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Distributed Consensus Control for a Network of Incommensurate Fractional-Order Systems

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***Abstract*—This letter presents a distributed solution for consensus control of a network of single-integrator incommensurate fractional-order systems with nonlinear and uncertain dynamics. To consider a broader class of fractional-order systems, compared to existing results, the fractional derivative orders of agents' dynamics are assumed non-identical, which makes the distributed control design more challenging. To cope with non-identical fractional derivative orders, the Mittag-Leffler function method is adopted to develop a novel distributed control scheme that guarantees consensus under mild assumptions. To deal with agents' dynamic uncertainties, the proposed approach is integrated with adaptive neural networks to make the distributed tracking errors for all follower agents converge to a small neighborhood of origin. A numerical example is provided to demonstrate the effectiveness of the proposed method.**

***Index Terms*—Multi-agent systems, Incommensurate fractional -order systems, Unknown nonlinearities, Distributed control.**

I. Introduction

DISTRIBUTED synchronization or consensus of a group of autonomous agents [1], [2] with the goal of achieving a common group objective through local exchange of information among neighboring agents, has been widely studied due to its scalability, flexibility, and low computational cost. Due to this mentioned advantages, distributed synchronization has been applied to numerous applications, including satellite cluster [3], rendezvous of mobile robots [4], formation of an unmanned aircraft team [5], automated highway systems [6] and so forth. Distributed synchronization of linear multi-agent systems (MASs) with first-integrator dynamics [7], second-integrator dynamics [8], and general linear case [9] are well studied and established in the literature. However, most of the real-world systems have agents with nonlinear and uncertain dynamics. Recently, distributed adaptive consensus schemes are presented for different classes of uncertain nonlinear MASs, for instance see [10]–[14]. These results, however, are limited to nonlinear integer-order MASs.

Due to the high precision of fractional-order representations for complex systems, many complex real-world network applications, such as social networks and a group of spacecraft can be regarded as fractional-order systems. However, the design of synchronization protocols and their stability analysis are more challenging for fractional-order systems, compared to integer-order systems. This is because some well-known mathematical tools, such as Leibniz rule, are not well established for fractional-order derivatives. As a result, it is not straightforward to adopt the classical stability analysis schemes, especially, the Lyapunov method, as well as its associated control strategies from integer-order systems to fractional-order systems.

Consensus control problem for fractional-order MASs has been recently studied for linear and nonlinear fractional-order MASs in [15]–[17] and [18]–[22], respectively. For nonlinear fractional-order systems, which are the focus of this letter, the results of [18] and [19] are limited to nonlinear first- and second- integrator fractional-order MASs, respectively, and both require some global information about the eigenvalues of the Laplacian matrix. In [20], the designed adaptive laws are obtained in a centralized manner to achieve consensus performance, and also are implemented based on the exact information of the Laplacian matrix. Moreover, in [18]–[20], unknown nonlinear system functions are assumed to satisfy the Lipchitz condition. To eliminate these shortcomings, in [21] and [22], two distributed adaptive control schemes are designed for a network of double-integrator uncertain nonlinear fractional-order MASs. However, all existing mentioned results, including [21] and [22] are only limited to the network of fractional-order systems with known and identical derivative orders. As mentioned in [23], the dynamics in incommensurate fractional-order systems (IFOSs) are represented by fractional-order differential equations subject to non-identical derivative orders. Moreover, network of IFOSs can more accurately describe the dynamics or behaviors of many practical and complex systems, including viscoelastic systems and biological systems. The control design and stability analysis for a network of IFOSs are more challenging than conventional fractional-order MASs due to non-identical derivative orders. To control design for single agent IFOSs with known derivative orders, some classical approaches such as frequency distributed model [23], [24], and designing fractional-order error surfaces [25]–[27] are used. However, the communication and interaction among agents in a MAS results in coupling terms in agents' dynamics and brings new challenges in the design of distributed control problem for a network of IFOSs. Besides, for a network of uncertain IFOSs with prior knowledge of only one of fractional derivative orders of follower agents, determining the order of fractional-order adaptive law is a technical challenge.

Motivated by the above discussions, this letter proposes a distributed control scheme for a network of nonlinear uncertain IFOSs with prior knowledge of only one of fractional derivative orders of follower agents. To facilitate the control design, first, we define α_k as one of fractional derivative orders of followers' agents (i.e., $\alpha_k \in \{\alpha_1, ..., \alpha_N\}$), and then it is shown that

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for all the follower agents, the differences between fractional derivative of tracking errors with orders α_l for $l = 1, ..., N$ and α_k are always equal to some unknown bounded disturbance-like function $\psi_l(t)$. Based on the definition of $\psi_l(t)$, the consensus stability for the network of IFOSs is proved in the sense of fractional derivative of the Lyapunov function with order α_k and the Mittag-Leffler function. Then, a composite uncertainty, composed of unknown functions, unknown control coefficients and fractional derivatives of leader output is defined. Finally, similar to [28] and [29] neural networks (NNs) and minimal learning parameter method are used to approximate the composite uncertainty. The upper bounds of the square of the norms of NNs weight vectors are adaptively estimated. Therefore, the proposed adaptive controller for each follower agent contains only one adaptive parameter and the existing curse of dimensionality drawback in [21] and [22] is relaxed. The proposed control design ensures that all the signals in the resulting closed-loop network system are bounded, and the tracking errors remain in an adjustable bound around the origin. The main contributions of this letter are as follows:

1) Unlike [15]–[27], this letter provides a generalization of the previous strategies for a broader class of multiple fractional-order systems in the presence of unknown disturbances, unknown time-varying control coefficients, unknown system functions and different fractional derivative orders of agents' dynamics.
2) Compared with [15]–[27], this letter designs fractional-order adaptive NN laws to approximate composite uncertainties. Moreover, to eliminate the algebraic-loop drawback appeared in the control design and also to deal with unknown time-varying control coefficients, a novel adaptive NN law is designed to estimate the upper bound of the square norms of NNs weight vectors such that only one parameter is updated for each follower agent.
3) In contrast with [23]–[27], the proposed control strategy guarantees the synchronization for a network of IFOSs without employing the frequency distributed model and fractional-order error surfaces. In this letter, a novel simple concept with regard to the network of IFOSs with prior knowledge of only one of fractional derivative orders of follower agents is provided by applying the Mittag-Leffler function to solve a coordination problem for a network of IFOSs.

Notations: In this letter, $\Re$ ($\Re^+$), $\Re^N$, and $\Re^{N\times N}$ indicate the set of real numbers (positive real numbers), the set of real N-vectors, and the set of real $N \times N$ matrices, respectively; $|x|$ indicates the absolute value of x; $||\bar{x}||$ denotes the 2-norm of a vector $\bar{x}$. The operator $\mathrm{diag}(\cdot)$ shows a diagonal matrix from its argument and $\arg(\cdot)$ denotes the argument of complex number. The superscript T denotes the transpose and $\sigma_{\min}(A)$ ($\sigma_{\max}(A)$) denote minimum (maximum) singular value of matrix $A \in \Re^{N\times N}$. Symbol $\otimes$ denotes the Kronecker product.

II. Preliminaries

A. Caputo fractional derivative

In this subsection, we provide some definitions and properties of fractional calculus, including Caputo fractional derivative and Mittag-Leffler function, alongside with a Lyapunov-based stability criterion for fractional-order systems.

There are mainly two widely used fractional operators, namely Caputo and Riemann-Liouville fractional operators. The popularity of the Caputo's fractional derivative is because its Laplace transform requires only integer-order derivatives for the initial conditions. Therefore, we use the Caputo fractional derivative in this letter to model the fractional-order agents.

For any real number $\alpha \in (0\ 1]$, the Caputo fractional derivative is defined as [15]

$$ {}_0^{\mathrm{C}}D_t^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{f^{'}(\tau)}{(t-\tau)^{\alpha}} \mathrm{d}\tau, \tag{1} $$

where ${}_0^{\mathrm{C}}D_t^{\alpha} f(t)$ denotes the Caputo fractional derivative with order α of the function $f(t)$, $f^{'}(t)$ is the first integer-order derivative of $f(t)$ and $\Gamma(1-\alpha) = \int_0^{\infty} \tau^{-\alpha}\exp(-\tau)\mathrm{d}\tau$. If $\alpha = 1$, then (1) is equivalent to the first integer-order derivative.

The Laplace transform of the Caputo fractional derivative $\alpha \in [0\ 1)$ is presented as [15]

$$ \mathcal{L}\{{}_0^{\mathrm{C}}D_t^{\alpha} f(t)\} = s^{\alpha}F(s) - s^{\alpha-1}f(0), \tag{2} $$

where $F(s)$ is the Laplace transform of $f(t)$.

In the following, we define the Mittag-Leffler function, which is used in Section 4 to analyze stability of a network system.

Definition 1 [15]: The *Mittag-Leffler* function is expressed as

$$ E_{(\alpha,\gamma)}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(ak+\gamma)}, \tag{3} $$

where $\alpha \in (0\ 1]$, $\gamma \in \Re^+$ and z is a complex number.

The Laplace transform of *Mittag-Leffler* is given by [15]

$$ \mathcal{L}\left\{t^{\gamma-1}E_{(\alpha,\gamma)}(-\zeta t^{\alpha})\right\} = \frac{s^{\alpha-\gamma}}{s^{\alpha}+\zeta}, \qquad \Re e(s) > |\zeta|^{\frac{1}{\alpha}}, \tag{4} $$

where $\Re e(s)$ is the real part of s and $\zeta \in \Re$.

The following Lemmas are used in Section 4 to analyze the stability of the closed-loop of network of fractional-order systems with model uncertainties and unknown disturbances.

Lemma 1 [22]: For $\gamma \in \Re$, $\alpha \in \Re^+$ and $\phi \in \Re^+$ satisfying $0 < \alpha \le 1$, and $\frac{\pi\alpha}{2} < \phi < \pi\alpha$ and $\Upsilon \in \Re^+$, the Mittag-Lefer function is bounded by

$$ E_{(\alpha,\gamma)}(z) \le \frac{\Upsilon}{1+|z|}, \quad \gamma \le |\arg(z)| \le \pi, \quad |z| \ge 0, \tag{5} $$

Lemma 2 [18]: If $\bar{x}(t) = [x_1(t), ..., x_n(t)]^{\mathrm{T}} \in \Re^n$ is a smooth vector function, $\alpha \in (0,1]$ and $t \ge 0$, then, there exists a positive definite matrix $P \in \Re^{n\times n}$ such that

$$ {}_0^{\mathrm{C}}D_t^{\alpha}\Big(\bar{x}^{\mathrm{T}}(t)P\bar{x}(t)\Big) \le 2\bar{x}^{\mathrm{T}}(t)\ P\ {}_0^{\mathrm{C}}D_t^{\alpha}\bar{x}(t). \tag{6} $$

Lemma 3 [22]: Let the α-order derivative of a smooth function $\mathcal{V}(t) : \Re^+ \to \Re$ satisfy

$$ {}_0^{\mathrm{C}}D_t^{\alpha}\mathcal{V}(t) + \varpi\mathcal{V}(t) \le \mu, \tag{7} $$

where $\alpha \in (0\ 1]$, $\varpi > 0$, and $\mu \ge 0$. Then, one has

$$ \mathcal{V}(t) \le \mathcal{V}(0)E_{(\alpha,1)}(-\varpi t^{\alpha}) + \frac{\mu\omega}{\varpi}, \qquad t \ge 0, \tag{8} $$

where ω is $\max = \{1, \Upsilon\}$ and Υ is defined in Lemma 1.

B. Graph theory

Algebraic graph theory is commonly employed as a mathematical tool to capture the communication and information flow between agents in a MAS. Let $\mathcal{G} = (\mathcal{V}, \mathcal{A}, \mathcal{E})$ be a weighted directed graph of order N, where $\mathcal{V} = \{v_l; l = 1, ..., N\}$ is considered as the set of the agents or nodes, $\mathcal{E} \subset \{e_{l,m} : l = 1, ..., N, m = 1, ..., N, l \neq m\}$ is the set of edges in which $e_{l,m} = (v_l, v_m) \in \mathcal{E}$ if and only if there exists information exchange from the l-th node to the m-th node. The adjacency matrix $\mathcal{A} = [a_{l,m}] \in \Re^{N\times N}$ describes the interactions among the nodes with $a_{l,m} \neq 0$ if $e_{m,l} \in \mathcal{E}$, and $a_{l,m} = 0$ if there is no direct path from the m-th node to the l-th node. The set of neighbors of the l-th node is defined as $N_l = \{m \mid a_{l,m} \neq 0\}$. The Laplacian matrix is defined as $\mathcal{L} = \mathcal{D} - \mathcal{A}$, where $\mathcal{D} = \mathrm{diag}(d_l)$ is the weighted in-degree matrix with $d_l = \sum_{m\in N_l} a_{l,m}$. We assume that the graph $\mathcal{G}$ captures the interaction among the followers and the communications between N followers and the leader is modeled through another directed graph $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$, where $\bar{\mathcal{V}} = \{v_l; l = 0, 1, ..., N\}$ and $\bar{\mathcal{E}} \subset \{e_{l,m} : l = 0, 1, ..., N, m = 1, ..., N, l \neq m\}$, and 0 denotes the label of the leader node. Define the leader adjutancy matrix as $\mathcal{B} = \mathrm{diag}(b_l) \in \Re^{N\times N}$, where $b_l > 0$ if only if l-th follower agent has access to leader output; otherwise $b_l = 0$. A directed graph has a directed spanning tree if there exists at least one agent (called root) that has directed paths to all other agents.

Assumption 1: The directed graph $\bar{\mathcal{G}}$ has a directed spanning tree with the leader as its root node.

III. Problem formulation

In this section, we formulate the network of incommensurate fractional-order systems with unknown nonlinearities and unknown external disturbances.

The mathematical models of the followers with single-integrator nonlinear fractional-order dynamics are given by

$$ {}_0^{\mathcal{C}}D_t^{\alpha_l} x_l = g_l(x_l)u_l + f_l(x_l) + d_l(t), \qquad l = 1, 2, ..., N, \tag{9}$$

where ${}_0^{\mathcal{C}}D_t^{\alpha_l} x_l = [{}_0^{\mathcal{C}}D_t^{\alpha_l} x_{l_1}, ..., {}_0^{\mathcal{C}}D_t^{\alpha_l} x_{l_n}]^{\mathrm{T}}$ denotes the Caputo fractional-order derivative of the l-th follower's state vector $x_l \in \Re^n$, with order α_l, where $0 < \alpha_l \leq 1$, $\alpha_l \neq \alpha_m$, for $l = 1, 2, ..., N, m = 1, 2, ..., N$, $l \neq m$, $u_l \in \Re^n$ corresponds to the control input vector, $f_l(x_l) : \Re^n \to \Re^n$ is unknown nonlinear smooth vector function, $g_l(x_l) = \mathrm{diag}\{g_{l_1}(x_l), ..., g_{l_n}(x_l)\}$, $g_{l_q}(x_l) : \Re^n \to \Re$ is unknown nonlinear smooth control coefficient for $q = 1, 2, ..., n$. $d_l(t) : \Re^+ \to \Re^n$ is an unknown external disturbance that is assumed bounded with unknown bound i.e., $||d_l(t)|| \leq d_l^*$, where d_l^* is an unknown constant.

The leader's dynamics are described by

$$ {}_0^{\mathcal{C}}D_t^{\alpha_0} x_0 = f_0(x_0, t), \tag{10}$$

where ${}_0^{\mathcal{C}}D_t^{\alpha_0} x_0 = [{}_0^{\mathcal{C}}D_t^{\alpha_0} x_{0_1}, ..., {}_0^{\mathcal{C}}D_t^{\alpha_0} x_{0_n}]^{\mathrm{T}}$ denotes the Caputo fractional-order derivative of the leader's state vector $x_0 \in \Re^n$, with order α_0, where $0 < \alpha_0 \leq 1$ is an unknown constant and $\alpha_0 \neq \alpha_l$ for $l = 1, 2, ..., N$, and $f_0(x_0, t) : \Re^n \times \Re^+ \to \Re^n$ is locally Lipschitz vector function in x_0 and piecewise continues in t, also it is unknown bounded vector function to all followers.

The control objective of this letter is to design a distributed control law for a network of followers (9) with a leader given in (10) such that all the closed-loop network variables are uniformly ultimately bounded and the followers' states synchronize to the leader's states.

Assumption 2: The sign of the control coefficients are known and $g_{l_q}(x_l) \neq 0$, $\forall x_l \in \Re^n$.

Assumption 3: Fractional derivatives of leader's state vector, i.e., ${}_0^{\mathcal{C}}D_t^{\alpha_l} x_0$ for $l = 1, 2, ..., N$ are smooth and bounded.

IV. Main results

In this section, the distributed control design is proposed for consensus of a network of IFOSs and its closed-loop consensus stability analysis is provided. Further, the ultimate bounds on the Lyapunov variables are also derived.

In order to construct the distributed controller for a network of IFOSs, the coordinate transformations are defined as

$$ e_l = \sum_{m\in N_l} a_{ml}(x_l - x_m) + b_l(x_l - x_0), \tag{11}$$

$$ z_l = x_l - x_0, \tag{12}$$

where $e_l \in \Re^n$ is distributed tracking error vector and $z_l \in \Re^n$ is tracking error vector.
With respect to (12), (11) is rewritten as

$$ e = ((\mathcal{L} + \mathcal{B}) \otimes I_n)z, \tag{13}$$

where $z = [z_1^{\mathrm{T}}, z_2^{\mathrm{T}}, ..., z_N^{\mathrm{T}}]^{\mathrm{T}} \in \Re^{Nn}$, $e = [e_1^{\mathrm{T}}, e_2^{\mathrm{T}}, ..., e_N^{\mathrm{T}}]^{\mathrm{T}} \in \Re^{Nn}$ and $I_n \in \Re^{n\times n}$ is the identity matrix.

To solve the coordination problem for a network of IFOSs, the fractional derivative of z_l with order α_k is proposed as

$$ {}_0^{\mathcal{C}}D_t^{\alpha_k} z_l = {}_0^{\mathcal{C}}D_t^{\alpha_l} z_l + \psi_l(t), \qquad l = 1, 2, ..., N, \tag{14}$$

where $\alpha_k \in \{\alpha_1, ..., \alpha_N\}$ is a known constant and $\psi_l(t) = [\psi_{l_1}(t), ..., \psi_{l_n}(t)]^{\mathrm{T}}$ is an unknown disturbance-like vector. Hence, it is assumed that only one of fractional derivative orders of follower agents is known, which is named α_k.

Assumption 4: The unknown disturbance-like vector $\psi_l(t)$ is bounded with an unknown bound. That is, one has

$$ ||\psi_l(t)|| \leq \psi_l^*, \qquad l = 1, 2, ..., N, \tag{15}$$

where ψ_l^* is an unknown positive parameter.

Remark 1: To take into account the non-identical fractional derivative orders in (9) and (10), the unknown disturbance-like term is added to ${}_0^{\mathcal{C}}D_t^{\alpha_l} z_l$ to construct equation (14). This simplifies the control design and performance analysis for the network of IFOSs.

From (9), (12) and (14) it follows that

$$ {}_0^{\mathcal{C}}D_t^{\alpha_k} z_l = g_l(x_l)u_l + f_l(x_l) + d_l(t) - {}_0^{\mathcal{C}}D_t^{\alpha_l} x_0 + \psi_l(t). \tag{16}$$

A Lyapunov function candidate is chosen as

$$ \mathcal{V}(t) = \frac{1}{2} z^{\mathrm{T}}((\mathcal{L} + \mathcal{B}) \otimes I_n)z + \sum_{l=1}^{N} \frac{1}{2\pi_l}\tilde{\zeta}_l^2, \tag{17}$$

where $\pi_l > 0$ is a design parameter and $\tilde{\zeta}_l = \zeta_l^* - \hat{\zeta}_l$.

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The fractional derivative of $\mathcal{V}(t)$ with order α_k along with (16) is

$$ {}_0^{\mathcal{C}}D_t^{\alpha_k}\mathcal{V}(t) \leq \sum_{l=1}^{N}\Bigg(e_l^{\mathrm{T}}\Big(u_l+\bar{f}_l(S_l)+d_l(t)+\psi_l(t)\Big) - \frac{1}{\pi_l}\tilde{\zeta}_l\ {}_0^{\mathcal{C}}D_t^{\alpha_k}\hat{\zeta}_l\Bigg), \tag{18}$$

where $\bar{f}_l(S_l) = g_l(x_l)u_l + f_l(x_l) - u_l - {}_0^{\mathcal{C}} D_t^{\alpha_l}x_0$ and $S_l = [x_l, u_l, {}_0^{\mathcal{C}} D_t^{\alpha_l}x_0]^{\mathrm{T}}$.
According to the universal approximation ability of radial basis function neural networks (RBF NNs) [30], one can write the function $\bar{f}(S_l)$ as $\bar{f}(S_l) = \Theta^*\varphi(\S_l)+\varepsilon$, and therefore (18) can be written as

$$ {}_0^{\mathcal{C}}D_t^{\alpha_k}\mathcal{V}(t) \leq \sum_{l=1}^{N}\Bigg(e_l^{\mathrm{T}}\Big(u_l+\Theta_l^*\varphi_l(S_l)+\varepsilon_l+d_l(t)+\psi_l(t)\Big) - \frac{1}{\pi_l}\tilde{\zeta}_l\ {}_0^{\mathcal{C}}D_t^{\alpha_k}\hat{\zeta}_l\Bigg), \tag{19}$$

where $\Theta_l^* = \mathrm{diag}\{\Theta_{l_1}^*, ..., \Theta_{l_n}^*\}$, $\varepsilon_l = [\varepsilon_{l_1}, ..., \varepsilon_{l_n}]^{\mathrm{T}}$, $\varphi_l(S_l) = [\varphi_{l_1}(S_l), ..., \varphi_{l_n}(S_l)]^{\mathrm{T}}$, and $||\varepsilon_l|| \leq \varepsilon_l^*$.
Using Cauchy's and Young's inequalities [10], Assumption 4, the following holds.

$$ e_l^{\mathrm{T}}\Big(\Theta_l^*\varphi_l(S_l)+\varepsilon_l+d_l(t)+\psi_l(t)\Big) \leq 1.5e_l^{\mathrm{T}}e_l + \frac{1}{2\kappa_l}e_l^{\mathrm{T}}e_l\zeta_l^* + \frac{1}{2}||\varepsilon_l^*||^2 + \frac{1}{2}||\psi_l^*||^2 + \frac{1}{2}||d_l^*||^2 + \frac{1}{2}\kappa_l, \tag{20}$$

where $\zeta_l^* = v_l||\Theta_l^*||_F^2$, $\varphi_l^{\mathrm{T}}(S)\varphi_l(S) \leq v_l$ and $\kappa_l > 0$. Moreover, $||\Theta_l^*||_F$ denotes Frobenius norm of matrix Θ_l^*.
By inserting (20) in (19), it yields

$$ {}_0^{\mathcal{C}}D_t^{\alpha_k}\mathcal{V}(t) \leq \sum_{l=1}^{N}\Bigg(e_l^{\mathrm{T}}u_l + 1.5e_l^{\mathrm{T}}e_l + \frac{1}{2\kappa_l}e_l^{\mathrm{T}}e_l\zeta_l^* + \frac{1}{2}||\varepsilon_l^*||^2 + \frac{1}{2}||\psi_l^*||^2 + \frac{1}{2}||d_l^*||^2 + \frac{1}{2}\kappa_l, - \frac{1}{\pi_l}\tilde{\zeta}_l\ {}_0^{\mathcal{C}}D_t^{\alpha_k}\hat{\zeta}_l\Bigg). \tag{21}$$

The control law and the adaptive law are now chosen as

$$ u_l = -c_le_l - 1.5e_l - \frac{1}{2\kappa_l}e_l\hat{\zeta}_l, \tag{22}$$

$$ {}_0^{\mathcal{C}}D_t^{\alpha_k}\hat{\zeta}_l = \frac{\pi_l}{2\kappa_l}e_l^{\mathrm{T}}e_l - \pi_l\sigma_l\hat{\zeta}_l, \tag{23}$$

where $c_l > 0$ and $\sigma_l > 0$ are design parameters.
Remark2: To deal with unknown control coefficients and to eliminate the assumption that the control coefficients are bounded functions in the existing literature, $\bar{f}_l(S_l)$ as the composite function is defined in (18) and approximated using universal approximation property of RBF NNs. Moreover, to eliminate the algebraic-loop drawback and avoid deriving the fractional derivatives of the leader output in the approximate unknown composite function, the following property of RBF NNs, i.e., $\varphi_l^{\mathrm{T}}(S_l)\varphi_l(S_l) \leq v_l$, where $\varphi_l(S_l)$ is Gaussian basis function vector of RBF NNs) as well as defining $\zeta_l^* = v_l||\Theta_l^*||_F^2$ as an unknown parameter are applied in approximating this function. This (applying minimal learning parameter method) makes the proposed controller effective in not only dealing with unknown control coefficients, but also in eliminating the possibility of occurrence the algebraic-loop.

Substituting (22) and (23) into (21) and applying Young's inequality results in

$$ {}_0^{\mathcal{C}}D_t^{\alpha_k}\mathcal{V}(t) \leq \sum_{l=1}^{N}\Bigg(-c_le_l^{\mathrm{T}}e_l - \frac{1}{2}\sigma_l\tilde{\zeta}_l^2 + \frac{1}{2}||\varepsilon_l^*||^2 + \frac{1}{2}||\psi_l^*||^2 + \frac{1}{2}||d_l^*||^2 + \frac{1}{2}\kappa_l + \frac{1}{2}\sigma_l\zeta_l^{*2}\Bigg). \tag{24}$$

Now, by defining (25) and (26) as

$$ \varpi = \min_{l=1,2,...,N}\left\{2c_l\frac{\sigma_{\min}^2((\mathcal{L}+\mathcal{B})\otimes I_n)}{\lambda_{\max}((\mathcal{L}+\mathcal{B})\otimes I_n)}, \pi_l\sigma_l\right\}, \tag{25}$$

$$ \mu = \sum_{l=1}^{N}\left(\frac{1}{2}||\varepsilon_l^*||^2 + \frac{1}{2}||\psi_l^*||^2 + \frac{1}{2}||d_l^*||^2 + \frac{1}{2}\kappa_l + \frac{1}{2}\sigma_l\zeta_l^{*2}\right), \tag{26}$$

one yields

$$ {}_0^{\mathcal{C}}D_t^{\alpha_k}\mathcal{V}(t) + \varpi\mathcal{V}(t) \leq \mu. \tag{27}$$

Consequently, according to Lemma 3, all the closed-loop signals are uniformly ultimately bounded. That is,

$$ \mathcal{V}(t) \leq \mathcal{V}(0)E_{(\alpha_k,1)}(\varpi t^{\alpha_k}) + \frac{\mu w}{\varpi}. \tag{28}$$

According to Lemma 1, the following ultimate bounds yield.

$$ \sum_{l=1}^{N}z_l^{\mathrm{T}}z_l \leq \frac{\mu w}{\lambda_{\min}((\mathcal{L}+\mathcal{B})\otimes I_n)\varpi}, \tag{29}$$

$$ \sum_{l=1}^{N}e_l^{\mathrm{T}}e_l \leq \frac{\sigma_{\max}^2((\mathcal{L}+\mathcal{B})\otimes I_n)\mu w}{\lambda_{\min}((\mathcal{L}+\mathcal{B})\otimes I_n)\varpi}, \tag{30}$$

where $\lambda_{\min}(\cdot)$ ($\lambda_{\max}(\cdot)$) denote minimum (maximum) eigenvalue from its argument. According to [10], based on Assumption 1, $(\mathcal{L}+\mathcal{B})$ is positive definite, and also $\varpi > 0$. Hence, in (29) and (30), singularity can not occur.
Assumption 4 is required for consensus stability analysis of the closed-loop network system. The following statement shows how this assumption is actually satisfied.

From (28), it follows that $\mathcal{V}(t) \in L_\infty$ which guarantees that $z_l \in L_\infty$ and $\tilde{\zeta}_l \in L_\infty$. Then, $z \in L_\infty$. Based on (13), e and e_l are bounded. Then, due to x_0 is bounded, based on (12), it implies that x_l is bounded. Because ζ_l^* is bounded and $\hat{\zeta}_l = \zeta_l^* + \tilde{\zeta}_l$, it is deduced that $\hat{\zeta}_l \in L_\infty$. Thus, the control input u_l and fractional-order derivative of adaptive parameter ${}_0^{\mathcal{C}}D_t^{\alpha_k}\hat{\zeta}_l$ are bounded according to (22) and (23), respectively. Therefore, all the closed-loop network signals are bounded. On the other hand, since $g_l(x_l)$ is a real smooth control coefficient matrix, $f_l(x_l)$ is a real smooth vector function and $d_l(t)$ is a bounded disturbance vector, according to (9) implies that ${}_0^{\mathcal{C}}D_t^{\alpha_l}x_l \in L_\infty$. Based on definition α_k, i.e., $\alpha_k \in \{\alpha_1, ..., \alpha_N\}$, it follows that ${}_0^{\mathcal{C}}D_t^{\alpha_k}x_l \in L_\infty$. Based on ${}_0^{\mathcal{C}}D_t^{\alpha_l}x_l \in L_\infty$, ${}_0^{\mathcal{C}}D_t^{\alpha_k}x_l \in L_\infty$, Assumption 3 and definition z_l in (12), one has ${}_0^{\mathcal{C}}D_t^{\alpha_l}z_l \in L_\infty$, ${}_0^{\mathcal{C}}D_t^{\alpha_k}z_l \in L_\infty$ and their difference is also

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bounded. Hence, according to (14), $\psi_l(t)$ is a bounded vector. Therefore, Assumption 4 is satisfied.

Remark 3: As observed in (29) and (30), one can make both tracking and distributed tracking errors converge to a small and bounded neighborhood around the origin by increasing c_l, π_l and decreasing σ_l, κ_l. In other words, undesirable effects of the disturbance and disturbance-like signals, approximation errors on the consensus performance can be arbitrarily attenuated by tuning some design parameters. However, if π_l and κ_l are chosen too large and too small, respectively, they may lead to drift in parameter estimation.

Theorem 1: Consider the network of incommensurate fractional-order nonlinear systems (9) and (10) under Assumptions 1-4. Let the control law in (22) and the fractional-order adaptive law in (23). Then, the tracking and distributed tracking errors of the controlled network system are driven to neighborhoods of the origin, which are given in (29) and (30), respectively, and all the signals of the closed-loop network system are bounded.

Proof: The Lyapunov function (17) along with control law (22) and fractional-order NN law (23) complete the proof. □

Remark 4: In this letter, the main objective is to deal with the network of IFOSs with prior knowledge of only one of derivative orders of follower agents. Although the effects of the approximation errors are not considered in the control design on Theorem 1, the following modified control law attenuates the effects of approximation errors, disturbance and disturbance-like signals (i.e., lumped unstructured uncertainties $\theta_l = \varepsilon_l + d_l(t) + \psi_l(t)$), by estimating them as $\hat{\theta}_l$ and taking them into account in the control design as

$$u_l = -c_l e_l - 1.5 e_l - \frac{1}{2\kappa_l} e_l \hat{\zeta}_l - \hat{\theta}_l, \tag{31}$$

$${}_0^{\mathcal{C}}D_t^{\alpha_k} \hat{\zeta}_l = \frac{\pi_l}{2\kappa_l} e_l^{\mathrm{T}} e_l - \pi_l \sigma_l \hat{\zeta}_l, \tag{32}$$

$${}_0^{\mathcal{C}}D_t^{\alpha_k} \hat{\theta}_l = \gamma_l e_l - \gamma_l \tau_l \hat{\theta}_l, \tag{33}$$

where $c_l > 0$, $\sigma_l > 0$ and $\tau_l >$ are design parameters.
The of boundedness of the closed-loop signals follows from the proof of Theorem 1 by considering the Lyapunov as $\mathcal{V}(t) = \frac{1}{2} z^{\mathrm{T}}((\mathcal{L}+\mathcal{B}) \otimes I_n) z + \sum_{l=1}^{N} (\frac{1}{2\pi_l} \tilde{\zeta}_l^2 + \frac{1}{2\gamma_l} \tilde{\theta}_l^{\mathrm{T}} \tilde{\theta}_l)$.

V. Simulation Results

In this section, a numerical example is provided to verify the effectiveness of the proposed consensus synchronization protocol. In order to solve to the fractional order differential equations, the numerical approach of predictor-corrector is used with a step time of 0.1 *ms*.

A network of IFOSs composed of one leader and three follower satellites is considered. The dynamics of the followers and the leader are, respectively, represented by,

$$\begin{cases} {}_0^{\mathcal{C}}D_t^{\alpha_l} x_l = \frac{y_l z_l}{3} - 0.4 x_l + \frac{z_l}{\sqrt{6}} + 0.7\sin(t) + u_{l1}, \\ {}_0^{\mathcal{C}}D_t^{\alpha_l} y_l = -x_l z_l + 0.175 y_l + 0.8\sin(t) + u_{l2}, \\ {}_0^{\mathcal{C}}D_t^{\alpha_l} z_l = -\sqrt{6} x_l - 0.4 z_l + x_l y_l + 0.5\sin(t) + u_{l3}, \end{cases} \tag{34}$$

and

$$\begin{cases} {}_0^{\mathcal{C}}D_t^{0.74} x_0 = \cos(t), \\ {}_0^{\mathcal{C}}D_t^{0.74} y_0 = 2\cos(t), \\ {}_0^{\mathcal{C}}D_t^{0.74} z_0 = 0.6\cos(t), \end{cases} \tag{35}$$

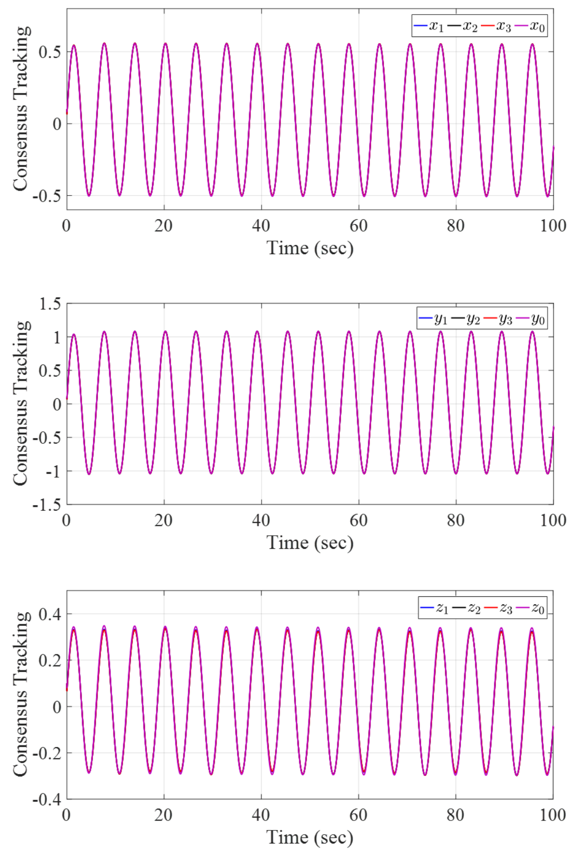

Figure 1: Consensus performance.

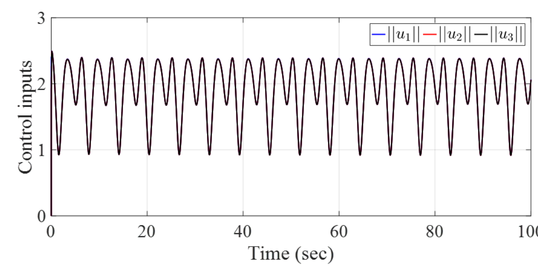

Figure 2: Control inputs.

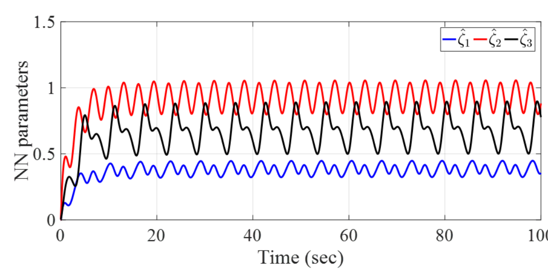

Figure 3: Neural network parameters.

The Laplacian matrix is given as

$$\mathcal{L} = \begin{pmatrix} 3.5 & -1 & -2.5 \\ -4 & 4 & 0 \\ 0 & -1.5 & 1.5 \end{pmatrix} \tag{36}$$

and the leader adjacency matrix is $\mathcal{B} = \mathrm{diag}\{1, 0, 0\}$.

For the simulation, the initial conditions are set as $[x_0(0), y_0(0), z_0(0)] = [0, 0, 0]$, $[x_l(0), y_l(0), z_l(0)] = [0.1, 0.2, 0.4]$, and $\zeta_l(0) = 0$ for $l = 1, 2, 3$, and $\alpha_1 = 0.98, \alpha_2 = 0.94, \alpha_3 = 0.93$. The initial values are set according to [31]. The controller parameters are chosen as $c_{l1} = 100, c_{l2} = 40, c_{l3} = 100, \sigma_l = 0.1, \pi_l = 1$ and $\kappa_l = 0.1$ for $l = 1, 2, 3$. To implement the proposed distributed control protocol, α_k is known as 0.94. The distributed control law and the fractional-order adaptive law are then constructed according to (22) and (23), respectively.

The simulation results are shown in Figs. 1-3. From Fig. 1, it is found that the attitude trajectory of the follower stellate systems synchronizes to the attitude of the leader stellate system. The norm of control input vector $u_l = [u_{l1}, u_{l2}, u_{l3}]^{\mathrm{T}}$ for $l = 1, 2, 3$ are displayed in Fig. 2. Fig. 3 shows the NN parameter of each follower. It is clear that the simulation performance verifies our theoretical result.

VI. CONCLUSION

In this letter, the distributed control strategy is proposed for the network of IFOSs. The main technical problem in this control strategy is that the networked system is represented through nonlinear uncertain fractional-order systems with prior knowledge of only one of fractional derivative orders. To deal with this control problem, first an unknown disturbance-like terms are added in the consensus analysis. Then, a composite uncertainty, composed of unknown system functions, unknown control coefficients and fractional derivatives of leader's state is formed to handle the network uncertainties. Finally, the composite uncertainties are identified using NNs with scalar adaptive laws, so that, the curse of dimension drawback is eliminated. Based on this control method, a distributed controller is constructed which guaranties that the states' followers synchronize the leader's state and also all the closed-loop network signals are bounded. The future work is how to extend the proposed approach to the output-feedback design for a network of IFOSs based on only the prior knowledge of one of fractional derivative orders.

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