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Invited Article

A bi-level model and solution methods for partial interdiction problem on capacitated hierarchical facilities

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ABSTRACT

Due to the importance of gaining high levels of customer satisfaction in today's competitive world, making appropriate decisions in the face of malicious attacks is valued highly by many organizations. In this paper, to predict and handle the destructive effects of an intentional attack on capacitated nested hierarchical facilities, a bi-level partial interdiction problem is proposed. In this problem, there is an interdictor who can attack facilities partially in different levels. Subsequently, the system defender could respond to the customers' demand in two different ways, namely through the remaining system facilities and the outsourcing option. The goal of the defender is to minimize the satisfaction cost of all customers' demand under the interdictor's attacking scenario. This problem can be modeled as a bi-level programming model in which an interdictor and the system defender play the role of the leader and the follower, respectively. Due to the inherent complexity of the bi-level programming models, we develop a heuristic approach, namely "FDS", to obtain near optimal solutions within a reasonable running time. In each iteration of the FDS, an interdiction scenario is produced heuristically and, thereupon CPLEX solver is called to solve the lower level of the model. To evaluate the effectiveness of the proposed model, a comparison between the cost of customers' demand satisfaction in both absence and presence of the bi-level model is drawn. Computational results show that for those instances in which the optimal solutions are available, the proposed model can, on average, achieve a saving of 7.94%.

1. Introduction

Risk of disruption is an inherent element of supply chain operations and depending on how prepared supply chains are, the severity of the consequences varies. A study on 151 supply chains reveals that 73% of these networks experienced disruptions over 7 years between 2002 and 2007 (Kouvelis et al., 2011). Moreover, the level of globalization of a supply chain may significantly contribute to the seriousness of the situation. Despite the widespread consequences of such disruptions in the service and production sectors, and the fact that part of the witnessed disruptions affected supply chains on a strategic level, most disruption risk management tools are tailored for operational level risks (Kleindorfer and Saad, 2005).

The literature categorizes the sources of disruption risks mainly into two groups, *premeditated* and *random*. In premeditated disruption, an intelligent attacker deliberately aims at causing a major damage in a system's performance. Labor strikes and terrorist attacks are examples of premeditated disruptions. In random disruption, however, the source of the risk is the nature, e.g. tsunami and volcanic eruption (Azad et al., 2013).

The major difference between these two sources of risk is the concept of deliberateness and premeditation versus haphazard occurrence (Church et al., 2004).

An overview of the literature reveals that the majority of the researchers have focused on the disruptions caused by the latter category, i.e. random disruption risks (Aksen and Aras, 2012). This is while studies show that since 2000 the proportion of the disruptions caused by premeditated risks has dramatically increased (Chalk et al., 2005). One of the main reasons of this growth is the fast and ongoing advancement in the information and communication technology which is misused by the attackers.

In this paper, we focus on disruptions caused by premeditated risks in the service sector, and models to interdict such evaders, a class of problems that in the literature is defined as "interdiction problem". Under such circumstances, "interdiction operations" are planned and run to prevent destructive attacks or minimize the detrimental effect (Pub, 1997).

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The primary motivation of interdiction problems is to assess the vulnerability of a system from a combatant force's perspective. The output of these problems makes the critical facilities of the system recognizable and may provide the system defender with appropriate policies to prevent the occurrence of the most destructive attacking scenario or mitigate its effects.

The application of interdiction problems is not limited to the military sphere and by reinterpreting the outputs, we can benefit from them even in the absence of the probability of any attacks on system components. Under the random disruption risks, this problem is able to warn the system designer about the worst possible failure in the system. To corroborate the wide variety of applications of interdiction problem, we refer to its utilization in telecommunication networks (Rai and Soh, 1991), nuclear smuggling (Morton et al., 2007), conflict resolution (Anandalingam and Apprey, 1991), supply networks (McMasters and Mustin, 1970), protection of supply systems (Zhu et al., 2013), hospital infection control (Assimakopoulos, 1987), critical infrastructure and key resources (Murray and Grubesic, 2012), border controls (Pan, 2005), electric grid security (Salmeron et al., 2004; Salmeron et al., 2009), highway transportation (Durbin, 1966), and military and homeland security (Lim and Smith, 2007).

The interdiction problem has a two-player game nature between an external enemy and a system defender. One of these players is considered as the leader who makes decisions independently and the other is considered as the follower whose decision is subordinated to the leader. Consequently, this problem predominantly is formulated as a bi-level optimization model whose first formulation dates back to 1934 when it has been formulated by Stackelberg in market economy (Dempe, 2002). The interested readers may refer to useful studies on applications of bi-level programming in interdiction problems which were carried out byWood (2011) and DeNegre,(2011). Bi-level programming is associated with inherent complexity and even the simplest category of these problems, i.e. linear bi-level programming problem is NP-hard (Jeroslow, 1985).

In this paper, inspired by many service systems, the facilities structure is considered to be hierarchical. In a non-hierarchical system all facilities provide the same service while, by contrast, in a hierarchical system to serve customers with different services, several levels of facilities are located. In a hierarchical system, facilities consist of k levels where the highest level is called level k and the lowest level is called level *1* (Şahin and Süral, 2007). Local clinics, hospitals and medical centers are examples of a hierarchical facility network in a healthcare system. At the lowest-level, the local clinic provides direct services to incoming patients. A hospital, at the middle level, provides services to local clinics and undertakes out-patient surgeries. Finally, the medical center, at the highest hierarchy level, provides services to the hospital and responds to in-patient cases (Zanjirani Farahani et al., 2014). There are a variety of classification schemes for hierarchical systems. According to the flow pattern attribute, facilities can be classified as single-flow or multi-flow pattern. In single-flow pattern, the goods shipment or customer's path starts from the lowest level and ends at the highest level or inversely. In multi-flow pattern, this restriction is obviated and the flow can pass through any levels (Şahin and Süral, 2007). Furthermore, both flow patterns can be segmented into two categories: referral or non-referral. In a referral system, a proportion of customers served at each level are referred to higher levels, in contrast, in a non-referral system referral between levels is considered impossible (Marianov and Serra, 2001). From service varieties point of view, a hierarchical system can be classified as nested or non-nested according to the service availability at the levels of the hierarchy. In non-nested facilities, each hierarchy provides a different service, whereas in nested systems, an upper level facility serves customers with all the services which a lower level facility can offer and at least one further service (Şahin and Süral, 2007).

In this paper, we study a partial interdiction problem on a capacitated nested hierarchical service system. In full interdiction, interdicting a component of a vulnerable system causes total destruction, whereas partial interdiction of a system component does not necessarily cause a complete loss of its functionality .Considering hierarchical structure in a partial interdiction problem introduces more complexity to the problem at hand since it increases the number of possible interdiction patterns. To formulate this problem, we use bi-level programming. Finally, to solve the model, we propose an enumeration algorithm and a heuristic procedure.

The remainder of this paper is as follows: Section 2 briefly reviews the relevant literature on interdiction problems. Definition of the problem and its assumptions are presented in Section 3. In Section 4, the mathematical formulation of the problem is given. The intricacies of the proposed heuristic solution procedure for large-scale real-world problems are discussed in Section 5. In Section 6, a two-stage enumeration solution procedure is presented. Computational experiments are conducted in Section 7, followed by some overall conclusions and recommendations for future research in Section 8.

2. Background

Interdiction problem models dates back to the early 1960s (Ramirez-Marquez and Rocco, 2009). In 1963, this problem named interdiction problem, for the first time (Wollmer, 1963). In that decade, Wollmer (1964), Durbin (1966) and Bellmore et al. (1967) carried out the first attempts to implement interdiction models for real world problems and they analyzed the vulnerability of a transportation system to military attacks and disruptions. The last effort in the decade, were made by Thomas and Models (1968) to offer several simple models for airstrikes.

The correspondence among these efforts is that all of them consider that interdiction only occurs in arcs of a network and if an arc is interdicted, its entire ability to serve customers will be lost. This category is termed "network interdiction problem". Constructing a safer transportation system and preventing trafficking are the primary motives for introducing this category of interdiction problem (Steinrauf, 1991). Objectives such as minimizing the max-flow through a network, see, e.g. Wollmer (1964) and Burch et al. (2003), maximizing the shortest path, see e.g. Losada et al. (2012) and Israeli and Wood (2002), and minimizing the interdiction cost of a network, see e.g. Nugent (1969) and Whiteman and Philip (1999), are the most frequent objective functions in the literature of network interdiction problem. For example Smith et al. (2007) benefit from building and fortifying network segments as a defensive strategy against the various interdiction scenarios. It is assumed that the attacker has a specified budget to disable any portion of arcs that are constructed on the network. More recently, Bidgoli and Kheirkhah (2018) propose a bi-level mathematical programming model for network interdiction vehicle routing problem. In their model interdictor interdicts a subset of arcs using limited resources and the distributor tries to maximize her/his profits in the interdicted network.

In some cases in interdiction problem, the vulnerability of facilities in a network is examined. The category is named "facility interdiction". The facility interdiction is first developed in 1982 by Corley and Sha (1982). In their proposed problem, the most vital nodes whose removal from the network causes the most elongation in the shortest distance between a source to a sink are identified.

Scaparra and Church (2012) study a service system that is formed from a set of capacitated facilities, which are vulnerable to both intentional and haphazard disruption risks. If a disaster occurs in a facility, its capacity will be reduced and some customers may face unsatisfactory service level. Hence, to prevent the occurrence of the worst-case scenario, advanced fortification of the facilities to the extent that the defender's resources allowed is suggested. For this problem, a tri-level model is proposed. Losada et al. (2012) assume that when a facility faces partial interdiction, the probability of its availability will be decreased. In partial interdiction, interdicting a component of a vulnerable system does not necessarily leads to a complete loss of its functionality. Zhang et al. (2014) present the random attack median fortification problem and the fortification median problem for disruptions caused by mixed types of attacks.

Aksen et al. (2014) publish the study of partial interdiction on a median system. In this problem, the interdictor acts first and is able to destroy any portion of facilities capacity with regard to her budget. Next, the defender acts and identifies the least-cost customer satisfaction pattern of all customers' demand. There are two ways of satisfying customers' demand, namely through system facilities or outsourcing option. This problem is formulated as a bi-level model in which in the upper level the most destructive interdiction scenario is identified and in the lower level the defender figures out the best customer satisfaction pattern.

Aliakbarian et al. (2015), for the first time, study the interdiction problem on a hierarchical system. This problem aims to fortify the critical facilities of a service system in order to mitigate the worst-destructive scenario impacts. Fortified facilities are not vulnerable to attacks anymore. This problem is formulated as a bi-level optimization model. The system defender is the leader of the problem and fortifies a specified number of facilities at each level. Subsequently, the interdictor, as the follower, chooses the most destructive full interdiction scenario among unprotected facilities. Since it is assumed that the facilities have infinite capacities, in the lower level of the problem through closest assignment constraints, the least-cost allocation pattern of satisfying all customers' demand is identified. It means that the demand at each customer zone is entirely supplied by the closest eligible facility to that zone and if that facility is lost due to interdiction, the demand is reassigned to the next closest eligible facility among the non-interdicted ones. Mahmoodjanloo et al. (2016) provide a tri-level defense facility location model and heuristic solution approaches for full coverage in r-interdiction median problem. Akbari-Jafarabadi et al. (2017) apply a tri-level facility location r-interdiction median based on bi-level programming.

More recently, Zhang et al. (2018) address the issue of decentralized supply chain fortification by proposing the r-interdiction median problem with fortification for decentralized supply systems. Fathollahi Fard and Hajiaghaei-Keshteli (2018) proposes a bi-objective, bi-level formulation for partial interdiction facilities problem considering different defensive systems.

Our paper is dedicated to the least common categories of interdiction problems (i.e. partial facility interdiction problems (Aksen et al., 2014)). For the first time, in this paper, partial interdiction problem is studied on capacitated hierarchical facilities. This study could be considered as a development of problems proposed in Aksen et al. (2014) and Aliakbarian et al. (2015), hence we provide the readers with the several important enhancements in the assumptions of the most related part of the literature which result in more compatibility with real-world problems.

Aksen et al. (2014) present the most related problem to ours. This paper is the first paper which examines a partial facility interdiction on a median system. The authors assume that the interdictor can result in any proportion of reduction in the capacity of the system facilities. This assumption is far from reality; therefore, in this paper, we benefit from the concept of different interdiction intensity levels to resolve this issue. Here, the interdictor can interdict facilities in discrete levels and these levels might be considered as the number of sorties or weapons which are available for interdicting each facility. Another restrictive assumption in Aksen et al. (2014) is how they satisfy the customers' demand. The authors presume each customer can be served only from one source, namely allocation to the system facilities or outsourcing, and if a customer benefits from the first source (i.e. allocation); all of her demand must be satisfied through exactly one facility. This assumption has two drawbacks. Firstly, since this pattern incurs a high amount of cost and leaves some fraction of the system capacity unused, most probably in real-world conditions the defender does not choose this pattern. In addition, in case that there are several facilities for which the differences of their aggregate distances from a customer are negligible and their total capacity is sufficient for satisfying the customer, it is not reasonable to allocate the customer to a far facility due to the insufficient capacity of each of these facilities. Hence, in this paper, we obviate these limitations and let the optimization model offer the optimal pattern.

In this paper, similar to Aliakbarian et al. (2015), we consider a nested and referral hierarchical system. However, to strengthen the conformity with reality, we consider that the facilities are capacitated and excessive demand must be outsourced. In Aliakbarian et al. (2015), the constraints on fortification and interdiction resources are cardinality constraints, i.e., a predetermined number of facilities in each level of the hierarchy can be attacked and hardened, while in real-world applications, for the sake of flexibility, these constraints are considered as budget constraints. Finally, in Aliakbarian et al. (2015) an expensive defensive strategy, i.e. fortification is applied in designing phase of the supply chain network to deal with worst-destructive scenario. In our paper we propose two cheaper options to cope with the situation in which the worst-destructive scenario happens, i.e. outsourcing and reassignment.

The major features of our proposed problem can be summarized as follows:

- The problem is classified as a partial facility interdiction problem.
- The system is median and the hierarchical facilities are nested and referral.
- Interdiction occurs in discrete levels.
- The budget constraint is imposed on interdiction resources.
- The customers' demand can be satisfied through allocating to the system facilities or outsourcing.
- A customer can be allocated to more than one facility.

3. Problem definition

This problem is studied on a region that is divided into smaller zones. For the sake of simplicity, it is assumed that the demand of each zone is concentrated at its center of gravity which is called "demand point" or "customer". To satisfy the demand of the customers, several capacitated nested hierarchical facilities are located in this region. Due to the nested nature of hierarchical facilities, a facility at level II can provide both types of service with regard to its capacity, whereas for facilities at level I, the capacity for service of type II is zero. By experience, it is known that the specific proportions of demand of each customer are required particular service levels, namely "type I" and "type II". However, it can be the case that a specific proportion of a customer's demand, which refers to a facility at level I or level II to receive service of type I, needs service of type II afterwards. We denote this service level by "referential type II".

It is assumed that the cost of fulfilling customer's demand through allocation to available facilities is a function of the distance travelled, the service type, and the amount of demand. The unit transportation costs for service of type I, type II, and referen-

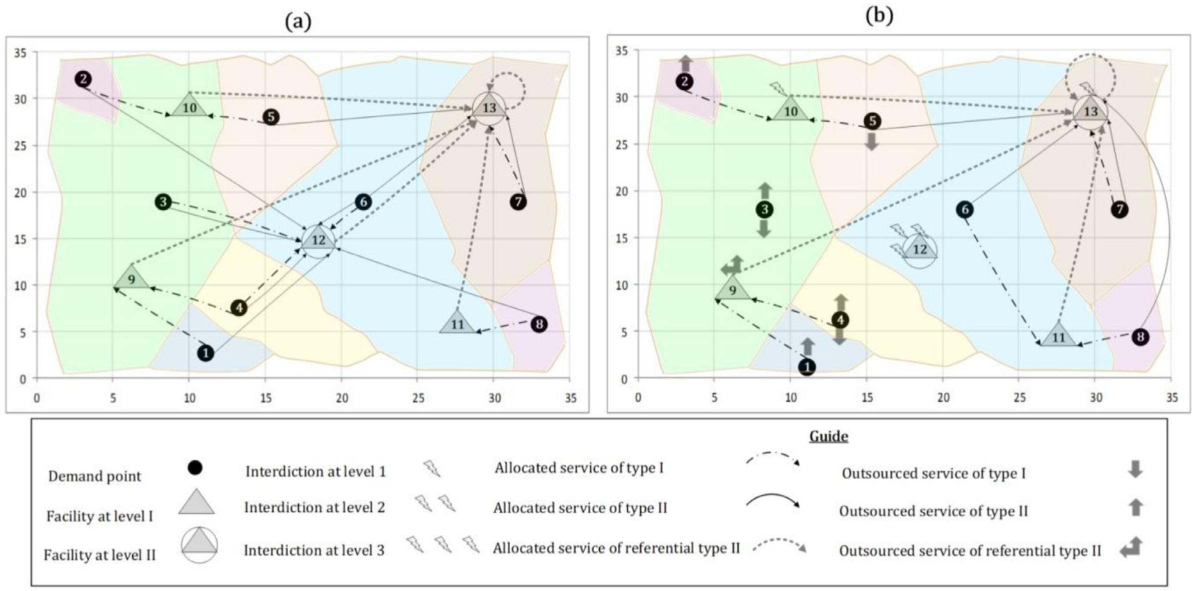

Fig. 1. An example of a vulnerable hierarchical service system.

tial type II are respectively denoted by α, β and γ, where $\alpha < \beta < \gamma$ in most real service systems.

Fig. 1(a) illustrates the allocation pattern of the customers to the facilities of a service system before interdiction. This hierarchical system consists of eight demand points (numbered from 1 to 8) which are allocated to three facilities at level I (numbered from 9 to 11) and two facilities at level II (numbered 12 and 13) to receive service type I and II. Moreover, facilities 9 to 12 are allocated to facility 13 to serve the proportion of their demand, which is in need of the service of referential type II. However, for facility 13, this proportion of demand is satisfied through self-referentiality.

In this problem, we assume that an interdictor can attack facilities partially. The discrete levels of interdiction are denoted by k and the reduction in the capacity of a facility at level I or II is a linear function of its interdiction level. After interdiction, due to the reduction in the capacity of the facilities, the system is not able to serve the customers through the previous assignment pattern. Therefore, the system defender benefits from two options: reassignment and outsourcing. The outsourcing cost is independent of the travelled distance and is calculated through multiplying the amount of outsourced demand by the outsourcing cost coefficient. The outsourcing cost coefficients for service of type I, type II and referential type II are indicated, respectively, by α', β' and γ'.

To promote clarity, see Fig. 1(b) which shows the hierarchal system after interdiction. In this system, it is assumed that the interdictor attack facilities at four levels (from 0 to 3). Facilities 9 and 11 are not interdicted (i.e. level 0), facilities 10 and 13 are interdicted at level 1 and facility 12 is fully interdicted (i.e. level 3).

4. Bi-level formulation

To give a formal description of the developed model, some notations and parameters are introduced as follows:

Sets:

I	set of customers,
S_1	set of existing facilities at level I,
S_2	set of existing facilities at level II,
K	set of different interdiction intensity levels.

Transportation cost parameters:

α	transportation cost from a customer to a facility at level I per unit of distance,
β	unit transportation cost from a customer to a facility at level II per unit of distance,
γ	unit transportation cost from a facility at level I to a facility at level II per unit of distance,

Outsourcing cost parameters:

α'	unit outsourcing cost for service of type I,
β'	unit outsourcing cost for service of type II,
γ'	unit outsourcing cost for referential type II service,

Interdiction cost parameters:

h_k^1	cost of attack on a facility at level I at interdiction level k,
h_k^2	cost of attack on a facility at level II at interdiction level k,

Demand-related parameters:

z_i	demand at customer i,
θ	proportion of the demand that is referred to receive service of type I,
σ	proportion of the demand that is referred to receive service of type II after receiving service of type I,

Interdiction budget and level parameters:

B	total interdiction budget.
k_{max}	the maximum interdiction intensity level (i.e. full interdiction),

Capacity-related parameters:

c_j^1	initial capacity of facility j for service of type I,
c_j^2	initial capacity of facility j for service of type II,
d_k^1	reduction ratio in capacity of a facility at level I after interdiction at level k,
d_k^2	reduction ratio in capacity of a facility at level II after interdiction at level k,

Distance parameters:

l_{ij}	shortest distance between customer i and facility j,
l'_{jf}	shortest distance between facility j and facility f,

The definition of the decision variables is as follows:

Reassignment variables:

u_{ij}^1	amount of demand of customer i that is allocated to facility j for service of type I,
u_{ij}^2	amount of demand of customer i that is allocated to facility j for service of type II,
u_{jf}^3	amount of demand of facility j that is referred to facility f to receive referential service of type II,

Outsourcing variables:

o_i^1	amount of demand of customer i that is outsourced to receive service of type I,
o_i^2	amount of demand of customer i that is outsourced to receive service of type II,

o_j^3 total amount of outsourced demand for referential service of type II that is allocated to facility j,

Interdiction variables:

x_{jk} binary variable, equal to one if facility j is interdicted at level k.

Finally, the mathematical bi-level formulation is as follows:

$$Max\ H(z) \tag{1}$$

$$\text{Subject to} \sum\nolimits_{k\in \boldsymbol{K}} x_{jk} = 1, \forall j \in \boldsymbol{S}_1, \boldsymbol{S}_2 \tag{2}$$

$$\sum\nolimits_{j\in \boldsymbol{S}_1} \sum\nolimits_{k\in \boldsymbol{K}} h_k^1 x_{jk} + \sum\nolimits_{j\in \boldsymbol{S}_2} \sum\nolimits_{k\in \boldsymbol{K}} h_k^2 x_{jk} \le B \tag{3}$$

$$x_{jk} \in \{0, 1\}, \forall j \in \boldsymbol{S}_1, \boldsymbol{S}_2, k \in \boldsymbol{K} \tag{4}$$

where x_{jk}, ($\forall j \in \boldsymbol{S}_1,\boldsymbol{S}_2, \forall k \in \boldsymbol{K}$) solve:

$$\begin{aligned} H(z) = Min\Big(&\sum\nolimits_{i\in \boldsymbol{I}} \sum\nolimits_{j\in \boldsymbol{S}_1} l_{ij}\alpha u_{ij}^1 + \sum\nolimits_{i\in \boldsymbol{I}} \sum\nolimits_{j\in \boldsymbol{S}_2} l_{ij}\beta u_{ij}^1 \\ &+ \sum\nolimits_{i\in \boldsymbol{I}} \sum\nolimits_{j\in \boldsymbol{S}_2} l_{ij}\beta u_{ij}^2 + \sum\nolimits_{j\in \boldsymbol{S}_1,\boldsymbol{S}_2} \sum\nolimits_{f\in \boldsymbol{S}_2} l'_{jf}\gamma u_{jf}^3 \\ &+ \sum\nolimits_{i\in \boldsymbol{I}} \alpha' o_i^1 + \sum\nolimits_{i\in \boldsymbol{I}} \beta' o_i^2 + \sum\nolimits_{j\in \boldsymbol{S}_1,\boldsymbol{S}_2} \gamma' o_j^3 + \sigma \sum\nolimits_{i\in \boldsymbol{I}} \gamma' o_i^1\Big) \end{aligned} \tag{5}$$

$$\text{Subject to}\ \theta z_i = \sum\nolimits_{j\in \boldsymbol{S}_1,\boldsymbol{S}_2} u_{ij}^1 + o_i^1, \forall i \in \boldsymbol{I} \tag{6}$$

$$(1-\theta) z_i = \sum\nolimits_{j\in \boldsymbol{S}_2} u_{ij}^2 + o_i^2, \forall i \in \boldsymbol{I} \tag{7}$$

$$\sigma \sum\nolimits_{i\in \boldsymbol{I}} u_{ij}^1 = \sum\nolimits_{f\in \boldsymbol{S}_2} u_{jf}^3 + o_j^3, \forall j \in \boldsymbol{S}_1, \boldsymbol{S}_2 \tag{8}$$

$$\sum\nolimits_{i\in \boldsymbol{I}} u_{ij}^1 \le c_j^1 - c_j^1 \sum\nolimits_{k=1}^{k_{max}} d_k^1 \widehat{x_{jk}}, \forall j \in \boldsymbol{S}_1 \tag{9}$$

$$\sum\nolimits_{i\in \boldsymbol{I}} u_{ij}^1 \le c_j^1 - c_j^1 \sum\nolimits_{k=1}^{k_{max}} d_k^2 \widehat{x_{jk}}, \forall j \in \boldsymbol{S}_2 \tag{10}$$

$$\sum\nolimits_{i\in I} u_{ij}^2 + \sum\nolimits_{f\in \boldsymbol{S}_1,\boldsymbol{S}_2} u_{fj}^3 \le c_j^2 - c_j^2 \sum\nolimits_{k=1}^{k_{max}} d_k^2 \widehat{x_{jk}}, \forall j \in \boldsymbol{S}_2 \tag{11}$$

$$u_{ij}^1, u_{ij}^2, u_{jf}^3, o_i^1, o_i^2, o_i^3 \ge 0, \forall i \in \boldsymbol{I}, \forall j \in \boldsymbol{S}_1, \boldsymbol{S}_2, \forall f \in \boldsymbol{S}_2 \tag{12}$$

This model consists of two levels. At the upper level (i.e., (1)-(4)), the most destructive interdiction scenario is identified, while at the lower level (i.e., (5)-(12)), the demand satisfaction pattern is optimized. The interdictor's objective function, as shown in (1), is to maximize the minimum total demand satisfaction cost which is shown in (5). In (2), choosing exactly one interdiction level, including level 0 (i.e. no interdiction), for each facility is enforced. Constraint (3) restricts the interdiction budget. Constraints (4) ensure binary condition for the interdiction decision variables. The defender's objective function, which is shown in (5), is similar to the attacker's objective functions but in the opposite direction. The first three terms in the objective function indicate the cost of satisfying customers' demand through available facilities, whereas the next five terms show the cost of satisfying customers' demand through outsourcing. Constraints (6)-(8) guarantee that all the customers' demand is met by the facilities in the network and/or outsourcing. Constraints (9)-(11) are facility-capacity constraints and enforce choosing an allocation pattern which respects the capacity constraints after interdiction. Lastly, constraints (12) ensure non-negativity condition for the defender's decision variables.

5. Heuristic procedure

Since bi-level programming problems belong to the class of NP-hard problems (Jeroslow, 1985), in this section, we propose a heuristic procedure, called FDS, to find near optimal solutions for large-sized instances. FDS is decomposed into two stages, namely (1) the constructive stage in which an initial solution is built; and (2) the improvement stage where the solution is iteratively improved through a local search procedure.

5.1. Constructive stage (CS-FDS)

In this section, we propose two algorithms to obtain an initial feasible solution. The details of these constructive algorithms are provided in the following subsections.

5.1.1. First constructive algorithm (CS1)

In order to build an initial solution, the facilities should be prioritized. In the first constructive algorithm, the priority of a facility, from the interdictor point of view, is proportional to the total weighted demand allocated to it in the absence of any attacks. We summarize the steps of the first constructive algorithm as follows:

Step 1:

Set $x_{jk}=1$ for all facilities at interdiction level $k=0$ and $x_{jk}=0$ for other interdiction levels. Subsequently, solve the follower level of the model to obtain the optimal amount of assignment variables (i.e., u_{ij}^{1*}, u_{ij}^{2*} and u_{ij}^{3*}). Following this step, relations (13)-(15) give, respectively, the aggregated demand of type I, type II and referential type II allocated to each facility.

$$t_j^1 = \sum\nolimits_{i\in I} u_{ij}^{1*}, \forall j \in \boldsymbol{S}_1, \boldsymbol{S}_2 \tag{13}$$

$$t_j^2 = \sum\nolimits_{i\in I} u_{ij}^{2*}, \forall j \in \boldsymbol{S}_2 \tag{14}$$

$$t_j^3 = \sum\nolimits_{f\in S_1,S_2} u_{fj}^{3*}, \forall j \in \boldsymbol{S}_2 \tag{15}$$

Step 2:

Calculate the weighted demand allocated to each facility at level I and level II, respectively, through Eqs. (16) and (17). Note that in an equal condition, allocating a customer for service of type I to a facility at level I is preferred to allocating it to a facility at level II. Hence, for the facilities at level I, t_j^1 is multiplied by the larger coefficient (i.e., β) and for those facilities at level II, t_j^1 is multiplied by the smaller coefficient (i.e., α).

$$w_j = \beta . t_j^1, \forall j \in \boldsymbol{S}_1 \tag{16}$$

$$w_j = \alpha . t_j^1 + \beta . t_j^2 + \gamma . t_j^3, \forall j \in \boldsymbol{S}_2 \tag{17}$$

Step 3:

Sort the facilities in a descending order according to w_j. Ties are broken by giving higher priority to the facility with the lower index.

Step 4:

Select the top priority facility and assign it the highest feasible interdiction level with regard to the interdiction budget. Then, update the residual interdiction budget and select the next top priority facility and continue the above-mentioned procedure until assigning an interdiction level (including level 0) to all available facilities.

5.1.2. Second constructive algorithm (CS2)

In the second algorithm, we develop an innovative imaginary interpretation of this two-player game. In particular, the assumption is that the interdictor and the defender agree to sign a peace treaty. This treaty forces the defender to give up some of her facilities to the interdictor. Obviously, according to the defender's objective function, she will give up the facilities with the lowest priority. We name this problem as "Reverse interdiction problem". Reverse interdiction problem is formulated as a non-linear single-level model. Since the application of this theoretical problem has been limited to identifying the importance of the facilities, we reformulate it as a linear one. For this purpose, we assume a continuous interdiction instead of discrete interdiction intensity levels. For this reformulation, some new notations must be defined. In particular, r_j shows the capacity fraction of facility j that is lost with regard to this imaginary treaty. In addition, ϱ_1 and ϱ_2 are two parameters which respectively show the cost of full interdiction of a facility at levels I and II. Finally, B' is the agreed price in the peace treaty and the defender must give up her facilities with regard to this parameter. In the reverse interdiction model, the objective function, presented in (5), and constraints (6)-(8) and (12) remain unchanged. The rest of the constraints read as follows:

$$\sum\nolimits_{j\in S_1} \varrho_1 r_j + \sum\nolimits_{j\in S_2} \varrho_2 r_j \ge B' \tag{18}$$

$$\sum\nolimits_{i\in \boldsymbol{I}} u_{ij}^1 \le c_j^1 - c_j^1 r_j, \forall j \in \boldsymbol{S}_1 \tag{19}$$

$$\sum\nolimits_{i\in \boldsymbol{I}} u_{ij}^1 \le c_j^1 - c_j^1 r_j, \forall j \in \boldsymbol{S}_2 \tag{20}$$

$$\sum_{i\in I} u_{ij}^2 + \sum_{f\in \boldsymbol{S}_1,\boldsymbol{S}_2} u_{fj}^3 \le c_j^2 - c_j^2 r_j, \forall\ j \in \boldsymbol{S}_2 \tag{21}$$

$$0 \le r_j \le 1, \forall j \in \boldsymbol{S}_1, \boldsymbol{S}_2 \tag{22}$$

In this model, constraint (18) forces the defender to give up several of her available facilities with regard to the agreed price (B'). Modified capacities of the facilities for service of type I and type II are shown in constraints (19)-(21). Constraints (22) impose the required limit on the decision variables.

In the second constructive algorithm, to generate an initial solution, we benefit from the solution of the reverse interdiction problem. Essentially, this solution is an anti-ideal solution for the interdictor with regard to her available budget. Therefore, from the interdictor's perspective, the priority of the facilities is in reverse order of the solution obtained by the reverse interdiction problem. Thus, the steps of the second constructive approach are summarized as follows:

Step 1:

Set $B' = B$ and use CPLEX to solve the reverse interdiction problem. For each $j \in \boldsymbol{S}_1,\boldsymbol{S}_2$, set $G_j = r_j^*$ in which r_j^* is the optimal value of the capacity proportion of facility j that is lost with regard to the agreed price (B'). Note that the facilities which are submitted at this agreed price are considered as the low priority facilities.

Step 2:

Set $B' = [(\varrho_1 \times |S_1| + \varrho_2 \times |S_2|) - B]$ and use CPLEX to solve the reverse interdiction problem. For each $j \in \boldsymbol{S}_1,\boldsymbol{S}_2$, set $(G_j = G_j + r_j^*)$. Note that those facilities which are not submitted at this agreed price are considered as the high priority facilities.

Step 3:

Sort the facilities in an ascending order according to G_j. If G_j of two facilities is the same, give higher priority to the facility with the lower index.

Step 4:

Similar to step 4 of *CS1.*

5.2. Improvement stage (IS-FDS)

In this section, we propose a heuristic which aims at improving the initial solution. The basic framework of the proposed heuristic algorithm is presented in Section 5.2.1, followed by the detailed description of the search policies in next subsections.

5.2.1. Basic framework of the heuristic algorithm

The proposed heuristic algorithm is composed of three polices. The first policy (policy I) is called "elite-based policy". This policy examines the neighborhood of the current interdiction pattern. In case of having an improvement in the objective function of the solution, it substitutes for the current pattern. During this procedure, a matrix of elite attribute of the improved solutions is constructed. This procedure continues until the stopping criterion of policy I is met. When the termination criterion of policy I is met, policy II, namely "diversification policy" is executed. The aim of this policy is to try to escape from local optimal solution by perturbing the solution. To do so, it will use the Elite matrix to guide the search toward the regions which are not investigated by applying policy I. Except for the acceptance criterion, the search procedure in policy III is similar to policy I. After running an iteration based on policy III, the stopping condition of the whole algorithm is checked. If this criterion is met, the algorithm ends and the best solution is reported. Otherwise, the acceptance criterion of policy III is checked. In this policy, acceptance of a worse neighbor solution under a particular condition is allowed with the purpose of encouraging the algorithm to change the neighborhood. Fig. 2 illustrates the basic framework of the proposed procedure.

5.2.2. Policy I: elite-based policy

In this procedure, three different methods, namely, depth I, depth II and depth III are introduced to generate a neighbor solution. The steps of constructing a neighbor solution through depth I are explained as follows:

Step 1:

Initialize the residual budget of the current interdiction pattern as$[B - \sum_{j\in \boldsymbol{S}_1} \sum_{k\in \boldsymbol{K}} h_k^1 x_{jk} + \sum_{j\in \boldsymbol{S}_2} \sum_{k\in \boldsymbol{K}} h_k^2 x_{jk}]$, in which x_{jk} is equal to 1 if facility j is interdicted at level k in the current solution.

Step 2:

Select two facilities j' and j''at level I. Note that these facilities are chosen based on "in-depth" strategies which are explained later. Following this selection, save their interdiction levels in k_1 and k_2, respectively, and add their corresponding interdiction budget to R, i.e. $R = R + h_{k_1}^1 + h_{k_2}^1$.

Step 3:

Select two interdiction levels k' and k'' that their aggregate interdiction cost does not exceed R (i.e. $R > h_{k'}^1 + h_{k''}^1$) and assign them to the selected facilities. Since in policy I, "best improvement" strategy is applied, all possible neighbor solutions for selected facilities are constructed and the interdiction pattern which maximizes the objective function (1) is accepted. As an example, see Fig. 3 in which facilities 1 and 2 from facilities at level I are selected and two new feasible interdiction levels are assigned to them.

In depth II, the procedure of constructing a neighbor solution is similar to depth I except that both facilities are chosen from facilities at level II.

The purpose of depth III is to encourage the algorithm to change the budget allocation contribution between both facilities at levels I and II. In particular, selecting the same number of facilities at each level most probably does not result in good neighbors. Assume v_1 and v_2 are, respectively, the number of selected facilities at levels I and II. In most cases, when the value of $\varrho_1 v_1$ and $\varrho_2 v_2$ are almost the same, the search will reach to better results. Hence in this procedure, we suggest the values of v_2 and v_1 to be, respectively, equal to 1 and $\frac{\varrho_2}{\varrho_1}$. For example, assume the cost of full interdiction of facilities at levels I and II are, respectively, equal to 4000 and 11,000. Thus, through depth III, to construct a neighbor solution three facilities are selected from facilities at level I

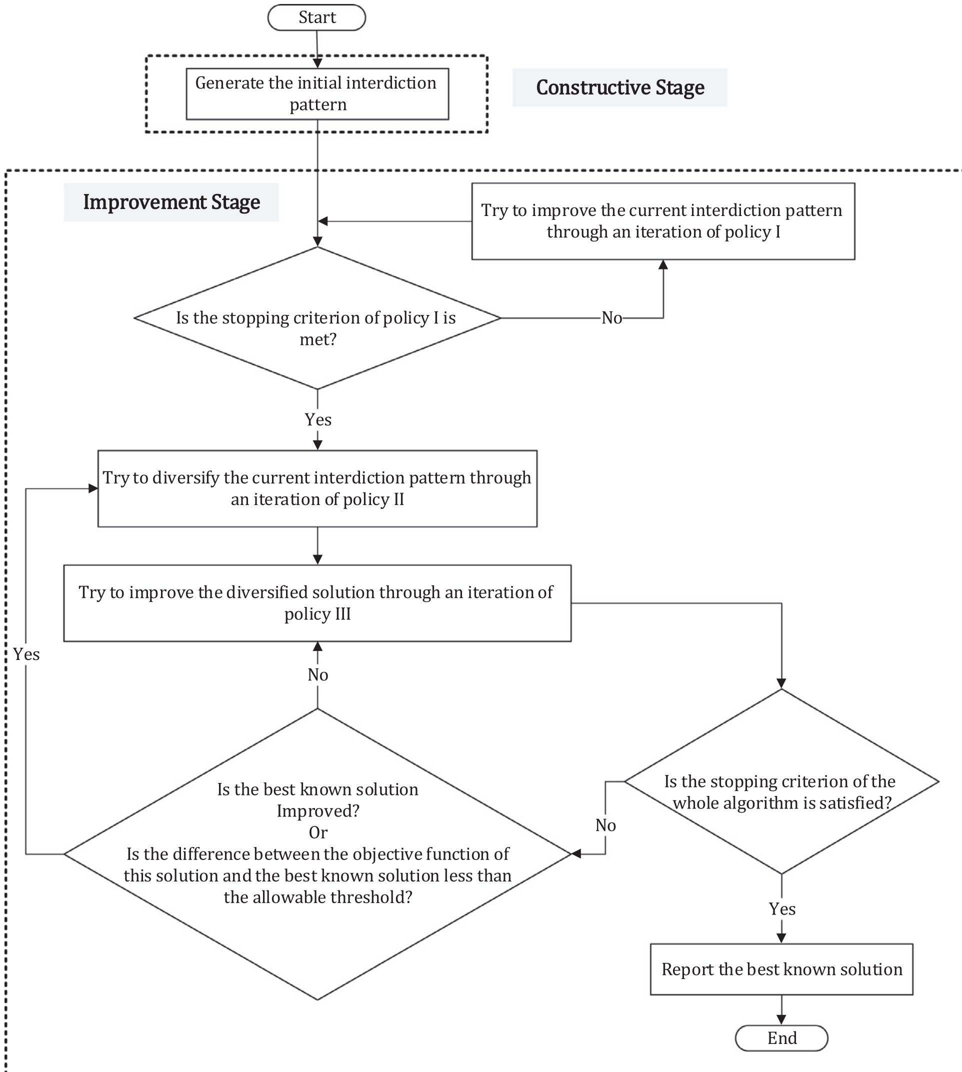

Fig. 2. Basic framework of the FDS algorithm.

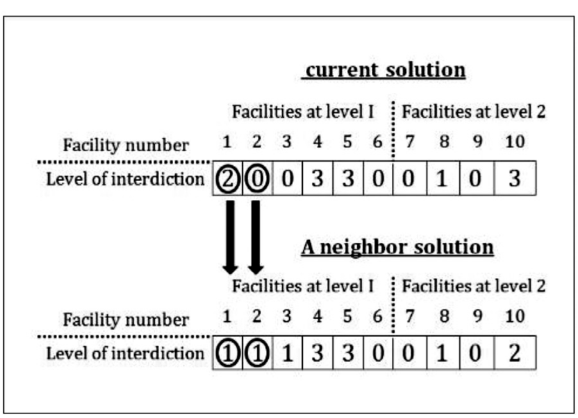

Fig. 3. An illustrative example of constructing a neighbor solution through depth I.

and one facility from facilities at level II. Other steps of the neighborhood constructing procedure are similar to that of depths I and II.

At the beginning of each iteration, the algorithm benefits from a long-term memory to select an appropriate depth with regard to its performance in prior iterations. The entrance probability to each depth is calculated through formula (23). In this relation, λ is the counter of this learning procedure, ρ_i counts the number of entrance to depth i and η_i counts the times that depth i improves the best known solution. In addition, e is a weight that is considered for entrance to each depth with equal probability. In early iterations, the historical data on the performance of each depth are not sufficient to decide which depth will result in a better neighbor structure. Hence, to improve the learning procedure an equal probability for entering each depth (i.e., $\frac{1}{3} \times e$) is considered. Over time, by increasing the corresponding value of λ, the impact of e upon the probability of selecting the depths decreases.

$$prob(i) = \frac{\left(\left(\frac{\eta_i}{\rho_i}\right) \times \lambda\right) + \left(\frac{1}{3} \times e\right)}{\sum_{i=1}^{3} \left(\left(\frac{\eta_i}{\rho_i}\right) \times \lambda\right) + \left(\frac{1}{3} \times e\right)}; i \in \{1, 2, 3\} \tag{23}$$

After selecting an appropriate depth through the learning procedure, two different strategies, namely "in-depth" strategies, can be used to decide upon the facilities which are selected for generating a neighbor solution. The probability of selecting facilities in each depth is modified based on the chosen in-depth strategy and a matrix, namely "Selection matrix", is constructed with regard to the current solution and the corresponding in-depth strategy.

New improved solution

Facility Level	Level I				Level II		
Facility Number	1	2	3	4	5	6	7
Interdiction Level	0	3	0	0	2	1	0

Current Elite matrix

Facility Level	Level I				Level II		
Facility Number	1	2	3	4	5	6	7
Number of presence	0	5	1	3	3	3	1

New Elite matrix

Facility Level	Level I				Level II		
Facility Number	1	2	3	4	5	6	7
Number of presence	0	6	1	3	4	4	1

Fig. 4. An example of updating Elite matrix.

In depth I and II, the first in-depth strategy has a tendency to choose one facility from those which are interdicted at high levels and another from the facilities which are non-interdicted or interdicted at low levels. However, the second in-depth strategy gives priority to selecting two facilities from ones which are both interdicted at high levels.

In depth III, through using the first in-depth strategy, there is a tendency to choose facilities at level I which are interdicted at low levels and one facility at level II which is interdicted at a high level. However, in the second in-depth strategy an inverse approach is applied.

In each depth, a short-term memory is applied to select an appropriate in-depth strategy. If in the previous entrance to a depth, the quality of the solution improves, again the same in-depth strategy will be selected. Otherwise, another strategy will be chosen.

In each iteration, if a new solution is accepted, its characteristics are saved in the "Best-solution matrix" and its objective function is saved in ξ variable. The history of accepted solution set is saved in a matrix which is named "Elite matrix". The algorithm will benefit from this matrix in policy II. In Elite matrix, for the accepted solution set, the number of presences of each facility at a non-zero interdiction level is counted. Hence, in each iteration of this policy, Elite matrix must be updated. In Fig. 4, an example of updating Elite matrix in an iteration of the algorithm is presented. In the new improved solution, facilities 2, 5 and 6 are interdicted at non-zero levels, thus, in the new Elite matrix, their corresponding values increase by one unit.

The algorithm takes advantage of a long-term memory for recognition of repetitious solutions and in order to decrease the processing time, CPLEX is only called for new interdiction patterns.

Finally, when the number of total unimproved iteration reaches its stop criterion (e_1), this policy is terminated.

5.2.3. Policy II: diversification

Local search methods can get stuck in local optima, where no improving neighbors are available. Hence, in this policy, the algorithm tries to diversify the attributes of improved solutions which were saved in Elite matrix. Since the learning procedure (see, Section 5.2.2) tries to intensifies the high-quality solutions, in policy II the depths are not chosen according to it. In this policy, the depths for constructing neighbor solutions are chosen in sequence. In each entrance to policy II, only one iteration is run and a short-term memory containing the number of pervious chosen depth is used to choose a new depth in the subsequent entrance.

Unlike policy I, in this policy, the chosen in-depth strategy tends to solution perturbation through decreasing the interdiction levels of those facilities with high respective values in Elite matrix and increasing the interdiction levels of those facilities with low respective values in the that matrix.

In each entrance to policy II, only one iteration is run and the solution is accepted unless it is considered as a repetitious one. Afterwards, the algorithm enters to policy III.

5.2.4. Policy III: modified neighborhood search

The description of the operators and the details of policy III are similar to which already explained in Section 5.2.2 except for the acceptance criterion for a created solution and the termination criteria.

To propel the search procedure into a modified neighborhood, the acceptance of a new solution even when its respective objective function is worse than the best solution (ξ) under a specific condition is allowed. This condition is that the distance between objective function of a worse solution and ξ does not exceed an allowable threshold (g).

In each entrance to policy III, with an increase in the number of iterations, this allowable threshold gradually decreases. The goal of considering a larger allowable threshold in initial iterations is to accelerate the procedure of escaping from local optima. However, in last iterations the concentration of this search policy centered upon improving the quality of the solutions. Eq. (24) shows how this adaptive threshold is calculated in each iteration of policy III. In this relation, ω denotes the initial threshold and q denotes the counter of policy III. Note that in each entrance to policy III, q is set to one.

$$g = \frac{\omega}{q \times 100} \tag{24}$$

This policy is terminated if one of the termination criteria is met. The first criterion is that in an iteration of this policy, the distance between the objective function of the neighbor solution and ξ be more than the allowable threshold or the best-known solution is improved. Under this condition, the algorithm enters to policy II. The second criterion is that the number of total unimproved iterations be equal to the stop criterion of the whole algorithm (e_2). Under this condition, the algorithm is terminated and the best solution is reported.

6. Enumeration solution procedure

We use an exhaustive enumeration method to find the optimal solution of the proposed bi-level model. In this method, all interdiction patterns are produced and for each pattern, we use CPLEX to solve the lower level problem. Finally, the optimal interdiction pattern is the solution which maximizes the objective function (1).

The following self-evident lemma is applied to reduce the number of CPLEX's calls:

Lemma. *If the residual budget of an interdiction pattern is sufficient for interdicting at least one non-interdicted facility or an interdicted facility at a higher level, the interdiction pattern is considered as a dominated solution and CPLEX is not called for this solution.*

In large-scale real-world problems, enumerating all interdiction patterns is inapplicable. Therefore, we develop a two-stage enumeration procedure to achieve high quality solutions within a limited time for computing.

6.1. Stage I: ordinary enumeration

The goal of this stage is to generate as much as interdiction patterns as possible and this stage of the algorithm does not give any priority to any specific interdiction patterns. The termination criteria for this stage are (1) to reach the time limit (stage I time limit) or (2) to enumerate all non-dominated patterns. If the first criterion is met, this stage is terminated and the algorithm proceeds by running the second stage. If the second criterion is met, the whole enumeration algorithm is terminated and the optimal solution is reported.

6.2. Stage II: modified enumeration

At first, the best solutions achieved by applying the first stage of the enumeration algorithm and the heuristic procedure are identified and the best one is selected as the input of the second stage. Following this step, we use a heuristic approach to improve the quality of the best-known solution. Essentially, for the selected solution, the interdiction levels of several facilities are fixed and for other facilities different interdiction patterns are enumerated. For this purpose, reverse interdiction problem and the attributes of the best solution are taken into consideration. The interdiction levels of two kinds of facilities are considered as the fixed ones:

- Fixed at level 0: the facilities which are not interdicted at the best-known solution and their respective G_j is equal to 2 in the reverse interdiction problem, i.e. are submitted completely under the both agreed prices (B').
- Fixed at the maximum level: the set of facilities which are interdicted at the maximum level in the best-known solution with low respective G_j in reverse interdiction problem are identified. As much as 50% percent of the available interdiction budget (B) assigns to full interdiction of those facilities which are chosen randomly from this facility set. The stopping criteria of this stage are to reach the time limit (stage II time limit) or enumerating all interdiction patterns for the non-fixed facilities. If one of the stopping criteria of this stage is met the enumeration algorithm is terminated and the best solution is reported.

7. Computational experiments

In this section, firstly the data structure for computational experiments is introduced and through analyzing the results of 18 sample data, the parameters of the algorithms are set. Next, in order to investigate the performance of the heuristic and the enumeration algorithm, they are tested on 54 further instances. Moreover, the behavior of the proposed bi-level model is analyzed and to prove the effectiveness of this model, a computational test is conducted on 21 instances for which the optimal solutions are available.

The introduced algorithms have been coded in C++ and compiled with Microsoft Visual Studio 2010. Moreover, to solve the lower level problem, ILOG CPLEX 12.3 is used. All proposed algorithms are tested on a 32-bit computer benefits from Intel Core i7 2.93 processor and 3.49 GB of RAM.

7.1. Random data generation

The generated instances are categorized in six series according to the number of facilities. In particular, in each series, the structure of data is the same and only the number of interdiction levels and the interdiction budget are different from each other. Finally, each series consists of nine instances leading to 54 instances. Table 1, reports the details for generating the instances.

In each series, for three instances, only full interdiction is allowed, thus two interdiction levels, i.e. full interdiction (level 1) and no interdiction (level 0) are considered. While for other instances, partial interdiction is allowed as well. Thus, for instances having three interdiction levels, the whole capacity of a facility is lost when it is interdicted at level 2, and half of its capacity is lost when it is interdicted at level 1 and no interdiction is shown with level 0. Analogously, when there are four interdiction levels, levels 3 and 0 show full interdiction and no interdiction, respectively. More precisely, levels 2 and 1 indicate 33% and 66% capacity loss, respectively.

Table 1
Data generation.

Parameters	Values
$\lvert S_2\rvert$	$\{4, 6, 8, 10, 12, 14\}$
$\lvert S_1\rvert$	$1.5 \times \lvert S_2\rvert$
$\lvert I\rvert$	$5 \times (\lvert S_1\rvert + \lvert S_2\rvert)$
$\lvert K\rvert$	$\{2, 3, 4\}$
B_L	$0.2 \times \lvert S_1\rvert \times \varrho_1 + 0.2 \times \lvert S_2\rvert \times \varrho_2$
B_M	$0.4 \times \lvert S_1\rvert \times \varrho_1 + 0.4 \times \lvert S_2\rvert \times \varrho_2$
B_H	$0.6 \times \lvert S_1\rvert \times \varrho_1 + 0.6 \times \lvert S_2\rvert \times \varrho_2$
(ϱ_1, ϱ_2)	(4000, 11000)
(R, L)	(1000, 1500)
$(x_i, y_i);\ i \in I$	$R_i = R \times U(0, 1),\ \theta_i = 2\pi \times U(0, 1);\ x_i = R_i \cos\theta_i, y_i = R_i \sin\theta_i$
$(x_j, y_j);\ j \in S_1$	$x_j = -0.5L \times \frac{L}{\lvert S_1\rvert} \times U(0, \lvert S_1\rvert); y_j = -0.5L \times \frac{L}{\lvert S_1\rvert} \times U(0, \lvert S_1\rvert)$
$(x_j, y_j);\ j \in S_2$	$x_j = -0.5L \times \frac{L}{\lvert S_2\rvert} \times U(0, \lvert S_2\rvert);\ y_j = -0.5L \times \frac{L}{\lvert S_2\rvert} \times U(0, \lvert S_2\rvert)$
z_i	$U(1000, 2000)$
(θ, σ)	(0.7, 0.1)
c_j^1	$\frac{\theta \times \sum_{i\in I} z_i}{\lvert S_1\rvert + \lvert S_2\rvert} + U(0, 0.15 \times [\frac{\theta \times \sum_{i\in I} z_i}{\lvert S_1\rvert + \lvert S_2\rvert}])$
c_j^2	$\frac{\sigma \times \theta \times \sum_{i\in I} z_i + \theta \times \sum_{i\in I} z_i}{\lvert S_2\rvert} + U(0, 0.15 \times [\frac{\sigma \times \theta \times \sum_{i\in I} z_i + \theta \times \sum_{i\in I} z_i}{\lvert S_2\rvert}])$
h_k^1	$\frac{k}{\lvert K\rvert - 1} \times \varrho_1$
h_k^2	$\frac{k}{\lvert K\rvert - 1} \times \varrho_2$
d_k^1	$\frac{k}{\lvert K\rvert - 1}$
d_k^2	$\frac{k}{\lvert K\rvert - 1}$
(α, β, γ)	(1, 2, 3)
$(\alpha', \beta', \gamma')$	$(2R\alpha, 2R\beta, 2R\gamma)$

Three different levels for available interdiction budget are considered. B_L shows the lowest level and the available budget in this level is sufficient for full interdiction of around 20% of facilities at level I and II. While B_M and B_H indicate medium and the highest level of available budget which is sufficient for full interdiction of around 40% and 60% of facilities of each level, respectively. In Table 1, ϱ_1 and ϱ_2 show the required budget for full interdiction of each facility at level I and II, respectively.

We benefit from the approach which is applied in Church et al. (2004) to identify the coordinates (x, y) of the customers and the facilities in each data series. The customers are uniformly dispersed on a circular area with radius (R) equal to 1000 which is centered at the origin (0,0). Those facilities at levels I and II are uniformly distributed on several imaginary equidistant vertical and horizontal lines. The number of these lines for the facilities at level I is $\lvert S_1\rvert + 1$ and for the facilities at level II is $\lvert S_2\rvert + 1$ and these imaginary lines dice a square centered at the origin (0,0) with a length (L) of 1500.

Each instance is made of a unique combination of three independent parameters (i.e., the number of facilities at level II, the level of available budget and the number of interdiction intensity levels) thus $6 \times 3 \times 3 = 54$ different instances can be generated. The main characteristic of each series is the number of its upper level and lower level facilities and the number of customers are proportional to it. For series one to six, the numbers of upper level facilities are 4, 6, 8, 10, 12 and 14, respectively.

7.2. Parameter setting

The most influential parameters of the heuristic algorithm are (1) e_1: stop criterion of policy, (2) e_2: stop criterion of the whole algorithm, and 3) ω: initial allowable threshold. Table 2, gives the

Table 2
Parameter tuning.

Parameters	Different Tested values	Selected value
e_1	$\{2, 5\} \times (\lvert S_1\rvert + \lvert S_2\rvert)$	5
e_2	$\{8, 10, 12, 14\} \times (\lvert S_1\rvert + \lvert S_2\rvert)$	8
ω	{10%, 50%}	10%

different tested values and the corresponding selected ones for each of these parameters. To select the best values of the main parameters in the heuristic algorithm, all 16 possible combinations of these parameters are tested on 18 sample instances. Thereupon, non-parametric Friedman test with p-value equal to 0.05 is used to compare the performance of each parameter combination. The test shows that there is no meaningful difference among the performance of these 16 parameter combinations. Thus, the combination which results in the least average running time is selected.

7.3. Computational results of the solution methods

In this section, to obtain the definitive structure of the enumeration algorithm and heuristic algorithm, computational results in both the absence and the presence of their crucial operators are analyzed. Subsequently, a comparison between the definitive structure of the heuristic and the enumeration procedures is drawn and the stability of the heuristic procedure is tested.

7.3.1. Identifying the enumeration algorithm structure

In comprehensive enumeration, all interdiction patterns are produced and for each pattern, CPLEX is called to solve the lower level problem. The number of interdiction patterns depends on the number of facilities in level I and II and the number of interdiction intensity levels. For example, in the largest problem size with 21 and 14 facilities for level I and II, respectively, and 4 interdiction intensity levels, there are 4^{35} combinations for interdiction that need a huge amount of time to be solved. Hence we considered 25 and 5 h time limit for stage I and stage II of the enumeration algorithm, respectively, in order to ensure getting high quality solutions for evaluation the proposed heuristic method.

Within the time limit of stage I of the enumeration algorithm for the instances of series 1 and 2 and instances 19, 20, 21 the optimal solutions are obtained. Thus, the second stage of the enumeration algorithm is not executed for these instances. For the large-scale instances, i.e. instances 22 to 54 in which their optimal solutions are not obtained, it is seen that the average improvement in the objective function after running the modified enumeration stage is 2.44%. In Fig. 5, this considerable improvement is shown.

Table 3
Performance of FDS with regard to the structure of its initial solution.

	CS1		CS2		FDS (CS1)		FDS (CS2)	
Series	Time	GAP%	Time	GAP%	Time	GAP%	Time	GAP%
1	0.05	6.00	0.09	4.55	17.87	0.00	13.85	0.00
2	0.09	5.98	0.17	5.30	38.64	0.00	34.51	0.00
3	0.15	7.46	0.26	5.39	89.99	0.20	77.72	0.16
4	0.22	7.34	0.43	5.76	176.3	0.21	151.6	0.11
5	0.30	11.43	0.56	7.48	265.0	0.68	241.5	0.46
6	0.40	8.06	0.71	5.88	424.8	0.81	413.2	0.38
Avg.	0.20	7.71	0.37	5.73	168.7	0.32	155.4	0.19

Table 4
Performance of the learning procedure.

	With Learning		Without Learning	
Series	Time	GAP%	Time	GAP%
1	13.85	0.00	14.88	0.04
2	34.51	0.00	37.40	0.01
3	77.72	0.16	88.19	0.06
4	151.63	0.11	159.15	1.38
5	241.56	0.46	244.52	1.38
6	413.16	0.38	473.05	0.50
Avg.	155.4	0.19	169.53	0.56

Hence, in the rest of the paper, for the computational analysis, both of these stages are inserted in the definitive structure of the enumeration algorithm. The solution which is obtained through this algorithm is called as "*BNS*" (Best Known Solution) and the solution of the FDS algorithm is saved in Δ. Hence, the differences between the objective functions of these two algorithms can be calculated through $\% = \frac{BNS-\Delta}{BNS} \times 100$.

7.3.2. Performance of the constructive algorithms

In Table 3, the performance of the FDS algorithm is evaluated with regard to the initial solutions which are obtained through the first and the second constructive algorithms and the corresponding results are reported, respectively, in the columns labeled by "FDS (CS1)" and "FDS (CS2)". For each constructive algorithm or the FDS algorithm, the columns labeled by "Time" and "GAP%" show, respectively, the average CPU running time (in second) and average gap between BNS and the solution obtained by the corresponding algorithm.

According to Table 4, CS2 algorithm is superior to CS1 algorithm and its average gaps over the six series are, respectively, around 2% and 0.13% less than the CS1, for the initial and the last obtained solutions. In addition, the difference between their running time is insignificant for initial solutions while the FDS algorithm when

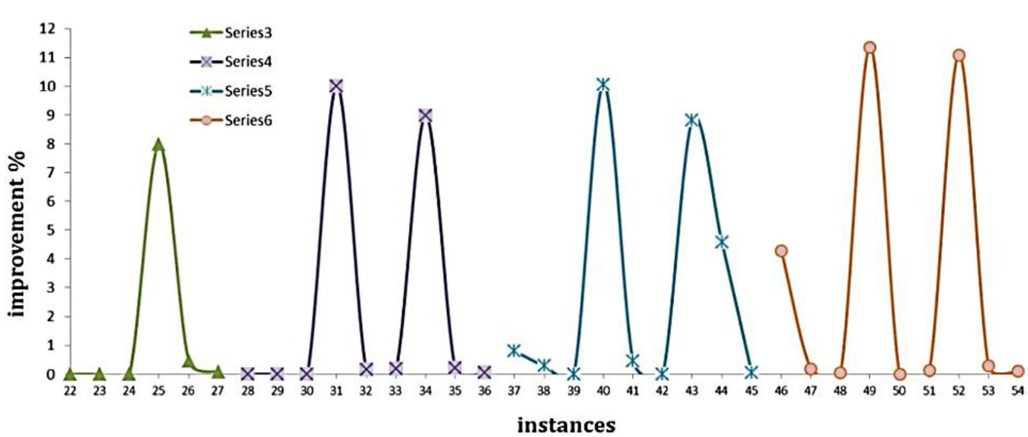

Fig. 5. Percentage improvement in stage II of the enumeration algorithm.

Table 5
Performance of diversification procedure.

Series	With diversification		Without diversification	
	Time	GAP%	Time	GAP%
1	13.85	0.00	4.80	0.00
2	34.51	0.00	10.27	0.00
3	77.72	0.16	27.99	0.20
4	151.63	0.11	50.06	0.21
5	241.56	0.46	90.64	0.68
6	413.16	0.38	195.09	0.81
Avg.	155.40	0.19	63.14	0.32

Table 6
Stability of the FDS algorithm.

Series	Avg. Time	GAP%			
		Best	Avg.	Worst	Std.
1	14.14	0.00	0.00	0.00	0.00
2	35.08	0.00	0.00	0.00	0.00
3	82.14	0.00	0.1	0.26	0.12
4	146.78	0.02	0.14	0.3	0.14
5	251.7	0.45	0.45	0.47	0.01
6	426.24	0.04	0.17	0.47	0.22
Avg.	159.35	0.09	0.14	0.25	0.08

starts from the initial solution obtained through CS2 algorithm is around 13 s faster in average. Thus, in the rest of this section, we benefit from the second constructive algorithm to obtain the initial solutions.

7.3.3. Performance of learning procedure

Through using the long-term memory, the probability of selecting a depth is affected by the performance of each depth in pervious iterations while in the absence of the learning procedure the algorithm selects each depth with the same probability. The computational results of the heuristic algorithm with and without applying the learning procedure are summarized in Table 4.

Table 4 demonstrates that through applying the learning procedure the running time and the gap are improved around 14 s and 0.37%, respectively. Thus, in rest of this section, the learning procedure is inserted in the structure of the FDS algorithm

7.3.4. Performance of the diversification procedure

Table 5 draws a comparison between the performance of the algorithm in both absence and the presence of the diversification procedure. It is observed that the diversification procedure improves the average running time by around 92 s and decreases the gap by 0.13%. Hence, the advantage of the diversification in term of both running time and quality of the solutions is verified. In this section, for the rest of computational analysis, diversification procedure is inserted in the FDS.

7.3.5. Stability of the heuristic algorithm

Since the developed FDS algorithm contains randomness, we perform a test to assess the impact of that over different runs of the algorithm. To do so, for each instance we run the algorithm four times and the results are summarized in Table 6. In this table, the columns which are labeled by “Best”, “Avg.” and “Worst” show, respectively the least, average and the worst gaps in the instances of each series, respectively. Moreover, the last column gives the standard deviation of the results for the instances in each series of data. The computational results clearly indicate the effectiveness and stability of FDS algorithm.

Table 7
The effect of interdiction budget on the selected interdiction pattern.

Series	Low (Budget)		Medium (Budget)		High (Budget)	
	LLF	ULF	LLF	ULF	LLF	ULF
1	100%	0%	95%	5%	19%	81%
2	100%	0%	5%	95%	8%	92%
3	100%	0%	9%	91%	3%	97%
4	100%	0%	2%	98%	9%	91%
5	100%	0%	5%	95%	0%	100%
6	100%	0%	2%	98%	3%	97%
Avg.	100%	0%	20%	80%	7%	93%

Table 8
Effect of partial interdiction on total budget consumption.

Series	Full interdiction	Partial interdiction
1	90%	98%
2	95%	99%
3	94%	99%
4	96%	99%
5	99%	99%
6	96%	99%
Avg.	95%	99%

7.4. Analyzing the behavior of the model

In each series of the generated instances, the number of customers and their demand, the number of facilities in each hierarchy and the coordinates of facilities and customers are the same. This integrated structure provides an appropriate framework to test the behavior of the proposed model in term of different number of interdiction intensity levels and available interdiction budget. In particular, to investigate the relation between the level of interdiction budget and its contribution toward the interdiction of each hierarchy, some results are reported in Table 7.

The instances of each series are categorized into three groups according to their available budget; i.e., Low, Medium and High. The columns which are labeled by “LLF” and “ULF” show the percentage of the available interdiction budget which is allocated to the interdiction of facilities at levels I and II, respectively. The computational results demonstrate that when the level of interdiction budget is low, the interdictor allocates her total budget to interdiction of the facilities at level I while when the level of her available budget is high; she prefers to allocate the most of her budget to interdiction of the facilities at level II. When the level of available interdiction budget is medium, the behavior of the model is not such clear but for most instances the interdictor prefers to allocate the biggest share of her available budget to interdiction of the facilities at level II.

As described in the definition of the problem, the interdictor is allowed to partially interdict the capacity of different facilities. To test the advantages of this assumption, some computational results are summarized in Tables 8 and 9. Table 8 shows the relation between the percentage of the total consumed interdiction budget and the type of interdiction problem, i.e., full or partial interdiction. The results demonstrate that when the interdictor can interdict facilities partially with four intensity levels, she can consume 99% of her available budget while in full interdiction only 95% of total budget can be consumed.

Table 9 shows the effect of considering different interdiction intensity levels on the quality of the selected interdiction pattern. For this purpose, the instances of each series are categorized in three new groups with respect to their level of available budget. Subsequently, for each category the objective function of the most destructive interdiction pattern is reported in “ObjPattern”. Each column which is labeled by “2Int”, “3Int” and “4Int” indicates the av-

Table 9
Effect of partial interdiction on destruction's depth of the selected interdiction pattern.

Series	Budget level	ObjPattern	Avg. GAP% 2Int	3Int	4Int
Series 1	Low	50,057,971	3.75%	3.75%	0.00%
	Medium	65,396,518	1.28%	1.28%	0.00%
	High	82,170,452	5.05%	0.55%	0.00%
		Average:	3.36%	1.86%	0.00%
Series 2	Low	55,178,155	0.00%	0.00%	0.00%
	Medium	83,935,997	6.26%	0.19%	0.00%
	High	117,030,188	2.39%	0.00%	0.80%
		Average:	2.88%	0.06%	0.27%
Series 3	Low	71,814,134	5.18%	1.32%	0.00%
	Medium	108,598,952	3.90%	0.95%	0.00%
	High	153,564,778	0.00%	0.00%	0.00%
		Average:	3.03%	0.76%	0.00%
Series 4	Low	89,562,255	3.28%	0.00%	1.12%
	Medium	136,730,099	2.04%	0.00%	0.69%
	High	188,721,011	1.69%	0.34%	0.00%
		Average:	2.34%	0.11%	0.60%
Series 5	Low	101,116,986	0.00%	0.00%	0.00%
	Medium	169,931,546	2.72%	2.72%	0.00%
	High	238,743,348	0.82%	0.82%	0.00%
		Average:	1.18%	1.18%	0.00%
Series 6	Low	264,965,454	1.88%	0.52%	0.00%
	Medium	179,452,683	2.34%	0.50%	0.00%
	High	107,266,139	3.83%	0.98%	0.00%
Average:			2.68%	0.67%	0.00%
Average (Total):			2.58%	0.77%	0.15%

erage difference between ObjPattern and the objective function of instances which have 2, 3 and 4 interdiction levels, respectively. For all data series, the average gap between the objective function of the most destructive interdiction pattern and the objective function of full interdiction is 2.58%, while for partial interdiction with 3 and 4 interdiction levels are 0.77% and 0.15%, respectively. Based on the results reported in Tables 8 and 9, partial interdiction results in more interdiction budget consumption and destructive interdiction pattern. Thus, in real-world problem most interdiction plans are benefit from partial interdiction versus full interdiction.

7.5. Investigating the effectiveness of the proposed model

In this paper, the main research question is "how much the proposed model can decrease the cost of this hierarchal service system?" and this section aims to answer it through a comparison between the computational results of this problem in both presence and absence of the bi-level model. In the absence of the proposed model, the system defender does not take the vulnerability of the facilities into consideration in the designing phase and no reassignment strategy is possible. Thus, when the system is faced with interdiction and some customers lose their pervious service level, only outsourcing option is available. To solve the test problems under this assumption, the following steps are done:

Step 1:

Set $x_{jk}=1$ for all facilities at interdiction level $k=0$ and $x_{jk}=0$ for other interdiction levels. Subsequently, solve the follower level of the proposed model to obtain the optimal amount of assignment variables (i.e., u_{ij}^{1*}, u_{ij}^{2*} and u_{ij}^{3*}).

Step 2:

Solve the bi-level model and obtain the most destructive interdiction pattern.

Step 3:

Fix the interdiction variables equal to the most destructive interdiction pattern. Moreover, for each facility, set the upper bound of the assignment variables equal to corresponding u_{ij}^{1*}, u_{ij}^{2*} and u_{ij}^{3*}. Subsequently, solve the lower level problem and report the objective function.

Among the six series of data which are solved through the enumeration algorithm, only for the first 21 instances the optimal solutions are obtained. Thus, the first 21 instances are solved in two ways; i.e., (1) through the above-mentioned steps, and (2) the proposed bi-level model. Computational results are reported in Table 10. The two columns labeled by "Cost" demonstrate the incurred cost to the service system, respectively, through applying the first and the second ways and the last two columns labeled by "Improvement" show respectively, the amount and the percentage of cost reduction through applying the proposed bi-level model. The "Time" column corresponds to the time required for solving the bi-level model in each instance. The results indicate that the proposed model is able to decrease the cost of customers' demand satisfaction around 7.94% on average.

Table 10
Investigating the ability of the model in cost reduction.

instance	Time	Cost With model	Cost Without model	Improvement Amount	Improvement Percentage
1	1.45	48,180,522	51,023,247	2,842,725	5.57
2	4.46	64,561,039	76,706,744	12,145,705	15.83
3	3.6	78,017,592	87,274,792	9,257,200	10.61
4	8.01	48,180,522	51,023,247	2,842,725	5.57
5	15.51	64,561,039	68,081,074	3,520,035	5.17
6	12.87	81,722,572	84,658,584	2,936,012	3.47
7	16.5	50,057,971	52,923,361	2,865,390	5.41
8	40.54	65,396,518	68,085,235	2,688,717	3.95
9	21.67	82,170,452	87,510,436	5,339,984	6.1
10	8.01	55,178,155	60,617,038	5,438,883	8.97
11	11.14	78,681,472	92,076,344	13,394,872	14.55
12	9.73	114,234,601	117,194,811	2,960,210	2.53
13	23.17	55,178,155	60,617,038	5,438,883	8.97
14	31.25	83,774,576	95,932,683	12,158,107	12.67
15	33.07	117,030,188	119,627,146	2,596,958	2.17
16	53.98	55,178,155	60,617,038	5,438,883	8.97
17	61.57	83,935,997	95,695,264	11,759,267	12.29
18	78.63	116,093,638	118,693,662	2,600,024	2.19
19	25.32	68,091,476	74,219,965	6,128,489	8.26
20	23.21	104,354,247	126,271,941	21,917,694	17.36
21	24.54	153,564,778	163,494,011	9,929,233	6.07
Average cost reduction:					7.94

8. Conclusion and further research

The purpose of this paper is to obtain an appropriate demand satisfaction pattern for hierarchical service facilities which face capacity lost due to partial interdiction. To formulate this problem a mixed-integer bi-level model is proposed. In the upper level problem, the interdictor chooses the most destructive interdiction pattern with regard to her available interdiction budget. Subsequently, in the lower level problem, the defender tries to minimize the total cost of demand satisfaction for all customers through benefiting from outsourcing and allocating the customers to the facilities with regard to their available capacity. The proposed model is a beneficial one for service facilities which face planned disasters and the defender should react intelligently and fast in order to incur the least possible cost to satisfy all customer demand.

To solve this problem, an enumeration procedure and a heuristic method are proposed. To speed up the enumeration procedure, in addition to ordinary enumeration, we proposed a modified enumeration stage. The computational results show that this extra stage is able to improve the solutions by around 3%. This proposed algorithm is not however a suitable one for large-scale real-world problems. We, therefore, develop a heuristic algorithm based on local search. To analyze the effectiveness of the proposed algorithms, several computational tests are conducted. Through these tests, firstly, the best possible structure for each algorithm is identified. Next, the effectiveness of the heuristic algorithm is verified and the behavior of partial interdiction model versus full interdiction model is analyzed. The results demonstrate that partial interdiction model through adding more flexibility in the process of selecting the most destructive interdiction pattern is more viable for interdictors. Thus, considering partial interdiction assist the proposed model to be more in touch with reality. Finally, the effectiveness of the proposed model is tested through comparing the computational results of handling the interdiction in both absence and presence of the proposed model. The findings also illustrate that this model is able to decrease the average cost of customers' demand satisfaction by around 7.94% for the instances with available optimal solutions.

This paper opens a new perspective for future studies in both military and non-military spheres, and many extensions can be considered for the proposed problem. Several examples are as follows:

- Consider the horizontal relations in addition to hierarchical relations among the facilities. For instance, "Goods sharing" strategy (see Azad et al., 2013) could be applied by using the excess commodity in a facility in the same service hierarchy to fulfill the demand at the interdicted facility,
- Develop a tri-level model to locate the hierarchal facilities in the system and identity appropriate capacities for them,
- Consider uncertainty in parameters of the model,
- Assume capacity expansion possibilities in the problem and extending the model as a multi-objective one,
- Consider reduction in the reliability and availability of the facilities as the potential outcomes of interdiction,
- Assume asymmetry in information sharing between the interdictor and the defender. Since in real-world problems the players try to hide their offensive and defensive strategies, this assumption is more in touch with reality.

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