Decentralised sliding mode control of RL-derivative based fractional-order large-scale nonlinear systems

Sajjad Shoja-Majidabad*,
Heydar Toosian-Shandiz and Amin Hajizadeh

Faculty of Electrical Engineering,
Shahrood University of Technology,
Semnan, Shahrood, Iran
Email: shoja.sajjad@gmail.com
Email: aminhajizadeh@shahroodut.ac.ir
Email: htshandiz@shahroodut.ac.ir
*Corresponding author

Abstract: The objective of this paper is to design decentralised sliding mode controllers for fractional-order large-scale nonlinear systems. In the first step, a fully-decentralised fractional-order sliding mode controller with a novel integral sliding manifold is developed. Practical stability of the closed-loop system is fulfilled under the assumption that the interconnections among the subsystems are bounded with known upper bounds. However, in reality the uncertainties and interconnections upper bounds are unknown. Therefore in the next step, an adaptive-fuzzy structure is applied to approximate the interactions and uncertainties. Since the states of neighbour subsystems are considered as the fuzzy system inputs, this technique is known as semi-decentralised control strategy. Due to using the fractional integral sliding surface, the zero convergence of the sliding manifold has been analysed based on integer-order stability theorems. In addition, system tracking errors convergence is deduced from fractional-order linear stability theorems. Computer simulations present the performance of the suggested controllers in the presence of uncertainties and interconnections.

Keywords: fractional-order large-scale nonlinear systems; fractional-order sliding mode control; FOSMC; fully-decentralised control; semi-decentralised control; adaptive-fuzzy approximator.

Reference to this paper should be made as follows: Shoja-Majidabad, S., Toosian-Shandiz, H. and Hajizadeh, A. (2015) 'Decentralised sliding mode control of RL-derivative based fractional-order large-scale nonlinear systems', *Int. J. Systems, Control and Communications*, Vol. 6, No. 3, pp.181–197.

Biographical notes: Sajjad Shoja-Majidabad is a PhD student in the Faculty of Electrical Engineering at the Shahrood University of Technology. His research interests include: fractional control and stability, power system analysis and control, robust control, and intelligent computing.

Heydar Toosian-Shandiz received his PhD from UMIST University. He is an Associate Professor in the Shahrood University of Technology. His research interests include: fractional calculus, industrial automation and control, intelligent computing, digital signal processing (DSP), systems identification, adaptive systems and information technology.

Amin Hajizadeh received his BSc degree from Ferdowsi University of Mashahd, Mashad, Iran in 2002, MSc and PhD degree in Electrical Engineering from the K.N. Toosi University of Technology, Tehran, Iran in 2005 and 2010. During Feb. 2009 until Aug. 2009, he worked as a Visiting PhD student with the Department of Electrical Power Engineering at the Norwegian University of Science and Technology (NTNU), Trondheim, Norway. Currently, he is an Assistant Professor at the Shahrood University of Technology in Shahrood, Iran. His current research interests include: control of distributed energy resources and grid-connected operation of hybrid power systems and distributed generation.

1 Introduction

Large-scale nonlinear systems are often composed of multiple low-dimensional subsystems, which are interconnected. Such dynamical systems are widely employed in industries like; electric power systems (Guo et al., 2000), chemical processes (Al-Gherwi et al., 2010), robotic manipulators (Zhu and Li, 2010), etc. These systems complexities lie in high nonlinearity, large dimensions and interconnections among the subsystems, which make the centralised control strategy computationally burden or hard to implement. Moreover, when the centralised controller fails, the entire system becomes out of control. In contrast, fully-decentralised control can be designed for local subsystems from local data with less computational efforts. But the fully-decentralised control may not provide pleasant performance and may not even guarantee system stability for systems with unknown interconnections. On the other side, the advancement of distributed control systems (DCS), fieldbus, and communication technologies in industry allows the engineers to introduce semi-decentralised and distributed control strategies as new control methodologies (Yousef et al., 2010; Stewart et al., 2011). The semi-decentralised structure means that the i^{th} subsystem's controller depends not only on the i^{th} subsystem variables, but also on neighbour subsystems variables. From cable connections and computation point of views, this technique is a bridge between the centralised and fully-decentralised control methods.

One of the dominant challenges in fully-decentralised and semi-decentralised techniques is to develop some robust methods for dealing with the interconnections and consequently global system stability. In Zhang and Feng (1997), Huang and Zhou (2010), Li et al. (2011), Yousef et al. (2006), Yan et al. (2003), Shyu et al. (2003), Chou and Cheng (2003), Cheng and Chang (2008), Da (2000) and Lin and Wang (2010), different decentralised control methods have been reported. In some of these studies, intelligent methods like adaptive-fuzzy (Zhang and Feng, 1997; Huang and Zhou, 2010) and adaptive-neural controllers (Li et al., 2011) are used to cope with the interconnections and nonlinearities. Yousef et al. (2010, 2006) have suggested a semi-decentralised technique based on direct and indirect adaptive-fuzzy techniques. In recent two decades, the sliding mode control (SMC) has been used in large-scale systems control (Yan et al., 2003; Shyu et al., 2003; Chou and Cheng, 2003; Cheng and Chang, 2008) due to its high precision and robust behaviour against model uncertainties and interactions. These literatures often assume that the interconnections are bounded by first-order or higher-order polynomials of states. However, some physical systems do not satisfy these conditions or finding such conditions is challenging. In Zhu and Li (2010), Da (2000) and

Lin and Wang (2010) some combinations of intelligent techniques and SMC have been reported to manage the mentioned problem. It is worthwhile to notify that the whole mentioned discussions on SMC of large-scale systems are developed based on integer-order (IO) calculus.

Fractional calculus is an old mathematical branch with a generalisation of ordinary differentiation-integration to an arbitrary order. Nearly 300 years, this field was viewed as an only theoretical topic with no practical applications (Podlubny, 1999). But in last three decades, it has been used in different branches of engineering and physics such as: reaction-diffusion system (Gafiychuk et al., 2008), electrical circuits (Luo et al., 2011), rotor-bearing system (Cao et al., 2011), finance system (Laskin, 2000), biological system (Petras and Magin, 2011), thermoelectric system (Ezzat, 2011), and finally designing fractional-order (FO) controllers on dynamical systems is a prominent case of these applications. Fractional-order sliding mode control (FOSMC) is a famous one of these FO controllers.

Recently, various forms of FOSMC have been used to control FO nonlinear systems especially the chaotic systems (Tavazoei and Haeri 2008; Hosseinnia et al., 2010; Aghababa, 2012, 2013a, 2013b; Chen et al., 2013; Binazadeh and Shafiei, 2013; Dadras and Momeni, 2013; Wang, 2013). In Tavazoei and Haeri (2008) and Hosseinnia et al. (2010), the FOSMC with a simple linear sliding manifold has been reported. Aghababa (2012, 2013a) has developed this method base on terminal sliding surfaces. To remove the chattering of FOSMC a non-chatter sliding manifold proposed in Aghababa (2013b), and a second-order structure is suggested in Chen et al. (2013). The sliding mode technique is designed for output tracking of a time-varying reference signal for FO nonlinear systems in Binazadeh and Shafiei (2013). In Dadras and Momeni (2013), a passivity-based integral sliding mode controller is considered. Also, Wang (2013) has tried to apply a backstepping SMC for uncertain chaotic systems. Most of the above literatures are common in the following cases:

1 Designing the FOSMC for small-scale systems: The main difficulty in the large-scale systems control in comparison with the small-scale ones, is the interconnections which makes the control process challenging. To the author's knowledge, there are few works on control of FO large-scale systems. Recently, robust decentralised control of FO large-scale linear systems is reported in Li et al. (2013). While most of physical systems have nonlinear dynamics and encountering with nonlinearities is a delicate matter. Therefore, the first novelty associated with this paper is developing a fully-decentralised FOSMC technique on fractional-order large-scale nonlinear systems with known uncertainty and interconnection bounds.

2 Applying the FOSMC with known uncertainty bounds. In the other word, lack of adaptive or adaptive-fuzzy structures for unknown uncertainty approximation in the proposed controllers is apparent. However, in practical point of view the uncertainties upper bounds are unknown. Based on our knowledge, there are few literatures on the adaptive and adaptive-fuzzy SMC of FO nonlinear systems with unknown uncertainties. In Lin and Lee (2011) and Lin et al. (2011), authors have proposed an adaptive-fuzzy sliding mode controller for synchronisation of FO chaotic systems. However, the final result of their work is questioned by Tavazoei (2012), because they were careless about some properties of FO calculus. In Wang et al. (2012) and Yin et al. (2013), two adaptive sliding mode controllers are constructed to facilitate the stability of systems with unknown uncertainties.

However, the presented methods are employed on simple FO small-scale systems. The main problem will arise when the system is large-scale with unknown interconnections. Hence, an algorithm is needed to approximate both uncertainty and interconnection. Therefore, the second novelty associated with this paper is employing the semi-decentralised FOSMC technique on fractional-order large-scale nonlinear systems with uncertainty and interconnection approximation.

The remainder of this paper is organised as follows: some preliminaries of fractional calculus are expressed in Section 2. In Section 3, FO large-scale nonlinear system model is introduced. A fully-decentralised FOSMC are developed in Section 4. Section 5 describes a semi-decentralised FOSMC strategy. An illustrative example is provided to approve the theoretical results in Section 6, and finally, conclusions are given in Section 7.

2 Preliminaries

In this section, some basic definitions, remarks and theorems of fractional calculus are expressed.

Definition 1 (Hewitt and Stromberg, 1963): The function $f(t)$: $R \rightarrow R$ is called C^k-class if its derivatives $f^{(1)}, f^{(2)}, \dots, f^{(k)}$ exist and be continuous (except for a finite number of points).

Based on this definition, $f(t) \in C^0$, C^1 are the classes of all continuous and continuously differentiable functions.

Definition 2 (Li and Deng, 2007): The α^{th} order Riemann-Liouville (RL) fractional integration of function $f(t)$ with respect to t is given by

$$I_{0,t}^{\alpha} f(t) = D_{0,t}^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t \frac{f(\tau)}{(t-\tau)^{1-\alpha}} d\tau \tag{1}$$

where $\Gamma(\cdot)$ is the Gamma function.

Definition 3 (Li and Deng, 2007): The Grunwald-Letnikov (GL) fractional derivative of function $f(t)$ with fractional order α is defined as

$${}_{GL}D_{0,t}^{\alpha} f(t) = \sum_{k=0}^{m-1} \frac{f^{(k)}(0) t^{-\alpha+k}}{\Gamma(-\alpha+k+1)} + \frac{1}{\Gamma(m-\alpha)} \int_0^t \frac{f^{(m)}(\tau)}{(t-\tau)^{1-m+\alpha}} d\tau \tag{2}$$

where $f(t) \in C^m[0, t]$ and $m - 1 \leq \alpha < m$, $m \in N$.

Definition 4 (Li and Deng, 2007): The RL fractional derivative of function $f(t)$ with fractional order α is defined as

$${}_{RL}D_{0,t}^{\alpha} f(t) = D^m D_{0,t}^{-(m-\alpha)} f(t) = \frac{1}{\Gamma(m-\alpha)} \frac{d^m}{dt^m} \int_0^t \frac{f(\tau)}{(t-\tau)^{1-m+\alpha}} d\tau \tag{3}$$

where $m - \leq \alpha < m$, $m \in N$.

Property 1 (Li and Deng, 2007): If $f(t) \in C^m[0, t]$ then ${}_{RL}D_{0,t}^{\alpha} f(t) = {}_{GL}D_{0,t}^{\alpha} f(t)$.

Theorem 1 (Qian et al., 2010): The autonomous linear system (4) with RL derivative, fractional order $0 < \alpha < 1$ and initial value $z_0 = {}_{RL}D_{0,t}^{\alpha-1} z(t)|_{t=0}$, is asymptotically stable if and only if all the non-zero eigenvalues of matrix A satisfy $|\arg(\lambda_i(A))| > \frac{\alpha\pi}{2}$.

$$ {}_{RL}D_{0,t}^{\alpha} z(t) = Az(t) \tag{4} $$

where $z(t) \in R^{n\times 1}$ is state vector, and λ_i is the i^{th} eigenvalue of the matrix $A \in R^{n\times n}$.

3 FO large-scale system dynamic

Consider a class of FO large-scale nonlinear system composed of N interconnected subsystems (S_i). All of the subsystems S_i can be described based on RL derivative as ($i = 1, 2,\ldots,N$):

$$ S_i : \begin{cases} {}_{RL}D^{\alpha} x_{i1}(t) = x_{i2}(t) \\ {}_{RL}D^{\alpha} x_{i2}(t) = x_{i3}(t) \\ \vdots \\ {}_{RL}D^{\alpha} x_{in}(t) = f_i(X_i) + g_i(X_i) u_i(t) + M_i(X_i, t) + I_i(X_1, \cdots, X_N, t) \end{cases} \tag{5} $$

where $\alpha \in (0, 1)$ is the order of system, $X_i = [x_{i1}, x_{i2},\ldots,x_{in}]^T$ is the state vector of i^{th} subsystem, $u_i \in R$ is the input, f_i: $R^n \to R$ and g_i: $R^n \to R$ are known functions ($g_i(X_i) \neq 0$), M_1: $R^{n+1} \to R$ is model uncertainty and external disturbance and I_i: $R^{n\times N+1} \to R$ represents the interconnection between the i^{th} subsystem and other subsystems. We consider $L_i(X, t) = M_i(X_i, t) + I_i(X_1,\ldots,X_N, t)$ which is called the lumped uncertainty. Where $X = [X_1^T, X_2^T, \ldots, X_N^T]^T$ is the global state vector.

Assumption 1: Full state vectors of the system are measurable.

By defining the tracking errors of the i^{th} subsystem as $e_{i1}(t) = x_{i1}(t) - x_{i1d}(t),\ldots,e_{in}(t) = x_{in}(t) - x_{ind}(t)$, the error dynamics of (5) will be in the following form:

$$ S_i : \begin{cases} {}_{RL}D^{\alpha} e_{i1}(t) = e_{i2}(t) \\ {}_{RL}D^{\alpha} e_{i2}(t) = e_{i3}(t) \\ \vdots \\ {}_{RL}D^{\alpha} e_{in}(t) = f_i(X_i) + g_i(X_i) u_i(t) + L_i(X, t) - {}_{RL}D^{\alpha} x_{ind}(t) \end{cases} \tag{6} $$

where sets $X_{id} = [x_{i1d}, x_{i2d},\ldots,x_{ind}]^T$ and $E_i = [e_{i1}, e_{i2},\ldots,e_{in}]^T$ are reference vector and tracking error vector of i^{th} subsystem, respectively, and the large-scale system error vector is given by $E = [E_1^T, E_2^T, \ldots, E_N^T]^T$. The goal is to design robust controllers for the FO systems (5) such that the state vectors $X_i(t)$, $i = 1, 2,\ldots,N$ track the time-varying reference vectors $X_{id}(t)$ (where $x_{i(j+1)d}(t) = {}_{RL}D^{\alpha} x_{ijd}(t)$, $1 \leq j \leq n - 1$).

4 Fully-decentralised fractional-order SMC

In this section, designing the fully-decentralised FOSMC for FO large-scale system (4) is developed. For this purpose, an integral sliding manifold is proposed.

Assumption 2: Assume that the lumped uncertainty $L_i(X, t)$ satisfies the following condition

$$\left|L_i(X,t)\right| \le \psi_{i1} \tag{7}$$

where ψ_{i1} is a known positive constant.

Consider the following integral FO sliding manifold:

$$s_i(t) = D^{-(1-\alpha)}\left(e_{in}(t) + \sum_{k=1}^{n-1} c_{ik} e_{ik}(t)\right) + c_0 D^{-1} e_{i1}(t) \tag{8}$$

where the positive constants c_{i0}, $c_{i1}, \ldots, c_{i(n-1)}$, are known as sliding manifold parameters. By differentiating from both sides of (8) and applying (6), one can obtain

$$\begin{aligned}
\dot{s}_i(t) &= D^1 D^{-(1-\alpha)}\left(e_{in}(t) + \sum_{k=1}^{n-1} c_{ik} e_{ik}(t)\right) + c_0 D^1 D^{-1} e_{i1}(t) \\
&= {}_{RL}D^{\alpha} e_{in}(t) + \sum_{k=1}^{n-1} c_{ik}\ {}_{RL}D^{\alpha} e_{ik}(t) + c_0 e_{i1}(t) \\
&= {}_{RL}D^{\alpha} e_{in}(t) + \sum_{k=1}^{n-1} c_{ik} e_{i(k+1)}(t) + c_0 e_{i1}(t) \\
&= {}_{RL}D^{\alpha} e_{in}(t) + \sum_{k=1}^{n-1} c_{ik} e_{i(k+1)}(t)
\end{aligned} \tag{9}$$

Putting (6) in (9), leads to

$$\dot{s}_i(t) = f_i\left(X_i\right) + g_i\left(X_i\right) u_i(t) - {}_{RL}D^{\alpha} x_{ind}(t) + \sum_{k=0}^{n-1} c_{ik} e_{i(k+1)}(t) + L_i(X,t) \tag{10}$$

If the sliding motion be on the sliding manifold ($s_i(t) = 0$), we can get

$$D^{-(1-\alpha)}\left(e_{in}(t) + \sum_{k=1}^{n-1} c_{ik} e_{ik}(t)\right) + c_0 D^{-1} e_{i1}(t) = 0 \tag{11}$$

By taking time derivative D^1 from both sides of (11) and using (6), results in

$$\begin{cases}
{}_{RL}D^{\alpha} e_{i1}(t) = e_{i2}(t) \\
{}_{RL}D^{\alpha} e_{i2}(t) = e_{i3}(t) \\
\quad \vdots \\
{}_{RL}D^{\alpha} e_{in}(t) = -\sum_{k=0}^{n-1} c_{ik} e_{i(k+1)}(t)
\end{cases} \tag{12}$$

or it can be rewritten in a matrix form as

$$
{}_{RL}D^{\alpha}E_i(t)=\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -c_{i0} & -c_{i1} & -c_{i2} & & -c_{i(n-1)} \end{bmatrix} E_i(t) \tag{13}
$$

The sliding manifold parameters (c_{i0}, $c_{i1}, \ldots, c_{i(n-1)}$) should be selected in such a way that the eigenvalues of (13) satisfy the condition of Theorem 1 (i.e., $|\arg(\lambda_i(A))| > \frac{\alpha\pi}{2}$). Note that, considering $c_{i0} = 0$ still can guarantee asymptotic stability of the tracking errors. For more details, see Remark 3.4(a) in Qian et al. (2010) and Lemma 1 from Pisano et al. (2010).

Theorem 2: Consider the RL derivative-based error dynamics (6) with the sliding manifold (8) and Assumptions 1 and 2, the decentralised control law

$$
\begin{aligned} u_i(t) = \frac{1}{g_i(X_i)} \Bigg(& -f_i(X_i) + {}_{RL}D^{\alpha}x_{ind}(t) - \sum_{k=0}^{n-1} c_{ik}e_{i(k+1)}(t) \\ & - \eta_i s_i(t) - K_{sw-i}\,\mathrm{sgn}(s_i(t)) \Bigg) \end{aligned} \tag{14}
$$

guarantees the closed-loop system asymptotical stability, if the switching gain K_{sw-i} be selected as

$$
K_{sw-i} \geq \psi_{i1} \geq |L_i(X,t)| \tag{15}
$$

then the tracking errors $E_1, E_2, \ldots, E_N$, will converge to zero. Where $\eta_i > 0$ is known as reaching rate.

Proof: Let select the following Lyapunov function:

$$
V(t, s(t)) = \|s(t)\|_1 = \sum_{i=1}^{N} V_i(t, s_i(t)) = \sum_{i=1}^{N} |s_i(t)| \tag{16}
$$

where $V_i(\bullet)$ is the Lyapunov function for each subsystem. Now, by taking time derivative from $V(\bullet)$, one has

$$
\dot{V}(t, s(t)) = \sum_{i=1}^{N} \mathrm{sgn}(s_i(t))\,\dot{s}_i(t) \tag{17}
$$

Substituting the sliding manifold dynamics (10), results in

$$
\begin{aligned} \dot{V}(t, s(t)) = \sum_{i=1}^{N} \mathrm{sgn}(s_i(t)) \Bigg(& f_i(X_i) + g_i(X_i)u_i(t) - {}_{RL}D^{\alpha}x_{ind}(t) \\ & + \sum_{k=0}^{n-1} c_{ik}e_{i(k+1)}(t) + L_i(X,t) \Bigg) \end{aligned} \tag{18}
$$

Using the control signal (14), one can obtain

$$\dot{V}\left(t, s(t)\right) = \sum_{i=1}^{N} \operatorname{sgn}\left(s_i(t)\right)\left(-\eta_i s_i(t) - K_{sw-i} \operatorname{sgn}\left(s_i(t)\right) + L_i(X, t)\right) \tag{19}$$

Since sgn($s_i(t)$) × sgn($s_i(t)$) = 1 and sgn($s_i(t)$) × $s_i(t)$ = | $s_i(t)$ |, then we have

$$\dot{V}\left(t, s(t)\right) = -\sum_{i=1}^{N}\left(\eta_i \left|s_i(t)\right| + K_{sw-i} - \operatorname{sgn}\left(s_i(t)\right) L_i(X, t)\right) \tag{20}$$

Choosing $K_{sw-i} \geq \psi_{i1}$, leads to

$$\begin{aligned} &\dot{V}\left(t, s(t)\right) \leq -\sum_{i=1}^{N} \eta_i \left|s_i(t)\right| \leq -\frac{1}{N}\sum_{i=1}^{N}\eta_i \sum_{i=1}^{N}\left|s_i(t)\right| = -\Omega \left\|s_i(t)\right\|_1, \\ &\Omega = \left(\frac{1}{N}\sum_{i=1}^{N}\eta_i\right) > 0 \end{aligned} \tag{21}$$

which implies the closed-loop system asymptotic stability. □

Remark 1: Inequality (21) is derived by the following Chebyshev's sum inequality (Toader, 1996):

For $a_1 \geq a_2 \geq \cdots \geq a_N$ and $b_1 \geq b_2 \geq \cdots \geq b_N$ then

$$\frac{1}{N}\sum_{i=1}^{N} a_i b_i \geq \left(\frac{1}{N}\sum_{i=1}^{N} a_i\right)\left(\frac{1}{N}\sum_{i=1}^{N} b_i\right) \tag{22}$$

Remark 2: Proposed decentralised FOSMC technique (14) based on Assumption 2, contain the following limitations:

1. Employing the sgn($s_i(t)$) function in (14) provoke the chattering phenomena, which can damage under control physical systems.
2. The control law (14), usually needs the upper bound of the interconnections and model uncertainties in order to assure the stability of the large-scale system. Generally, it is not easy to obtain this knowledge in practice because of the complexity of the system. Moreover, when an unknown perturbation occurs in one subsystem, it may causes large changes in interaction bounds, which makes the calculation of the switching gain K_{sw-i} difficult. Therefore, a plan is needed to approximate the interconnections and uncertainties.

5 Semi-decentralised FOSMC

In this section, an adaptive-fuzzy structure is employed to approximate the interconnections and uncertainties, and then the aim of stability can be obtained.

5.1 Fuzzy logic system brief review

The basic configuration of the fuzzy system composed of a collection of fuzzy IF-THEN rules, which can be written as follows (Wang, 1997):

Rule 1: If x_1 is F_1^l and…and x_p is F_p^l then y is A^l.

where the input vector $X = [x_1,\ldots,x_p]^T \in R^p$ and the output variable $y \in R$ denote the linguistic variables of the fuzzy system, $i = 1, 2,\ldots,p$ denotes the number of input for the fuzzy system and $l = 1, 2,\ldots M$ denotes the number of the fuzzy rules, F_i^l and A^l are labels of the input and output fuzzy sets, respectively.

By using the product inference, singleton fuzzification and centre average defuzzification, the fuzzy system output will be as

$$y\left(X_p\right) = \frac{\sum_{l=1}^{M} y^l \left(\prod_{i=1}^{p} \mu_{F_i^l}\left(x_i\right)\right)}{\sum_{l=1}^{M}\prod_{i=1}^{p} \mu_{F_i^l}\left(x_i\right)} \tag{23}$$

where $\mu_{F_i^l}(x_i)$ and $\mu_{A^l}(y^l) = 1$ are the membership function of the linguistic variables x_i and y, respectively. By introducing the concept of fuzzy basis function, (23) can be rewritten in the following form

$$y(X) = \theta^T \xi\left(X_p\right) \tag{24}$$

where $\theta = [y^1,\ldots,y^M]^T$ is the parameter vector and $\xi(X) = [\xi^1(X_p),\ldots,\xi^M(X_p)]^T$ is a regressive vector which can be defined as

$$\xi^l\left(X_p\right) = \frac{\prod_{i=1}^{p} \mu_{F_i^l}\left(x_i\right)}{\sum_{l=1}^{M}\prod_{i=1}^{p} \mu_{F_i^l}\left(x_i\right)} \tag{25}$$

5.2 Interconnection and uncertainty approximation strategy

Based on the universal approximation property of fuzzy systems it is possible to approximate the interconnections and uncertainties.

Now, consider the rewritten form of the sliding manifold dynamics (10) as follows:

$$\dot{s}_i(t) = f_i\left(X_i\right) + g_i\left(X_i\right)u_i(t) - {}_{RL}D^\alpha x_{ind}(t) + \sum_{k=0}^{n-1} c_{ik} e_{i(k+10)}(t) + \theta_i^T \xi_i(X) \tag{26}$$

where $\theta_i = [\theta_{i1},\ldots,\theta_{iM}]^T$ is the parameter vector, $\xi_i(X)$ is a regressive vector.

Theorem 3: Choosing the semi-decentralised control law as

$$u_i(t) = \frac{1}{g_i\left(X_i\right)}\left(-f_i\left(X_i\right) + {}_{RL}D^\alpha x_{ind}(t) - \sum_{k=0}^{n-1} c_{ik} e_{i(k+1)}(t) - \eta_i s_i(s) - \hat{\theta}_i^T \xi_i(X)\right) \tag{27}$$

guarantees the large-scale systems (4) stability with the following adaptation mechanism

$$\dot{\tilde{\theta}}_i = -\mu_i \xi_i(X) s_i(t) \tag{28}$$

where $\tilde{\theta} = \theta - \hat{\theta}$ is the parameter error vector, $\hat{\theta}$ is the estimation vector of the unknown parameter vector θ, and μ_i is a positive constant used for adaptation.

Proof: Using (26) and (27) the closed-loop dynamic becomes

$$\dot{s}_i(t) = \theta_i^T \xi_i(X) - \eta_i s_i(t) - \hat{\theta}_i^T \xi_i(X) = \tilde{\theta}_i^T \xi_i(X) - \eta_i s_i(t) \tag{29}$$

To study the stability and derive the adaptation law for $\tilde{\theta}$, we consider the following Lyapunov function:

$$V\left(t, Y(t)\right) = \alpha\left(\|Y\|\right) = \|Y\|_2^2 = \sum_{i=1}^{N} V_i\left(t, Y_i(t)\right) = \sum_{i=1}^{N}\left(\frac{1}{2}s_i^2(t) + \frac{1}{2\mu_i}\tilde{\theta}_i^T\tilde{\theta}_i\right) \tag{30}$$

where $Y_i = [s_i \quad \tilde{\theta}_i^T]$. Differentiating (30) along the trajectory (29), results in

$$\begin{aligned}
\dot{V}\left(t, Y(t)\right) &= \sum_{i=1}^{N}\left(s_i(t)\dot{s}_i(t) + \frac{1}{\mu_i}\tilde{\theta}_i^T\dot{\tilde{\theta}}_i\right) \\
&= \sum_{i=1}^{N}\left(s_i(t)\left(\tilde{\theta}_i^T\xi_i(X) - \eta_i s_i(t)\right) + \frac{1}{\mu_i}\tilde{\theta}_i^T\dot{\tilde{\theta}}_i\right) \\
&= \sum_{i=1}^{N}\left(-\eta_i s_i^2(t) + \tilde{\theta}_i^T\left(\xi_i(X)s_i(t) + \frac{1}{\mu_i}\dot{\tilde{\theta}}_i\right)\right)
\end{aligned} \tag{31}$$

Inserting the adaptation law (28), leads to

$$\dot{V}\left(t, Y(t)\right) = \sum_{i=1}^{N} -\eta_i s_i^2(t) \le 0 \tag{32}$$

which assures the large-scale system (4) stability. Therefore, $s_i(t)$ and $\tilde{\theta}_i$ are bounded. Although $s_i(t)$ converges to zero (Barbalat's lemma), but the system is not asymptotically stable, because $\tilde{\theta}_i$ is only bounded.

Remark 3: By substituting the proposed control laws in the corresponding sliding manifold dynamics, we can get the following closed-loop sliding manifold dynamics:

- decentralised strategy

$$\dot{s}_i(t) = -\eta_i s_i(t) - K_{sw-i}\,\mathrm{sgn}\left(s_i(t)\right) + L_i(X, t) \tag{33}$$

- semi-decentralised strategy

$$\begin{aligned}
\dot{s}_i(t) &= \tilde{\theta}_i^T\xi_i(X) - \eta_i s_i(t) \\
\dot{\tilde{\theta}}_i &= -\mu_i\xi_i(X)s_i(t)
\end{aligned} \tag{34}$$

from equations (33) and (34), it is obvious that there is no FO operators. Therefore, the IO stability analysis can be applied for (33) and (34). Moreover, it is worthwhile to notify that the error dynamics (13) is FO, and the sliding manifold parameters should be selected in such a way that the eigenvalues of (13) satisfy the condition of Theorem 1.

Remark 4: Although the i^{th} subsystem interconnection $L_i(X, t)$ and regressive vector $\xi_i(X)$ input vector is considered $X_{(n\times N)\times 1}$, but in reality the dimension of input vector is lower than $(n \times N) \times 1$ (input vector is not full dimension). In the other word, the dimension of input vector is the number of i^{th} subsystem states plus the states of neighbour subsystems which are present in the i^{th} interconnection. Therefore, the non-interconnected subsystems or states should be removed from the input vector. This fact is considered in the simulation process.

6 Simulation results

In this section, an illustrative example is presented to reveal the effectiveness of the proposed control strategies. Simulation results are presented for fully-decentralised and semi-decentralised controllers (14) and (27).

Consider the following FO large-scale nonlinear system composed of two subsystems which are described by:

$$S_1 : \begin{cases} {}_{RL}D^{0.8} x_{11} = x_{12} \\ {}_{RL}D^{0.8} x_{12} = -x_{11}^3 - x_{12} + \left(1 + e^{-x_{11} + x_{12}^2}\right) u_1 + L_1(X, t) \end{cases}$$

$$S_2 : \begin{cases} {}_{RL}D^{0.8} x_{21} = x_{22} \\ {}_{RL}D^{0.8} x_{22} = -x_{21} - x_{22}^2 + \left(2 + \sin\left(x_{21}\right)\right) u_2 + L_2(X, t) \end{cases}$$

where the lumped uncertainty terms and the reference values are considered as:

$$L_1(X, t) = 0.6\cos(t) + 0.4x_{12}\sin(0.5t) + 0.2x_{21}\sin(3t) + 0.5x_{22}\cos(10t)$$

$$L_2(X, t) = 0.5\sin(5t) + 2x_{11}\sin\left(x_{21}\right) + 0.4x_{22} \times \sin(0.2t)$$

$$x_{11d}(t) = \sin\left((\pi/20)t\right), \quad x_{12d}(t) = {}_{RL}D^{\alpha} \sin\left((\pi/20)t\right)$$

$$x_{21d}(t) = \sin\left((\pi/15)t\right), \quad x_{22d}(t) = {}_{RL}D^{\alpha} \sin\left((\pi/15)t\right)$$

Also, the initial conditions are selected as below to avoid the initial value interpretation problem of RL derivative.

$$\left(x_{11}(0), x_{12}(0)\right) = (0, 0), \left(x_{21}(0), x_{22}(0)\right) = (0, 0)$$

For RL-derivative-based dynamics (5) and control laws (14) and (27), the ${}_{RL}D^{\alpha}$ operator has been approximated (from Property 1) by GL-derivative discrete-time algorithm with the sampling interval h = 0.005 (Monje et al., 2010). This algorithm is executed by S-function blocks in MATLAB.

6.1 Fully-decentralised FOSMC

Based on (14), the fully-decentralised FOSMC parameters are selected as:

$$\rho_1 = \rho_2 = 0.5, \quad c_{10} = c_{20} = 1, \quad c_{11} = c_{21} = 2,$$
$$\eta_1 = \eta_2 = 20, \quad K_{sw-1} = K_{sw-2} = 50$$

Also we replaced $sgn(s_i(t))$ by $\tan(s_i(t) / \rho_i)$ in order to remove the chattering and produce a continuously differentiable signal (C^1).

Figure 1 illustrates the state responses and control signals of the large-scale system for the fully-decentralised FOSMC strategy. From this figure, it is evident that the system states (x_{11}, x_{12}, x_{21}, x_{22}) are tracking the desired trajectories robustly with small control signals in the presence of uncertainties and interconnections.

Figure 1 The responses of system under the fully-decentralised FOSMC (see online version for colours)

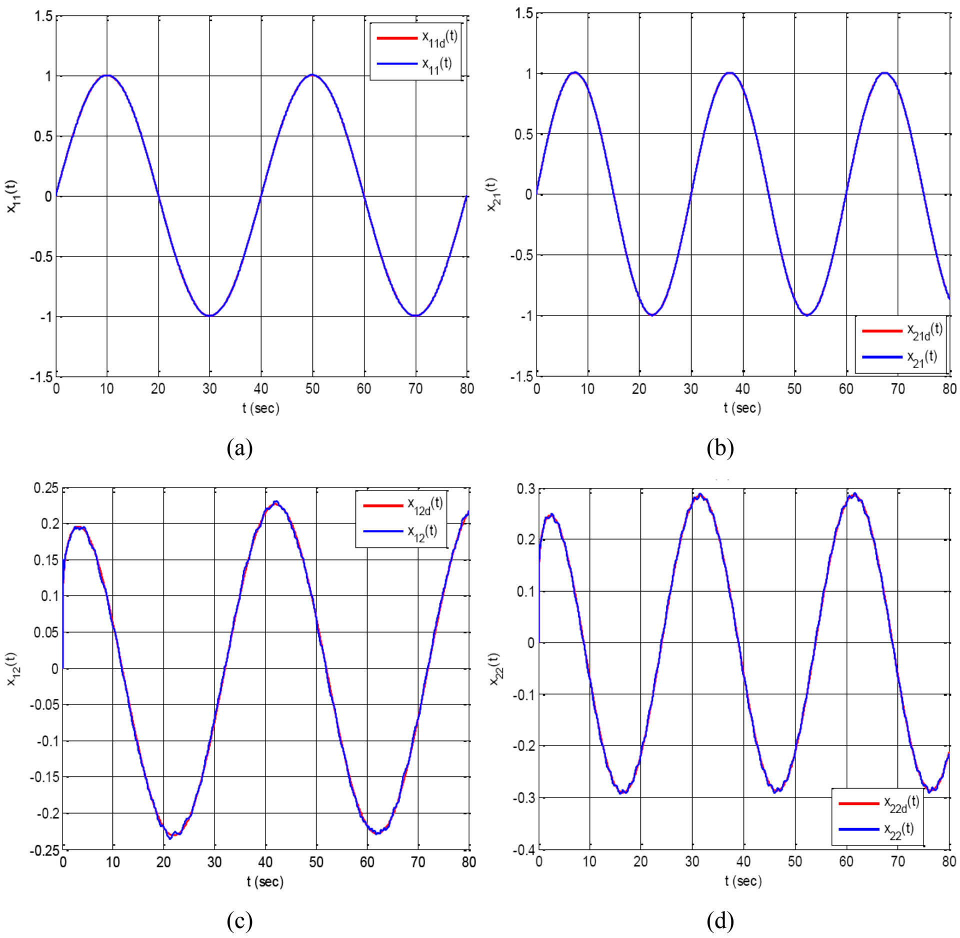
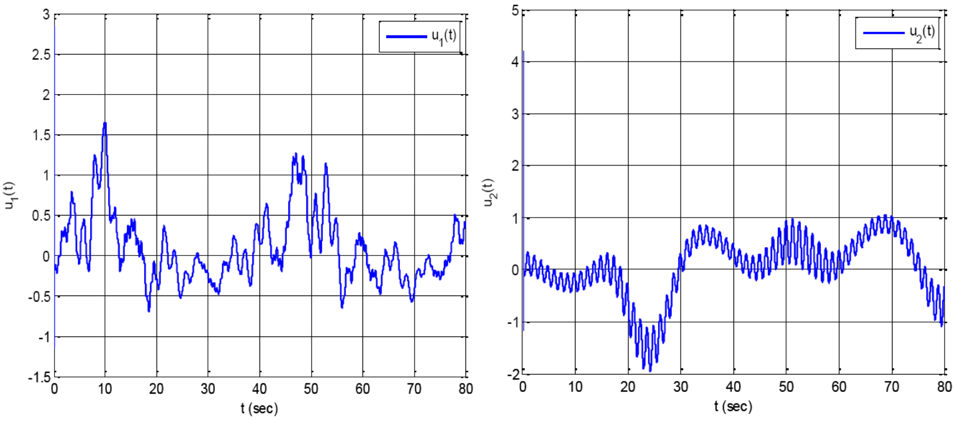


Figure 1 The responses of system under the fully-decentralised FOSMC (continued) (see online version for colours)



(e) (f)

6.2 Semi-decentralided FOSMC

Based on (27), the controller parameters are picked as follows:

$$c_{10} = c_{20} = 1, \quad c_{11} = c_{21} = 2, \quad \eta_1 = \eta_2 = 20, \quad \mu_1 = \mu_2 = 100$$

All the states of i^{th} subsystem and neighbour subsystem which are effective in $L_i(X, t)$ are considered as the fuzzy system input variables (three variables for each subsystem). Fuzzy sets for input variables are defined according to the following membership functions which are depicted in Figure 2.

$$\mu_1(x_{ik}) = \exp\left(-10x_{ik}^2\right), \quad \mu_2(x_{ik}) = 1 - \mu_1(x_{ik})$$
$$i, k = 1, 2$$

Two fuzzy sets for each input variable have been found sufficient. Therefore, the number of fuzzy rules will be 2 × 2 × 2 = 8.

Figure 2 Fuzzy sets assigned to input variables

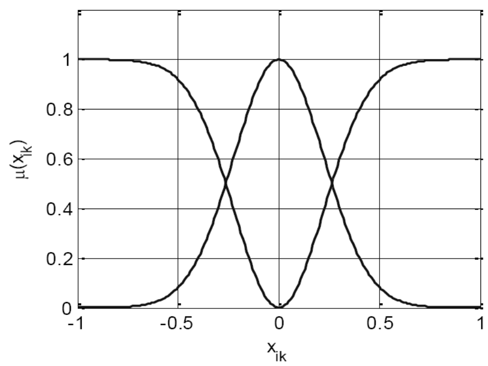
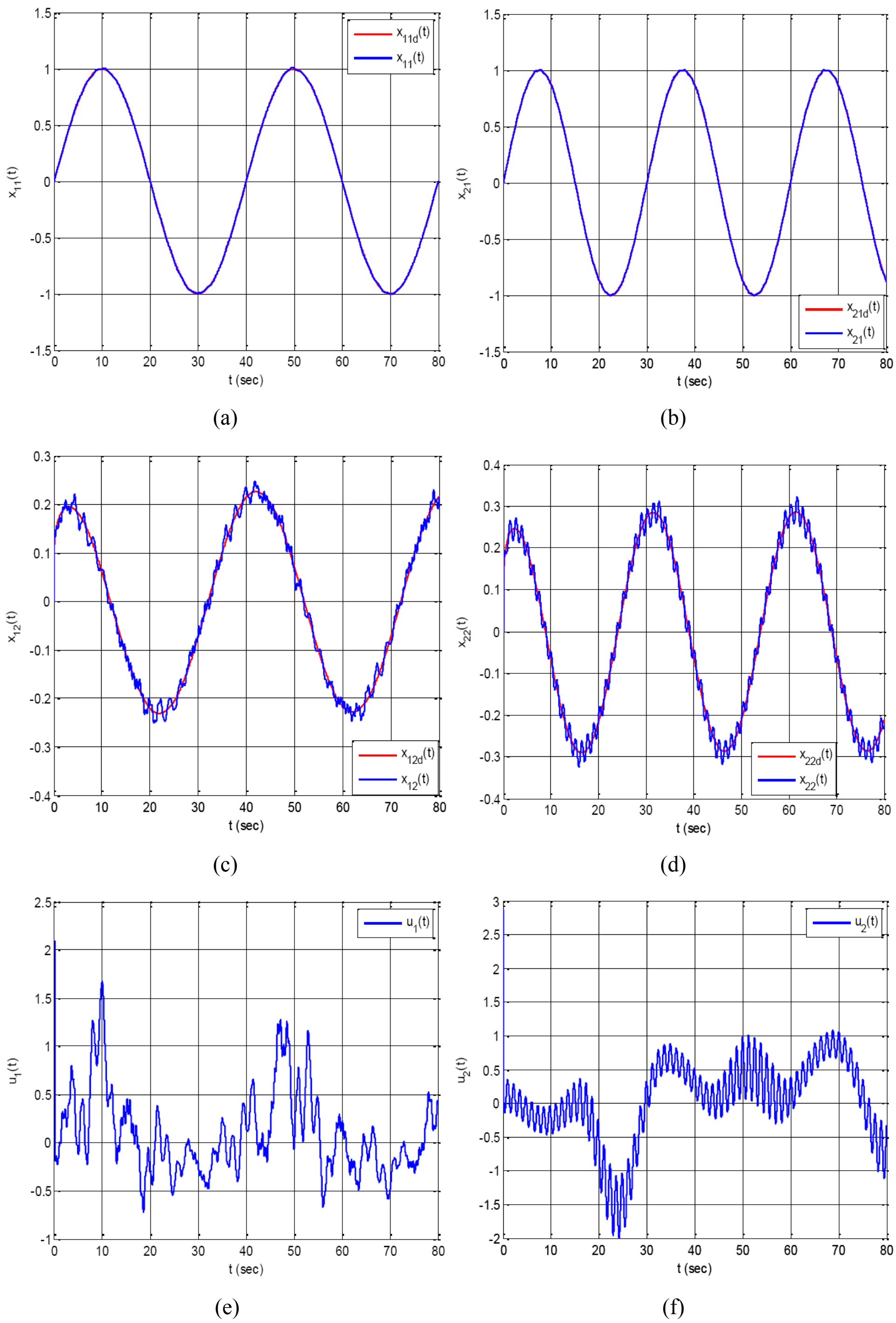


Figure 3 The responses of system under the semi-decentralised FOSMC (see online version for colours)



The simulation results of the semi-decentralised FOSMC strategy are presented in Figure 3. As can be seen from this figure, the responses of the semi-decentralised FOSMC appear to be satisfactory, since they track desired trajectories with low deviation and small control signals.

Moreover, there are few differences in the control signals amplitude and the second state deviations. For instant: the states x_{12} and x_{22} oscillations in the semi-decentralised FOSMC are little high.

7 Conclusions

In this paper, the fully-decentralised and the semi-decentralised FOSMC strategies have been developed for FO large-scale nonlinear systems for the first time. First, we designed a fully-decentralised controller for FO system with known interconnections and uncertainties. In the second step, the semi-decentralised control strategy is proposed to deal with unknown interconnections and uncertainties. The sliding manifold zero convergence is proved using the IO stability theorems due to the proposed integral sliding surface. Moreover, the system tracking errors convergence is concluded from Theorem 1. Simulation results expose the high performance of the suggested control techniques in trajectory tracking of FO large-scale case study. For both control strategies, the system states are converged to the desired values in the presence of uncertainties and interconnections. The proposed controllers are applicable for vast range of fractional-order systems.

References

Aghababa, M.P. (2012) 'Finite-time chaos control and synchronization of fractional-order nonautonomous chaotic (hyperchaotic) systems using fractional nonsingular terminal sliding mode technique', *Nonlinear Dynamics*, Vol. 69, No. 1, pp.247–261.

Aghababa, M.P. (2013a) 'A novel terminal sliding mode controller for a class of non-autonomous fractional-order systems', *Nonlinear Dynamics*, Vol. 73, No. 1, pp.679–688.

Aghababa, M.P. (2013b) 'No-chatter variable structure control for fractional nonlinear complex systems', *Nonlinear Dynamics*, Vol. 73, No. 4, pp.2329–2342.

Al-Gherwi, W., Budman, H. and Elkamel, A. (2010) 'Selection of control structure for distributed model predictive control in the presence of model errors', *Journal of Process Control*, Vol. 20, No. 3, pp.270–284.

Binazadeh, T. and Shafiei, M.H. (2013) 'Output tracking of uncertain fractional-order nonlinear systems via a novel fractional-order sliding mode approach', *Mechatronics*, Vol. 23, No. 7, pp.888–892.

Cao, J., Ma, C., Jiang, Z. and Liu, S. (2011) 'Nonlinear dynamic analysis of fractional order rub-impact rotor system', *Communications in Nonlinear Science and Numerical Simulation*, Vol. 16, No. 3, pp.1443–1463.

Chen, H., Chen, W., Zhang, B. and Cao, H. (2013) 'Robust synchronization of incommensurate rractional-order chaotic systems via second-order sliding mode technique', *Journal of Applied Mathematics*, Article ID 321253, 11pp, http://dx.doi.org/10.1155/2013/321-253.

Cheng, C.C. and Chang, Y. (2008) 'Design of decentralised adaptive sliding mode controllers for large-scale systems with mismatched perturbations', *International Journal of Control*, Vol. 81, No. 10, pp.1507–1518.

Chou, C.H. and Cheng, C.C. (2003) 'A decentralized model reference adaptive variable structure controller for large-scale time-varying delay systems', *IEEE Transactions on Automatic Control*, Vol. 48, No. 7, pp.1213–1217.

Da, F. (2000) 'Decentralized sliding mode adaptive controller design based on fuzzy neural networks for interconnected uncertain nonlinear systems', *IEEE Transactions on Neural Networks*, Vol. 11, No. 6, pp.1471–1480.

Dadras, S. and Momeni, H.R. (2013) 'Passivity-based fractional-order integral sliding-mode control design for uncertain fractional-order nonlinear systems', *Mechatronics*, Vol. 23, No. 7, pp.880–887.

Ezzat, M.A. (2011) 'Theory of fractional order in generalized thermoelectric MHD', *Applied Mathematic Modelling*, Vol. 35, No. 10, pp.4965–4978.

Gafiychuk, V., Datsko, B. and Meleshko, V. (2008) 'Mathematical modeling of time fractional reaction-diffusion systems', *Journal of Computational and Applied Mathematics*, Vol. 220, No. 1, pp.215–225.

Guo, Y., Hill, D.J. and Wang, Y. (2000) 'Nonlinear decentralized control of large-scale power systems', *Automatica*, Vol. 36, No. 9, pp.1275–1289.

Hewitt, E. and Stromberg, K. (1963) *Real and Abstract Analysis*, Springer-Verlag, Berlin.

Hosseinnia, S. H., Ghaderi, R., Ranjbar, A., Mahmoudian, M. and Momani, S. (2010) 'Sliding mode synchronization of an uncertain fractional order chaotic system', *Computers and Mathematics with Applications*, Vol. 59, No. 5, pp.1637–1643.

Huang, Y.S. and Zhou, D.Q. (2010) 'Decentralized adaptive output feedback fuzzy controller for a class of large-scale nonlinear systems', *Nonlinear Dynamics*, Vol. 65, No. 1, pp.85–101.

Laskin, N. (2000) 'Fractional market dynamics', *Physica A*, Vol. 287, No. 3, pp.482–492.

Li, C. and Deng, W. (2007) 'Remarks on fractional derivatives', *Applied Mathematics and Computation*, Vol. 187, No. 2, pp.777–784.

Li, J., Lu, J.G. and Chen, Y.Q. (2013) 'Robust decentralized control of perturbed fractional-order linear interconnected systems', *Computers and Mathematics with Applications*, Vol. 66, No. 5, pp.844–859.

Li, T., Li, R. and Li, J. (2011) 'Decentralized adaptive neural control of nonlinear interconnected large-scale systems with unknown time delays and input saturation', *Neurocomputing*, Vol. 74, No. 14, pp.2277–2283.

Lin, D. and Wang, X. (2010) 'Observer-based decentralized fuzzy neural sliding mode control for interconnected unknown chaotic systems via network structure adaptation', *Fuzzy Sets and Systems*, Vol. 161, No. 15, pp.2066–2080.

Lin, T.C. and Lee, T.Y. (2011) 'Chaos synchronization of uncertain fractional-order chaotic systems with time delay based on adaptive fuzzy sliding mode control', *IEEE Transactions on Fuzzy Systems*, Vol. 19, No. 4, pp.623–635.

Lin, T.C., Lee, T.Y. and Balas, V.E. (2011) 'Adaptive fuzzy sliding mode control for synchronization of uncertain fractional order chaotic systems', *Chaos, Solitons & Fractals*, Vol. 44, No. 10, pp.791–801.

Luo, Y., Chen, Y.Q. and Pi, Y. (2011) 'Experimental study of fractional order proportional derivative controller synthesis for fractional order systems', *Mechatronics*, Vol. 21, No. 1, pp.204–214.

Monje, C.A., Chen, Y.Q., Vinagre, B.M., Xue, D. and Feliu, V. (2010) *Fractional-order Systems and Controls Fundamentals and Applications*, Springer-Verlag, London.

Petras, I. and Magin, R.L. (2011) 'Simulation of drug uptake in a two compartmental fractional model for a biological system', *Communications in Nonlinear Science and Numerical Simulation*, Vol. 16, No. 12, pp.4588–4595.

Pisano, A., Rapaic, M.R., Jelicic, Z.D. and Usai, E. (2010) 'Sliding mode control approaches to the robust regulation of linear multivariable fractional-order dynamics', *International Journal of Robust Nonlinear Control*, Vol. 20, No. 18, pp.2045–2056.

Podlubny, I. (1999) *Fractional Differential Equations*, Academic Press, New York.

Qian, D., Li, C., Agarwal, R.P. and Wong, P.J.Y. (2010) 'Stability analysis of fractional differential system with Riemann-Liouville derivative', *Mathematical and Computer Modelling*, Vol. 52, No. 5, pp.862–874.

Shyu, K.K., Liu, W.J. and Hsu, K.C. (2003) 'Decentralized variable structure control of uncertain large-scale systems containing a dead-zone', *IEE Proc.-Control Theory Applications*, Vol. 150, No. 6, pp.467–475.

Stewart, B.T., Wright, S.J. and Rawlings, J.B. (2011) 'Cooperative distributed model predictive control for nonlinear systems', *Journal of Process Control*, Vol. 21, No. 5, pp.698–704.

Tavazoei, M.S. (2012) 'Comments on 'Chaos synchronization of uncertain fractional-order chaotic systems with time delay based on adaptive fuzzy sliding mode control'', *IEEE Transactions Fuzzy Systems*, Vol. 20, No. 5, pp.993–995.

Tavazoei, M.S. and Haeri, M. (2008) 'Synchronization of chaotic fractional-order systems via active sliding mode controller', *Physica A*, Vol. 387, No. 1, pp.57–70.

Toader, G. (1996) 'Note on Chebyshev's inequality for sequences', *Discrete Mathematics*, Vol. 161, No. 1, pp.317–322.

Wang, L.X. (1997) *A Course in Fuzzy System and Control*, Prentice-Hall, New Jersey.

Wang, Z. (2013) 'Synchronization of an uncertain fractional-order chaotic system via backstepping sliding mode control', *Discrete Dynamics in Nature and Society*, Article ID 732503, 6pp, http://dx.doi.org/10.1155/2013/732503.

Wang, Z., Huang, X. and Shen, H. (2012) 'Control of an uncertain fractional order economic system via adaptive sliding mode', *Neurocomputing*, Vol. 83, No. 15, pp.83–88.

Yan, X.G., Spurgeon, S.K. and Edwards, C. (2003) 'Decentralized output feedback sliding mode control of nonlinear large-scale systems with uncertainties', *Journal of Optimization Theory and Applications*, Vol. 119, No. 3, pp.597–614.

Yin, C., Dadras, S., Zhong, S.M. and Chen, Y.Q. (2013) 'Control of a novel class of fractional-order chaotic systems via adaptive sliding mode control approach', *Applied Mathematical Modeling*, Vol. 37, No. 4, pp.2469–2483.

Yousef, H., El-Madbouly, E., Eteim, D. and Hamdy, M. (2006) 'Adaptive fuzzy semi-decentralized control for a class of large-scale nonlinear systems with unknown interconnections', *International Journal of Robust and Nonlinear Control*, Vol. 16, No. 15, pp.687–708.

Yousef, H., Hamdy, M. and El-Madbouly, (2010) 'E. robust adaptive fuzzy semi-decentralized control for a class of large-scale nonlinear systems using input-output linearization concept', *International Journal of Robust and Nonlinear Control*, Vol. 20, No. 1, pp.27–40.

Zhang, T.P. and Feng, C.B. (1997) 'Decentralized adaptive fuzzy control for large-scale nonlinear systems', *Fuzzy Sets and Systems*, Vol. 92, No. 1, pp.61–70.

Zhu, M. and Li, Y. (2010) 'Decentralized adaptive fuzzy sliding mode control for reconfigurable modular manipulators', *International Journal of Robust and Nonlinear Control*, Vol. 20, No. 4, pp.472–488.