^2 | 0 \leq x, z \leq 4000\},
\]

with units in \(m\). The grid spacing is 20 m in all directions. A Ricker wavelet with a dominant frequency of 10 Hz is used. The source is located at one of the corners of the model. The receivers are located on a square grid. Figure 3 shows the model and acquisition geometry.

Five different models are used in the ray tracing and finite difference modelling:

1. \(\Omega_1\): a homogeneous isotropic medium, with \(v_p = 3000\) m/s, \(\epsilon = \delta = 0\);
2. \(\Omega_2\): a homogeneous VTI medium, with \(v_p = 3000\) m/s, \(\epsilon = 0.2\) and \(\delta = 0.1\);
3. \(\Omega_3\): a vertically varying VTI medium, with \(v_p = 1500 + z\), \(\epsilon = 0.2\) and \(\delta = 0.1\);
4. \(\Omega_4\): an isotropic medium with random heterogeneity, \(\epsilon = 0\) and \(\delta = 0\);
5. \(\Omega_5\): a VTI medium with random heterogeneity, \(\epsilon = 0.2\) and \(\delta = 0.1\).

Figures 4–8 show the models as well as the ray tracing and modelling results. Figures 4(b)–8(b) in particular show the ray paths and Figs 4(c)–8(c) show the travel times. It can be seen from these figures that the ray paths mildly bend in the linearly increasing velocity model (Fig. 6(b)) and that they get bent by the low- and high-velocity regions (Fig. 8(c)) as expected.

Figure 9 shows the travel times at the nine receiver locations obtained by the above ray tracing procedure. These are compared to the travel times computed using the finite difference modelling.

Overthrust model

In order to test the fidelity of the new equation, we examine the solution of the compact form eikonal equation on the smooth SEG/EAGE overthrust model (Aminzadeh, Jean and Kunz 1997) shown in Fig. 10. We assume that the value of the Thomsen parameters is constant in the entire model with \(\epsilon = 0.3\) and \(\delta = 0.05\). We increase the anisotropy for this model, as compared to the previous test models, and use a higher value of \(\epsilon\) and a lower value of \(\delta\). We perform this test to evaluate whether our methodology still works in this case (\(\eta = \frac{\epsilon^2 - \delta^2}{1 + 2\delta} = 0.23\)). The source is located at (250, 2500) m and a set of 16 receivers has been used to record the wavefield as shown in Fig. 10. Figure 11 shows the synthetic seismogram that is created by solving equation (1), using the finite difference method (eighth order in space and second order in time) with a Ricker wavelet as source function. The results from the travel time calculation based on equation (16) are shown using a black solid line. As can be seen, the solution of the compact eikonal equation is still quite accurate even in the case of a relatively high effective anisotropy.

LIMITATIONS

We observe two limitations in the numerical implementation of the compact eikonal equation formulation, which should be mentioned. First, in the hybrid ray tracing algorithm implementation when switching from tangent formulation (16) to the cotangent formulation (17) at a 45° shooting angle, we see a slight gap between the rays with adjacent shooting angles as can be seen in Figs 6(b), 7(b) and 8(a). We do not know what the exact cause of this gap is, but we do not see any disturbance in the travel time map when switching from one form to other. We can overcome this problem with a simple interpolation (Figs 6(c) and 7(c)).

The other limitation is more severe. On some occasions when the rays approach 0°, 90°, 180°, 270° angles, they become unstable (behave erratically, e.g. the fourth ray in Fig. 6(b)) or their length shorten significantly (on approaching 90°; the Figs 6(b), 7(b) and 8(a)). We should be able to solve the latter problem by increasing the step length exponentially when shooting angle approaches the 90°. A solution for former problem could be to locally rotate the
CONCLUSIONS

In this paper, a compact form of eikonal equation is derived for the coupled system of second-order partial differential equations that approximate the propagation of P waves through an anisotropic medium. These eikonal equations give rise to new ray tracing equations. It is shown that for a particular choice of Hamiltonian, the ray equations turn out to be relatively simple. They are therefore the basis for efficient numerical computation of travel times. Several examples of the computations of travel times are given. These examples include simple velocity models as well as a smoothed version of the SEG/EAGE overthrust model. The ray-shooting implementation of our proposed method shows a mild ray-bending effect comparing with historical reports of ray shooting for both isotropic and anisotropic media with strong velocity heterogeneity. Our numerical calculations confirm that there is good agreement between the computed travel times using our eikonal equation and the travel time estimated using full finite difference PDE solver for VTI heterogeneous velocity models.

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