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Leader-Follower Formation Control of Uncertain USV Networks Under Stochastic Disturbances

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ABSTRACT

The robust adaptive leader-follower formation control of uncertain unmanned surface vehicles (USVs) under stochastic disturbances is studied in this paper. The additive noises are considered in the kinematics that stands for the un-modeled dynamic. The disturbances induced by wind, waves and ocean currents are also separated into stochastic and deterministic components. A comprehensive model for each agent is then derived by stochastic differential equations including standard Wiener processes. Thus, the problem definition is more challenging since both the environmental disturbances and kinematics states are defined by stochastic differential equations. Quartic Lyapunov functions synthesis, dynamic surface control (DSC) technique, the projection algorithm, and neural networks (NNs) are employed to guarantee that all the tracking errors converge to a ball centered at the origin in probability. Finally, the simulation experiments quantify the effectiveness of proposed approach.

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I. Introduction

The control of nonlinear systems has been among the hot topics in the past decades [1]. The control and stability analysis of cooperative nonlinear agents has been given considerable attention in the past several years such as in formation control of robot agents [2], synchronization (consensus) of multiple robots [3], and etc. Among these approaches, the formation of agents leads to maintaining a desired configuration between the neighboring systems [4], [5]. The leader-follower formation control of USVs was studied in several works (see for example, [5], [6]).

The stability analysis of nonlinear control systems has led to different controlling architectures and schemes such as sliding mode control (SMC) [7], feedback linearization [8], [9], backstepping [1], and etc. An adaptive robust tracking controller based on backstepping method is presented for uncertain electrically-driven robotic manipulators in the framework of voltage control strategy in [10]. The *explosion of complexity* for the backstepping approach was resolved using dynamic surface control (DSC) method in [11] for deterministic cases. The DCS approach is utilized for deterministic types of nonlinear systems [12], [13] and different classes of stochastic nonlinear systems [14]–[16].

On the other hand, the existence of stochastic environmental loads highlights the importane of analyzing the stability of nonlinear stochastic systems [17]–[19] and correlated stochastic agents [20], [21]. We have to note that the distructive effects of environmnetal loads on the performance of marine vehicles is necessary to be considered in practice [22]. The platoon and leader-follower formation of multiple USVs are investigated in [4] and [5], respectively. Time

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varying external disturbances are considered and compensated in [4], [5]. Nevertheless, these environmental loads are deterministic. We have to note that ocean loads contain stochastic components in practice [23]. An ocean vehicle is supposed to track a reference trajectory under stochastic exogenous disturbances in [24]. A backstepping path-tracking method is investigated in [25] under stochastic loads. Although [24] and [25] have investigated the effects of stochastic disturbances, a single path tracking vehicle is only addressed. In [20], the formation is achieved among the agent. In earlier problem formulations for formation control of MASs the kinematics is considered deterministic [20]. To be precise, even in stochastic problem formulations, the kinematics is considered deterministic. More understandably, uncertain states and un-modeled dynamics in all the existing literature for the kinematics are ignored, which makes a control design inapplicable in practice.

The main contributions of the proposed approach in this paper are listed below:

- Compared to existing results, the kinematics in our proposed approach is described by ordinary stochastic differential equations.
- The proposed architecture in this paper considers both state uncertainty and stochastic exogenous loads denoted by standard Wiener process.

The remaining paper is organized as follows. The first section gives the preliminaries. Section two, studies the formation control architecture. In Section four, the effectiveness of the implementation is investiagateds. Finally, Section five concludes the paper.

II. Preparatory Work

The used notation in this paper is reported in Table I.

TABLE I.

USED NOTATION

$\mathbb{R}$	The sets of real numbers
$\mathbb{R}^N$	The sets of real *N*-vectors
$\mathbb{R}^{N\times N}$	The sets of real $N\times N$ matrices
$\lvert x\rvert$	The absolute value of $x\in\mathbb{R}$
$\lVert\mathbf{x}\rVert$	The 2-norm of a given vector $\mathbf{x}$
$\lVert\mathbf{x}\rVert_p$	The p-norm of a given vector $\mathbf{x}$
$diag(\bullet)$	A diagonal matrix of its argument
$\mathbf{I}$	The identity matrix of appropriate dimensions
$(\bullet)^{\mathcal{T}}$	The transpose operation on vectors or matrices
$\mathrm{Tr}(\bullet)$	The trace of a given matrix
$\mathcal{C}\mathrm{ov}(\bullet)$	The covariance matrix of its arguments
$\mathcal{C}\mathrm{ol}(\bullet)$	The column spaces of its arguments
$\mathcal{R}\mathrm{ow}(\bullet)$	The row spaces of its arguments
$\mathbf{x}\circ\mathbf{x}$	The Hadamard production
$\lambda_m(\bullet)$	the minimum eigenvalue of its arguments
$\lambda_M(\bullet)$	the maximum eigenvalue of its arguments

A. Preliminaries

Neural Networks: In order to approximate the continuous function $g(\mathbf{Z}):\mathbb{R}^q\rightarrow\mathbb{R}$, RBF neural networks is utilized as

$$g_{nn}(\mathbf{Z})=\mathbf{W}^{\mathcal{T}}\mathcal{S}(\mathbf{Z}). \tag{1}$$

The networks node number and input dimension are denoted by $l>1$ and q, respectively. Therefore, $\mathbf{Z}\in\mathbb{R}^q$ is the input vector and the weight vector is indicated by $\mathbf{W}=[w_1,w_2,\ldots,w_l]^{\mathcal{T}}$. The elements of basis function vector $\mathcal{S}(\mathbf{Z})=[s_1(\mathbf{Z}),s_2(\mathbf{Z}),\ldots,s_l(\mathbf{Z})]^{\mathcal{T}}$ is chosen as Gaussian functions of the form

$$s_i(\mathbf{Z})=\exp\left[-\frac{(\mathbf{Z}-\varkappa_i)^{\mathcal{T}}(\mathbf{Z}-\varkappa_i)}{\eta_i^2}\right]\quad, i=1,2,\ldots,l, \tag{2}$$

where $\varkappa_i=[\varkappa_{i1},\varkappa_{i2},\ldots,\varkappa_{iq}]^{\mathcal{T}}$ is the center of receptive field, and η_i is the width of Gaussian function. The RBF neural networks approximate almost any function $g(\mathbf{Z})$ with sufficiently large number of nodes l over a compact set Ω_Z to arbitrary any accuracy $\epsilon>0$ as

$$g(\mathbf{Z})=\mathbf{W}^{*\mathcal{T}}\mathcal{S}(\mathbf{Z})+\delta(\mathbf{Z}),\ \forall z\in\Omega_Z\subset\mathbb{R}^q, \tag{3}$$

where $\mathbf{W}^*$ is the ideal constant weight vector and defined as

$$\mathbf{W}^*=\arg\min_{\mathbf{W}\in\mathbb{R}^l}\left\{\sup_{\mathbf{Z}\in\Omega_Z}\lvert g(\mathbf{Z})-\mathbf{W}^{\mathcal{T}}\mathcal{S}(\mathbf{Z})\rvert\right\}, \tag{4}$$

and $\delta(\mathbf{Z})$ is the approximation error which satisfies $\lvert\delta(\mathbf{Z})\rvert\le\epsilon$.

Definition 1 [26]: The projection algorithm over two arguments ϖ and $\hat{\varrho}$ is denoted by $\mathcal{J}(\varpi,\hat{\varrho})$ and defined as

$$\begin{cases}\mathcal{J}(\varpi,\hat{\varrho})=(1-\Omega(\hat{\varrho}))\varpi:\Omega(\hat{\varrho})>0\,\&\,\bar{\Omega}(\hat{\varrho})\varpi>0,\\ \mathcal{J}(\varpi,\hat{\varrho})=\varpi:\Omega(\hat{\varrho})>0\,\&\,\bar{\Omega}(\hat{\varrho})\varpi\le 0,\\ \mathcal{J}(\varpi,\hat{\varrho})=\varpi:\Omega(\hat{\varrho})\le 0,\end{cases} \tag{5}$$

with

$$\Omega(\hat{\varrho})=\frac{\lVert\hat{\varrho}\rVert^2-\varrho_M^2}{\kappa^2+2\kappa\varrho_M},\ \bar{\Omega}(\hat{\varrho})=\frac{\partial\Omega(\hat{\varrho})}{\partial\hat{\varrho}} \tag{6}$$

where κ is a small positive constant and $\lVert\varrho\rVert\le\varrho_M$.

The projection algorithm is such that if

$$\dot{\hat{\varrho}}=\Gamma\mathcal{J}(\varpi,\hat{\varrho})$$

For some positive symmetric positive definite matrix Γ and $\lVert\hat{\varrho}(t_0)\rVert\le\varrho_M$ then

$$a)\ \hat{\varrho}(t) \le \varrho_M + \kappa, \quad t_0 \le t \le \infty,$$
$$b)\ \mathcal{J}(\varpi, \hat{\varrho}) \quad \text{is continuous},$$
$$c)\ \| \mathcal{J}(\varpi, \hat{\varrho}) \| \le \| \varpi \|,$$
$$d)\ \tilde{\varrho}^T \mathcal{J}(\varpi, \hat{\varrho}) \ge \tilde{\varrho}^T \varpi, \ \ \tilde{\varrho} = \varrho - \hat{\varrho}$$

Young's Inequality [1]: For any vector set $(\mathbf{k}, \mathbf{m}) \in \mathbb{R}^n$, the following inequality holds

$$\mathbf{k}^{\mathrm{T}}\mathbf{m} \le \frac{\varepsilon^p}{p} \| \mathbf{k} \|^p + \frac{1}{q\varepsilon^q} \| \mathbf{m} \|^q, \tag{7}$$

where $\varepsilon > 0,\ p > 1, q > 1$, and $(p-1)(q-1) = 1$.

B. Stochastic dynamics

The dynamics of a particular stochastic nonlinear system is described by [24]

$$\mathrm{d}\mathbf{x} = \mathbf{h}(\mathbf{x}, t)\mathrm{d}t + \mathbf{G}(\mathbf{x}, t)\mathbf{\Lambda}(t)\mathrm{d}\mathbf{w} \tag{8}$$

where $\mathbf{x} \in \mathbb{R}^n$ is the system state. $\mathbf{w}$ is an independent standard Wiener process of dimension r. $\mathbf{h} : \mathbb{R}^n \times \mathbb{R}_+ \to \mathbb{R}^n$ and $\mathbf{G} : \mathbb{R}^n \times \mathbb{R}_+ \to \mathbb{R}^{n \times r}$ are locally bounded and Lipschitz continuous in $\mathbf{x} \in \mathbb{R}^n$. $\mathbf{\Lambda}(t) : \mathbb{R}_+ \to \mathbb{R}^{r \times r}$ is nonnegative definite and Borel measurable for all $t \in \mathbb{R}_+$ and also bounded.

Lemma 1 [27]: Let a C^2 function $\mathcal{V}(\mathbf{x}(t))$ is defined over the stochastic system (8). Then the infinite generator $\mathcal{LV}(\mathbf{x}(t))$ is derived as

$$\mathcal{LV}(\mathbf{x}(t)) = \left(\frac{\partial \mathcal{V}}{\partial \mathbf{x}}\right)^{\mathrm{T}} \mathbf{h}(\mathbf{x}, t) + \frac{1}{2}\mathrm{Tr}\left(\mathbf{\Lambda}^{\mathrm{T}}(t)\mathbf{G}^{\mathrm{T}}(\mathbf{x}, t)\frac{\partial^2 \mathcal{V}}{\partial \mathbf{x}^{\mathrm{T}} \partial \mathbf{x}}\mathbf{G}(\mathbf{x}, t)\mathbf{\Lambda}(t)\right). \tag{9}$$

Lemma 2 [28]: Suppose that a C^2 function $\mathcal{V}(\mathbf{x}(t)) : \mathbb{R}^n \to \mathbb{R}_+$ is defined. If there exist class K_∞ functions $\varsigma_1(\bullet)$ and $\varsigma_2(\bullet)$, constant c, and a Borel measurable increasing function $\varsigma_3(\bullet)$ such that $\forall \mathbf{x}(t) \in \mathbb{R}^n$ and $\forall t \ge t_0 \ge 0$, we have

$$\begin{cases} \varsigma_1(\| \mathbf{x}(t) \|) \le \mathcal{V}(\mathbf{x}(t)) \le \varsigma_2(\| \mathbf{x}(t) \|), \\ \mathcal{LV}(\mathbf{x}(t)) \le -c\mathcal{V}(\mathbf{x}(t)) + \varsigma_3(\| \mathbf{\Lambda}(t)\mathbf{\Lambda}^{\mathrm{T}}(t) \|), \end{cases} \tag{10}$$

then there exists a unique strong solution of the system (8) for each $\mathbf{x}(t_0) \in \mathbb{R}^n$ which satisfies

$$\mathbf{E}[\mathcal{V}(\mathbf{x}(t))] \le \exp(-c(t - t_0))\mathcal{V}(\mathbf{x}(t_0)) + \frac{1}{c}\varsigma_3\left(\sup_{t_0 \le \tau \le t} \| \mathbf{\Lambda}(\tau)\mathbf{\Lambda}^{\mathrm{T}}(\tau) \|\right). \tag{11}$$

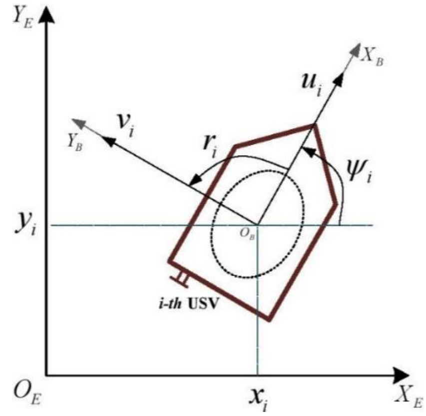

Fig. 1: The schematic motion behavior of i-th hypothetical USV.

C. Problem Statement

The motion of each agent is depicted in Fig. 1. The kinematics of one particular agent is described by

$$\begin{cases} dx_i = (u_i \cos(\psi_i) - v_i \sin(\psi_i))dt + g_{x_i} dw, \\ dy_i = (u_i \sin(\psi_i) + v_i \cos(\psi_i))dt + g_{y_i} dw, \\ d\psi_i = r_i dt + g_{\psi_i} dw, \end{cases} \tag{12}$$

where w denotes a one-dimensional standard Wiener process defined on a complete probability space. Furthermore, $g_{x_i} : \mathbb{R} \to \mathbb{R}$, $g_{y_i} : \mathbb{R} \to \mathbb{R}$ and $g_{\psi_i} : \mathbb{R} \to \mathbb{R}$ are smooth unknown nonlinear functions of $\boldsymbol{\eta}_i$ with zero initial conditions where $\boldsymbol{\eta}_i = [x_i, y_i, \psi_i]^{\mathcal{T}}$.

With $\mathbf{v}_i = [u_i, v_i, r_i]^{\mathcal{T}}$, the kinetics of i-th USV agent is usually derived using Lagrangian mechanics represented by

$$\begin{aligned} d\mathbf{v}_i = \mathbf{M}_i^{-1}[&-C_i(\mathbf{v}_i)\mathbf{v}_i - D_i(\mathbf{v}_i)[\mathbf{v}_i + \mathbf{v}_c] \\ &- \mathbf{M}_A \dot{\mathbf{v}}_r + \phi_i(\boldsymbol{\eta}_i, \mathbf{v}_i)\xi_i + \boldsymbol{\tau}_{\mathrm{wd}} + \boldsymbol{\tau}_{\mathrm{wv}} + \boldsymbol{\tau}_i]dt, \end{aligned} \tag{13}$$

where $\mathbf{M}_i$ is the vehicle inertia matrix as

$$\mathbf{M}_i = \begin{bmatrix} m - X_{\dot{u}} & 0 & 0 \\ 0 & m - Y_{\dot{v}} & mx_g - Y_{\dot{r}} \\ 0 & mx_g - N_{\dot{v}} & I_z - N_{\dot{r}} \end{bmatrix}. \tag{14}$$

$\mathbf{M}_A$ is added mass inertia matrix. The control input is $\boldsymbol{\tau}_i = [\tau_{i,1}, \tau_{i,2}, \tau_{i,3}]^{\mathcal{T}}$. The total Coriolis matrix is $C_i(\mathbf{v}_i)$ with

$$C_i(\mathbf{v}_i) = \begin{bmatrix} 0 & 0 & C_{i13} \\ 0 & 0 & C_{i23} \\ -C_{i13} & -C_{i23} & 0 \end{bmatrix} \tag{15}$$

The damping matrix $D_i(\mathbf{v}_i)$ is

$$D_i(\mathbf{v}_i) = \begin{bmatrix} -X_u - X_{|u|u}|\bar{u}| - X_{uuu}\bar{u}^2 & 0 & 0 \\ 0 & -Y_v - Y_{|v|v}|\bar{v}| & -Y_r \\ 0 & -N_v - N_{|v|v}|\bar{v}| & 0 \end{bmatrix} \tag{16}$$

Note that here, $C_{i13} = -(m - Y_{\dot{v}})v - (mx_g - Y_{\dot{r}})r$, $C_{i23} = (m - X_{\dot{u}})u$, $\bar{u} = u - \bar{\bar{u}}_c$, $\bar{v} = v - \bar{\bar{v}}_c$ and $\bar{r} = r$.

$\boldsymbol{\xi}_i = \left[Y_{|r_i|v_i}, Y_{|v_i|r_i}, Y_{|r_i|r_i}, N_{|r_i|v_i}, N_{r_i}, N_{|v_i|r_i}, N_{|r_i|r_i}, b_{1i}, b_{2i}, b_{3i} \right]^{\mathcal{T}}$ is also a constant uncertain parameter vector and $\phi_i(\boldsymbol{\eta}_i, \mathbf{v}_i)$ is described by

$$\phi_i(\boldsymbol{\eta}_i, \mathbf{v}_i) =$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cos(\psi_i) & \sin(\psi_i) & 0 \\ |r_i|v_i & |v_i|r_i & |r_i|r_i & 0 & 0 & 0 & 0 & -\sin(\psi_i) & \cos(\psi_i) & 0 \\ 0 & 0 & 0 & |r_i|v_i & r_i & |v_i|r_i & |r_i|r_i & 0 & 0 & 1 \end{bmatrix} \quad (17)$$

The relative velocity vector is $\mathbf{v}_r = \mathbf{v} - \mathbf{v}_c$, where $\mathbf{v}_c = [u_c, v_c, r_c]^{\mathcal{T}}$ is the sea current velocity. Let $\widehat{S}$ and $\breve{S}$ be the deterministic and stochastic elements of S, respectively. Then, one has $\boldsymbol{\tau}_{\text{wd}} = \widehat{\boldsymbol{\tau}}_{\text{wd}} + \breve{\boldsymbol{\tau}}_{\text{wd}}$, $\boldsymbol{\tau}_{\text{wv}} = \widehat{\boldsymbol{\tau}}_{\text{wv}} + \breve{\boldsymbol{\tau}}_{\text{wv}}$, $\mathbf{v}_c = \widehat{\mathbf{v}}_c + \breve{\mathbf{v}}_c$, and $\dot{\mathbf{v}}_c = \dot{\widehat{\mathbf{v}}}_c + \dot{\breve{\mathbf{v}}}_c$. Accordingly, one has

$$\mathbf{M}_A \dot{\mathbf{v}}_r = \mathbf{M}_A (\dot{\mathbf{v}} - \dot{\mathbf{v}}_c) = \mathbf{M}_A (\dot{\mathbf{v}} - \dot{\widehat{\mathbf{v}}}_c - \dot{\breve{\mathbf{v}}}_c). \quad (18)$$

Therefore, by defining $\mathbf{M}_t = \mathbf{M} - \mathbf{M}_A$, the stochastic kinetics of i-th USV agent is derived as

$$d\mathbf{v}_i = [\mathbf{g}_i + \mathbf{G}_i \mathbf{Y}_i + \phi_i(\boldsymbol{\eta}_i, \mathbf{v}_i)\boldsymbol{\xi}_i + \boldsymbol{\tau}_i^*]dt + [\mathbf{G}_i \boldsymbol{\Lambda}_i] d\mathbf{w}, \quad (19)$$

where $\mathbf{w}$ is a 6-dimensional Wiener standard process vector and

$$\begin{cases} \boldsymbol{\tau}_i^* = \mathbf{M}_t^{-1} \boldsymbol{\tau}_i, \\ \mathbf{g}_i = \mathbf{M}_t^{-1}(-C_i(\mathbf{v}_i)\mathbf{v}_i - D_i(\mathbf{v}_i)\mathbf{v}_i), \\ \mathbf{G}_i = \mathbf{M}_t^{-1} \mathcal{R}\text{ow}(D_i(\mathbf{v}_i), \mathbf{I}_3), \\ \mathbf{Y}_i = \mathcal{C}\text{ol}(\widehat{\mathbf{v}}_c, \mathbf{M}_A \dot{\widehat{\mathbf{v}}}_c + \widehat{\boldsymbol{\tau}}_{\text{wd}} + \widehat{\boldsymbol{\tau}}_{\text{wv}}). \end{cases} \quad (20)$$

Furthermore, $\boldsymbol{\Lambda}_i = \text{diag}(\boldsymbol{\Lambda}_{1i}, \boldsymbol{\Lambda}_{2i})$, with

$$\begin{cases} \boldsymbol{\Lambda}_{1i} = \mathcal{C}\text{ov}(\breve{\mathbf{v}}_c), \\ \boldsymbol{\Lambda}_{2i} = \mathcal{C}\text{ov}(\mathbf{M}_A \dot{\breve{\mathbf{v}}}_c + \breve{\boldsymbol{\tau}}_{\text{wd}} + \breve{\boldsymbol{\tau}}_{\text{wv}}). \end{cases} \quad (21)$$

III. MAIN RESULTS

The following assumptions hold true.

Assumption 1: The reference signal is deterministic, bounded, and also satisfies $\boldsymbol{\eta}_0 \in C^2$

Assumption 2: There exists a predefined preceding USV i-1 associated with agent i.

Assumption 3: The vectors $\mathbf{Y}_i$, $\boldsymbol{\xi}_i$ and covariance matrix $\boldsymbol{\Lambda}_i(t)$ are all bounded, i.e., for $\mathrm{Y}_i^{\mathbf{M}}$, $\xi_i^{\mathbf{M}}$, and $\Lambda_i^{\mathbf{M}}$, one has

$$\| \mathbf{Y}_i \| \le \mathrm{Y}_i^{\mathbf{M}}, \| \boldsymbol{\xi}_i \| \le \xi_i^{\mathbf{M}}, \| \boldsymbol{\Lambda}_i \boldsymbol{\Lambda}_i^{\mathcal{T}} \|_\infty \le \Lambda_i^{\mathbf{M}}. \quad (22)$$

D. Leader-follower formation control design

Consider again the team of N fully actuated USVs described in (12) and (19). The schematic of two agents is shown in Fig. 2.

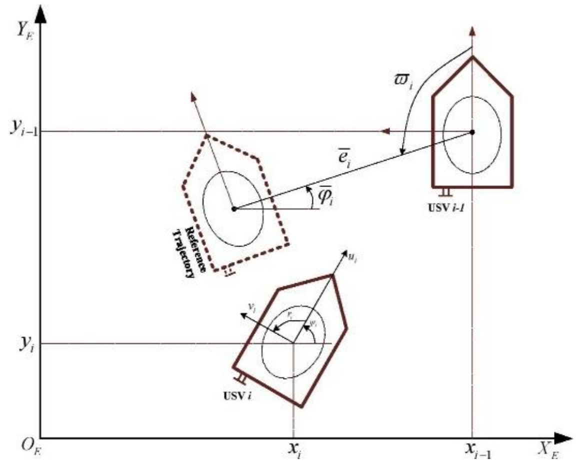

Fig. 2: The schematic of two hypothetical agents.

A desired predetermined reference trajectory $\boldsymbol{\eta}_0 = [x_0, y_0, \psi_0]^{\mathcal{T}}$ is accessible by the leading USV of the team. In this configuration, each follower is the leader for another agent. More precisely, each two consecutive USVs act as a pair of leader-follower. According to Fig. 2, the reference trajectory to be followed by the i-th vehicle is determined by USV i-1, which is shifted longitudinally and circularly by predefined parameters $\bar{e}_i$ and ϖ_i, respectively, and then rotated relatively through $\bar{\varphi}_i$. These predetermined formation parameters can be either time-invariant or time-varying.

The reference trajectories for the i-th follower agent in x, y, and ψ directions are described as

$$\begin{cases} x_{ri} = x_{i-1} + \bar{e}_i[\cos(\psi_{i-1})\cos(\varpi_i) - \sin(\psi_{i-1})\sin(\varpi_i)], \\ y_{ri} = y_{i-1} + \bar{e}_i[\sin(\psi_{i-1})\cos(\varpi_i) + \cos(\psi_{i-1})\sin(\varpi_i)], \\ \psi_{ri} = \psi_{i-1} + \bar{\varphi}_i, \end{cases} \quad (23)$$

We define the leader-follower formation errors as

$$s_{xi} = x_i - x_{ri}, \; s_{yi} = y_i - y_{ri}, \; s_{\psi i} = \psi_i - \psi_{ri}. \quad (24)$$

Then according to Lemma 1, we have

$$\begin{aligned} ds_{xi} &= [u_i \cos(\psi_i) - v_i \sin(\psi_i) - \bar{x}_{i-1} + f_{x_i}]dt + \bar{g}_{x_i} dw_i, \\ ds_{yi} &= [u_i \sin(\psi_i) + v_i \cos(\psi_i) - \bar{y}_{i-1} + f_{y_i}]dt + \bar{g}_{y_i} dw_i, \\ ds_{\psi_i} &= [r_i - \bar{\psi}_{i-1}]dt + \bar{g}_{\psi_i} dw_i, \end{aligned} \quad (25)$$

where

$$\begin{cases} \bar{x}_{i-1} = u_{i-1}\cos(\psi_{i-1}) - v_{i-1}\sin(\psi_{i-1}) \\ \quad - \bar{e}_i r_{i-1}[\cos(\varpi_i)\sin(\psi_{i-1}) + \sin(\varpi_i)\cos(\psi_{i-1})], \\ \bar{y}_{i-1} = u_{i-1}\sin(\psi_{i-1}) + v_{i-1}\cos(\psi_{i-1}) \\ \quad - \bar{e}_i r_{i-1}[\cos(\varpi_i)\cos(\psi_{i-1}) - \sin(\varpi_i)\sin(\psi_{i-1})], \\ \bar{\psi}_{i-1} = r_{i-1} \end{cases} \quad (26)$$

and

$$f_{x_i} = 0.5\bar{e}_i g_{\psi_{i-1}}^2 [\cos(\varpi_i)\cos(\psi_{i-1}) - \sin(\varpi_i)\sin(\psi_{i-1})],$$
$$f_{y_i} = 0.5\bar{e}_i g_{\psi_{i-1}}^2 [\cos(\varpi_i)\sin(\psi_{i-1}) + \sin(\varpi_i)\cos(\psi_{i-1})] \quad (27)$$

Additionally

$$\bar{g}_{x_i} = \bar{e}_i g_{\psi_{i-1}} [\cos(\varpi_i)\sin(\psi_{i-1}) + \sin(\varpi_i)\cos(\psi_{i-1})] + g_{x_i} - g_{x_{i-1}},$$
$$\bar{g}_{y_i} = \bar{e}_i g_{\psi_{i-1}} [\sin(\varpi_i)\sin(\psi_{i-1}) - \cos(\varpi_i)\cos(\psi_{i-1})] + g_{y_i} - g_{y_{i-1}}, \quad (28)$$
$$\bar{g}_{\psi_i} = g_{\psi_i} - g_{\psi_{i-1}}$$

Step I:

Let

$$\begin{cases} \boldsymbol{\zeta}_i = \mathbf{v}_i - \boldsymbol{\alpha}_i^f, \\ \boldsymbol{\chi}_i = \boldsymbol{\alpha}_i^f - \boldsymbol{\alpha}_i. \end{cases} \quad (29)$$

A Lyapunov function candidate is selected as

$$\mathcal{V}_{0,i} = \frac{1}{4}s_{xi}^4 + \frac{1}{4}s_{yi}^4 + \frac{1}{4}s_{\psi i}^4. \quad (30)$$

By employing Lemma 1, we have

$$\begin{aligned} \mathcal{L}\mathcal{V}_{0,i} &= s_{xi}^3[\alpha_{1i}\cos(\psi_i) - \alpha_{2i}\sin(\psi_i) - \bar{x}_{i-1} + f_{1i}(\bullet)] \\ &+ s_{yi}^3[\alpha_{1i}\sin(\psi_i) + \alpha_{2i}\cos(\psi_i) - \bar{y}_{i-1} + f_{2i}(\bullet)] \\ &+ s_{\psi i}^3[\alpha_{3i} - \bar{\psi}_{i-1} + f_{3i}(\bullet)] + \boldsymbol{\zeta}_i^{\mathcal{T}}\mathbf{E}_i + \boldsymbol{\chi}_i^{\mathcal{T}}\mathbf{E}_i \\ &- \frac{3}{4}s_{xi}^4(1 + 2(\frac{\beta_{1i}^2 + \beta_{2i}^2}{\beta_{1i}^2\beta_{2i}^2}) \| h_{1i}^* \|^4) + \frac{3}{2}s_{xi}^2\bar{g}_{x_i}^2 \\ &- \frac{3}{4}s_{yi}^4(1 + 2(\frac{\beta_{1i}^2 + \beta_{2i}^2}{\beta_{1i}^2\beta_{2i}^2}) \| h_{1i}^* \|^4) + \frac{3}{2}s_{yi}^2\bar{g}_{y_i}^2 \\ &- \frac{3}{4}s_{\psi i}^4(1 + \frac{1}{\beta_{3i}^2} \| h_{2i}^* \|^4) + \frac{3}{2}s_{\psi i}^2\bar{g}_{\psi_i}^2, \end{aligned} \quad (31)$$

where $\mathbf{E}_i = [\mathcal{E}_{1i}, \mathcal{E}_{2i}, \mathcal{E}_{3i}]^{\mathcal{T}}$, with

$$\begin{cases} \mathrm{E}_{1i} = s_{xi}^3\cos(\psi_i) + s_{yi}^3\sin(\psi_i), \\ \mathrm{E}_{2i} = s_{yi}^3\cos(\psi_i) - s_{xi}^3\sin(\psi_i), \\ \mathrm{E}_{3i} = s_{\psi i}^3 \end{cases} \quad (32)$$

with unknown smooth nonlinear functions $h_{1i}^*(s_{xi})$, $h_{2i}^*(s_{yi})$, and $h_{3i}^*(s_{\psi i})$ to be introduced later. Moreover, we have the following unknown continuous functions

$$f_{1i}(\bullet) = \frac{3}{4}s_{xi}(1 + 2(\frac{\beta_{1i}^2 + \beta_{2i}^2}{\beta_{1i}^2\beta_{2i}^2}) \| h_{1i}^* \|^4 + \frac{1}{l_{1i}^2} \| \bar{g}_{x_i} \|^4) + f_{x_i},$$
$$f_{2i}(\bullet) = \frac{3}{4}s_{yi}(1 + 2(\frac{\beta_{1i}^2 + \beta_{2i}^2}{\beta_{1i}^2\beta_{2i}^2}) \| h_{2i}^* \|^4 + \frac{1}{l_{2i}^2} \| \bar{g}_{y_i} \|^4) + f_{y_i}, \quad (33)$$
$$f_{3i}(\bullet) = \frac{3}{4}s_{\psi i}(1 + \frac{1}{\beta_{3i}^2} \| h_{3i}^* \|^4 + \frac{1}{l_{3i}^2} \| \bar{g}_{\psi_i} \|^4)$$

One has

$$\begin{cases} \frac{3}{2}s_{xi}^2\bar{g}_{x_i}^2 \le \frac{3}{4l_{1i}^2}s_{xi}^4 \| \bar{g}_{x_i} \|^4 + \frac{3}{4}l_{1i}^2, \\ \frac{3}{2}s_{yi}^2\bar{g}_{y_i}^2 \le \frac{3}{4l_{2i}^2}s_{yi}^4 \| \bar{g}_{y_i} \|^4 + \frac{3}{4}l_{2i}^2, \\ \frac{3}{2}s_{\psi i}^2\bar{g}_{\psi_i}^2 \le \frac{3}{4l_{3i}^2}s_{\psi i}^4 \| \bar{g}_{\psi_i} \|^4 + \frac{3}{4}l_{3i}^2. \end{cases} \quad (34)$$

Define the approximation errors $\tilde{\Theta}_{1i}$, $\tilde{\Theta}_{2i}$, and $\tilde{\Theta}_{3i}$. Then, we have

$$\begin{cases} s_{xi}^3 f_{1i} \le \frac{s_{xi}^6}{2a_{1i}^2}\mathcal{S}_{1i}^{\mathcal{T}}\mathcal{S}_{1i}\Theta_{1i}^* + \frac{1}{2}a_{1i}^2 + \frac{3}{4}s_{xi}^4 + \frac{1}{4}\epsilon_{1i}^{*4}, \\ s_{yi}^3 f_{2i} \le \frac{s_{yi}^6}{2a_{2i}^2}\mathcal{S}_{2i}^{\mathcal{T}}\mathcal{S}_{2i}\Theta_{2i}^* + \frac{1}{2}a_{2i}^2 + \frac{3}{4}s_{yi}^4 + \frac{1}{4}\epsilon_{2i}^{*4}, \\ s_{\psi i}^3 f_{3i} \le \frac{s_{\psi i}^6}{2a_{3i}^2}\mathcal{S}_{3i}^{\mathcal{T}}\mathcal{S}_{3i}\Theta_{2i}^* + \frac{1}{2}a_{3i}^2 + \frac{3}{4}s_{\psi i}^4 + \frac{1}{4}\epsilon_{3i}^{*4}, \end{cases} \quad (35)$$

By expanding the Lyapunov function candidate in (30) one has

$$\mathcal{V}_{1,i} = \mathcal{V}_{0,i} + \frac{1}{2\gamma_{1i}}\tilde{\Theta}_{1i}^2 + \frac{1}{2\gamma_{2i}}\tilde{\Theta}_{2i}^2 + \frac{1}{2\gamma_{3i}}\tilde{\Theta}_{3i}^2, \quad (36)$$

which, results in

$$\mathcal{L}\mathcal{V}_{1,i} = \mathcal{L}\mathcal{V}_{0,i} - \frac{1}{\gamma_{1i}}\tilde{\Theta}_{1i}\dot{\hat{\Theta}}_{1i} - \frac{1}{\gamma_{2i}}\tilde{\Theta}_{2i}\dot{\hat{\Theta}}_{2i} - \frac{1}{\gamma_{3i}}\tilde{\Theta}_{3i}\dot{\hat{\Theta}}_{3i}. \quad (37)$$

The virtual control laws α_{1i}, α_{2i}, and α_{3i} are then designed as

$$\begin{aligned} \alpha_{1i} &= [-\mathcal{K}_{xi}s_{xi} - \frac{1}{2a_{1i}^2}s_{xi}^3\mathcal{S}_{1i}^{\mathcal{T}}\mathcal{S}_{1i}\hat{\Theta}_{1i} + \bar{x}_{i-1}]\cos(\psi_i) \\ &+ [-\mathcal{K}_{yi}s_{yi} - \frac{1}{2a_{2i}^2}s_{yi}^3\mathcal{S}_{2i}^{\mathcal{T}}\mathcal{S}_{2i}\hat{\Theta}_{2i} + \bar{y}_{i-1}]\sin(\psi_i), \\ \alpha_{2i} &= [\mathcal{K}_{xi}s_{xi} + \frac{1}{2a_{1i}^2}s_{xi}^3\mathcal{S}_{1i}^{\mathcal{T}}\mathcal{S}_{1i}\hat{\Theta}_{1i} - \bar{x}_{i-1}]\sin(\psi_i) \\ &- [\mathcal{K}_{yi}s_{yi} + \frac{1}{2a_{2i}^2}s_{yi}^3\mathcal{S}_{2i}^{\mathcal{T}}\mathcal{S}_{2i}\hat{\Theta}_{2i} - \bar{y}_{i-1}]\cos(\psi_i), \\ \alpha_{3i} &= -\mathcal{K}_{\psi i}s_{\psi i} - \frac{1}{2a_{3i}^2}s_{\psi i}^3\mathcal{S}_{3i}^{\mathcal{T}}\mathcal{S}_{3i}\hat{\Theta}_{3i} + \bar{\psi}_{i-1}, \end{aligned} \quad (38)$$

where $\mathcal{K}_{xi}, \mathcal{K}_{yi}, \mathcal{K}_{\psi i}$, $i = 1,\ldots,N$, are control gains. We design

$$\begin{cases} \dot{\hat{\Theta}}_{1i} = \frac{\gamma_{1i}}{2a_{1i}^2}s_{xi}^6\mathcal{S}_{1i}^{\mathcal{T}}\mathcal{S}_{1i} - \gamma_{1i}\sigma_{1i}\hat{\Theta}_{1i}, \\ \dot{\hat{\Theta}}_{2i} = \frac{\gamma_{2i}}{2a_{2i}^2}s_{yi}^6\mathcal{S}_{2i}^{\mathcal{T}}\mathcal{S}_{2i} - \gamma_{2i}\sigma_{2i}\hat{\Theta}_{2i}, \\ \dot{\hat{\Theta}}_{3i} = \frac{\gamma_{3i}}{2a_{3i}^2}s_{\psi i}^6\mathcal{S}_{3i}^{\mathcal{T}}\mathcal{S}_{3i} - \gamma_{3i}\sigma_{3i}\hat{\Theta}_{3i}. \end{cases} \quad (39)$$

By taking all the aforementioned into account, (37) leads to

$$
\begin{aligned}
\mathcal{LV}_{1,i} \le & -\mathcal{K}_{xi}s_{xi}^4 - \mathcal{K}_{yi}s_{yi}^4 - \mathcal{K}_{\psi i}s_{\psi i}^4 - \frac{1}{2}\sigma_{1i}\tilde{\Theta}_{1i}^2 - \frac{1}{2}\sigma_{2i}\tilde{\Theta}_{2i}^2 \\
& - \frac{1}{2}\sigma_{3i}\tilde{\Theta}_{3i}^2 + \boldsymbol{\zeta}_i^{\mathcal{T}}\mathbf{E}_i - \frac{3}{2}s_{xi}^4\Big(\frac{\beta_{1i}^2+\beta_{2i}^2}{\beta_{1i}^2\beta_{2i}^2}\Big)\| h_{1i}^*\|^4 + \boldsymbol{\chi}_i^{\mathcal{T}}\mathbf{E}_i \\
& - \frac{3}{2}s_{yi}^4\Big(\frac{\beta_{1i}^2+\beta_{2i}^2}{\beta_{1i}^2\beta_{2i}^2}\Big)\| h_{1i}^*\|^4 - \frac{3}{4}s_{\psi i}^4\frac{1}{\beta_{3i}^2}\| h_{2i}^*\|^4 + \frac{1}{2}a_{1i}^2 \\
& + \frac{1}{4}\epsilon_{1i}^{*4} + \frac{1}{2}a_{2i}^2 + \frac{1}{4}\epsilon_{2i}^{*4} + \frac{1}{2}a_{3i}^2 + \frac{1}{4}\epsilon_{3i}^{*4} + \frac{3}{4}l_{1i}^2 + \frac{3}{4}l_{2i}^2 \\
& + \frac{3}{4}l_{3i}^2 + \frac{1}{2}\sigma_{1i}\Theta_{1i}^{*2} + \frac{1}{2}\sigma_{2i}\Theta_{2i}^{*2} + \frac{1}{2}\sigma_{3i}\Theta_{3i}^{*2}
\end{aligned} \tag{40}
$$

Consider the first-order filter $\boldsymbol{\mu}_i\dot{\boldsymbol{\alpha}}_i^f + \boldsymbol{\alpha}_i^f = \boldsymbol{\alpha}_i$ such that $\boldsymbol{\alpha}_i^f(0) = \boldsymbol{\alpha}_i(0)$. Then for $q = 1,2,3,$ we have

$$
\begin{cases}
d\chi_{qi} = [\dfrac{-\chi_{qi}}{\mu_{qi}} - \mathcal{B}_{qi}(\bullet)]dt - \mathcal{C}_{qi}(\bullet)dw, \\
d\alpha_{qi}^f = \dfrac{-\chi_{qi}}{\mu_{qi}},
\end{cases} \tag{41}
$$

where $\mathcal{B}_{1i}$ and $\mathcal{B}_{2i}$ are functions of $\bar{\boldsymbol{\eta}}_{i-1}$, $\dot{\bar{\boldsymbol{\eta}}}_{i-1}$, $\ddot{\bar{\boldsymbol{\eta}}}_{i-1}$, s_{xi}, s_{yi}, χ_{1i}, χ_{2i}, ζ_{1i}, ζ_{2i}, $\tilde{\Theta}_{1i}$, and $\tilde{\Theta}_{2i}$. The arguments of $\mathcal{B}_{3i}$ on the other hand are $\bar{\boldsymbol{\eta}}_{i-1}$, $\dot{\bar{\boldsymbol{\eta}}}_{i-1}$, $\ddot{\bar{\boldsymbol{\eta}}}_{i-1}$, $s_{\psi i}$, χ_{3i}, ζ_{3i}, and $\tilde{\Theta}_{3i}$. Note that $\bar{\boldsymbol{\eta}}_{i-1} = [\bar{x}_{i-1}, \bar{y}_{i-1}, \bar{\psi}_{i-1}]^{\mathcal{T}}$. Furthermore, we have

$$
\begin{cases}
\mathcal{C}_{1i} = \dfrac{\partial\alpha_{1i}}{\partial s_{xi}}h_{1i} + \dfrac{\partial\alpha_{1i}}{\partial s_{yi}}h_{2i},\ \mathcal{C}_{2i} = \dfrac{\partial\alpha_{2i}}{\partial s_{xi}}h_{1i} + \dfrac{\partial\alpha_{2i}}{\partial s_{yi}}h_{2i}, \\
\mathcal{C}_{3i} = \dfrac{\partial\alpha_{3i}}{\partial s_{\psi i}}h_{3i}.
\end{cases} \tag{42}
$$

We can write, $h_{1i} = s_{xi}h_{1i}^*$, $h_{2i} = s_{yi}h_{2i}^*$, and $h_{3i} = s_{\psi i}h_{3i}^*$. The Lyapunov function candidate is expanded to give

$$
\mathcal{V}_{2,i} = \mathcal{V}_{1,i} + \sum_{j=1}^{3}\frac{1}{4}\chi_{ji}^4, \tag{43}
$$

which yields in

$$
\mathcal{LV}_{2,i} = \mathcal{LV}_{1,i} + \sum_{j=1}^{3}\chi_{ji}^3\Big[\frac{-\chi_{ji}}{\mu_{ji}} - \mathcal{B}_{ji}(\bullet)\Big] + \sum_{j=1}^{3}\frac{3}{2}\chi_{ji}^2\|\mathcal{C}_{ji}\|^2. \tag{44}
$$

One has

$$
\begin{aligned}
\frac{3}{2}\chi_{1i}^2\|\mathcal{C}_{1i}\|^2 \le & \frac{3}{2\beta_{1i}^2}(s_{xi}^4\|h_{1i}^*\|^4 + s_{yi}^4\|h_{2i}^*)\|^4) \\
& + \frac{3}{2}\beta_{1i}^2\chi_{1i}^4\Big[\Big(\frac{\partial\alpha_{1i}}{\partial s_{xi}}\Big)^4 + \Big(\frac{\partial\alpha_{1i}}{\partial s_{yi}}\Big)^4\Big], \\
\frac{3}{2}\chi_{2i}^2\|\mathcal{C}_{2i}\|^2 \le & \frac{3}{2\beta_{2i}^2}(s_{xi}^4\|h_{1i}^*\|^4 + s_{yi}^4\|h_{2i}^*\|^4) \\
& + \frac{3}{2}\beta_{2i}^2\chi_{2i}^4\Big[\Big(\frac{\partial\alpha_{2i}}{\partial s_{xi}}\Big)^4 + \Big(\frac{\partial\alpha_{2i}}{\partial s_{yi}}\Big)^4\Big], \\
\frac{3}{2}\chi_{3i}^2\|\mathcal{C}_{3i}\|^2 \le & \frac{3}{4}\beta_{3i}^2\chi_{3i}^4\Big(\frac{\partial\alpha_{3i}}{\partial s_{\psi i}}\Big)^4 + \frac{3}{4\beta_{3i}^2}s_{\psi i}^4\|h_{3i}^*\|^4.
\end{aligned} \tag{45}
$$

We have

$$
\begin{aligned}
\mathcal{LV}_{2,i} \le & -\mathcal{K}_{xi}s_{xi}^4 - \mathcal{K}_{yi}s_{yi}^4 - \mathcal{K}_{\psi i}s_{\psi i}^4 - \frac{1}{2}\sigma_{1i}\tilde{\Theta}_{1i}^2 \\
& - \frac{1}{2}\sigma_{2i}\tilde{\Theta}_{2i}^2 - \frac{1}{2}\sigma_{3i}\tilde{\Theta}_{3i}^2 + \boldsymbol{\zeta}_i^{\mathcal{T}}\mathbf{E}_i + \boldsymbol{\chi}_i^{\mathcal{T}}\mathbf{E}_i \\
& - \sum_{j=1}^{3}\frac{\chi_{ji}^4}{\mu_{ji}} + \sum_{j=1}^{3}\frac{1}{4\beta_{4i}^4}\mathcal{B}_{qi}^4(\bullet) + \sum_{j=1}^{3}\frac{3}{4}\beta_{4i}^{\frac{4}{3}}\chi_{ji}^4 \\
& + \mathbb{A}_i + \frac{1}{2}a_{1i}^2 + \frac{1}{4}\epsilon_{1i}^{*4} + \frac{1}{2}a_{2i}^2 + \frac{1}{4}\epsilon_{2i}^{*4} \\
& + \frac{1}{2}a_{3i}^2 + \frac{1}{4}\epsilon_{3i}^{*4} + \frac{3}{4}l_{1i}^2 + \frac{3}{4}l_{2i}^2 + \frac{3}{4}l_{3i}^2 \\
& + \frac{1}{2}\sigma_{1i}\Theta_{1i}^{*2} + \frac{1}{2}\sigma_{2i}\Theta_{2i}^{*2} + \frac{1}{2}\sigma_{3i}\Theta_{3i}^{*2}.
\end{aligned} \tag{46}
$$

with

$$
\begin{aligned}
\mathbb{A}_i = & \frac{3}{2}\beta_{1i}^2\chi_{1i}^4\Big[\Big(\frac{\partial\alpha_{1i}}{\partial s_{xi}}\Big)^4 + \Big(\frac{\partial\alpha_{1i}}{\partial s_{yi}}\Big)^4\Big] \\
& + \frac{3}{2}\beta_{2i}^2\chi_{2i}^4\Big[\Big(\frac{\partial\alpha_{2i}}{\partial s_{xi}}\Big)^4 + \Big(\frac{\partial\alpha_{2i}}{\partial s_{yi}}\Big)^4\Big] \\
& + \frac{3}{4}\beta_{3i}^2\chi_{3i}^4\Big(\frac{\partial\alpha_{3i}}{\partial s_{\psi i}}\Big)^4
\end{aligned} \tag{47}
$$

This completes the first step of designing the leader-follower formation control approach.

Step II:

From (19) and (29), one obtains that

$$
d\boldsymbol{\zeta}_i = [\mathbf{g}_i + \mathbf{G}_i\mathbf{Y}_i + \phi(\boldsymbol{\eta}_i, \mathbf{v}_i)\boldsymbol{\xi}_i + \boldsymbol{\tau}_i^* + \boldsymbol{\mu}_i^{-1}\boldsymbol{\chi}_i]dt + [\mathbf{G}_i\boldsymbol{\Lambda}_i]dw. \tag{48}
$$

Define $\mathbf{Y}_i = \mathrm{Y}_i^* - \mathbf{Y}_i$, $\tilde{\boldsymbol{\xi}}_i = \boldsymbol{\xi}_i^* - \hat{\boldsymbol{\xi}}_i$, and $\tilde{\delta}_i = \|\boldsymbol{\Lambda}_i\boldsymbol{\Lambda}_i^{\mathcal{T}}\|_\infty^2 - \hat{\delta}_i,\ i = 1,\ldots,N$.

Then, expand the Lyapunov function to

$$
\mathcal{V}_{3,i} = \mathcal{V}_{2,i} + \frac{1}{4}\|\boldsymbol{\zeta}_i\|_4^4 + \frac{1}{2}\mathbf{Y}_i^{\mathcal{T}}\boldsymbol{\Gamma}_{1i}^{-1}\mathbf{Y}_i + \frac{1}{2}\tilde{\boldsymbol{\xi}}_i^{\mathcal{T}}\boldsymbol{\Gamma}_{2i}^{-1}\tilde{\boldsymbol{\xi}}_i + \frac{1}{2\gamma_{3i}}\tilde{\delta}_i^2. \tag{49}
$$

where $\boldsymbol{\Gamma}_{1i}$ and $\boldsymbol{\Gamma}_{2i}$ are symmetric positive definite design matrices and $\gamma_{3i},\ i = 1,\ldots,N$ are positive constants. Hence

$$\begin{aligned}\mathcal{L}\mathcal{V}_{3,i} \le \mathcal{L}\mathcal{V}_{2,i} &+ \zeta_i^{\mathcal{T}} \circ |\zeta_i^2|^{\mathcal{T}} [\mathbf{g}_i + \mathbf{G}_i \mathbf{Y}_i \\ &+ \phi_i(\boldsymbol{\eta}_i, \mathbf{v}_i)\boldsymbol{\xi}_i + \boldsymbol{\tau}_i^* + \boldsymbol{\mu}_i^{-1}\boldsymbol{\chi}_i] \\ &+ \frac{1}{2}\mathrm{Tr}\{\boldsymbol{\Lambda}_i^{\mathcal{T}} \mathbf{G}_i^{\mathcal{T}} \frac{\partial^2(\bullet)}{\partial \zeta_i^{\mathrm{T}} \partial \zeta_i} \mathbf{G}_i \boldsymbol{\Lambda}_i\} - \mathbf{Y}_i^{\mathcal{T}} \Gamma_{1i}^{-1} \dot{\mathbf{Y}}_i \\ &- \tilde{\boldsymbol{\xi}}_i^{\mathcal{T}} \Gamma_{2i}^{-1} \dot{\hat{\boldsymbol{\xi}}}_i - \frac{1}{\gamma_{3i}} \tilde{\delta}_i \dot{\hat{\delta}}_i,\end{aligned} \tag{50}$$

where

$$\frac{\partial^2(\bullet)}{\partial \zeta_i^{\mathrm{T}} \partial \zeta_i} = \mathrm{diag}(3\zeta_{1i}^2, 3\zeta_{2i}^2, 3\zeta_{3i}^2),\ i = 1, \ldots, N$$

By Young's inequality, one obtains

$$\zeta_i^{\mathcal{T}} \mathbf{E}_i \le \frac{1}{2}\beta_{5i}^4 \|\zeta_i\|_4^4 + \frac{3}{2}\beta_{5i}^{\frac{-4}{3}} s_{xi}^4 + \frac{3}{2}\beta_{5i}^{\frac{-4}{3}} s_{yi}^4 + \frac{3}{4}\beta_{5i}^{\frac{-4}{3}} s_{\psi i}^4, \tag{51}$$

and

$$\chi_i^{\mathcal{T}} \mathbf{E}_i \le \frac{1}{2}\beta_{7i}^4 \sum_{j=1}^{3} \chi_{ji}^4 + \frac{3}{2}\beta_{7i}^{\frac{-4}{3}} s_{xi}^4 + \frac{3}{2}\beta_{7i}^{\frac{-4}{3}} s_{yi}^4 + \frac{3}{4}\beta_{7i}^{\frac{-4}{3}} s_{\psi i}^4. \tag{52}$$

Furthermore, the following holds true

$$\begin{aligned}&\frac{1}{2}\mathrm{Tr}\{\boldsymbol{\Lambda}_i^{\mathcal{T}} \mathbf{G}_i^{\mathcal{T}} \frac{\partial^2(\bullet)}{\partial \zeta_i^{\mathrm{T}} \partial \zeta_i} \mathbf{G}_i \boldsymbol{\Lambda}_i\} \\ &\le 3\beta_{6i} \|\mathbf{G}_i\|^4 \|\zeta_i\|^4 \|\boldsymbol{\Lambda}_i \boldsymbol{\Lambda}_i^{\mathcal{T}}\|_\infty^2 + \frac{3}{4\beta_{6i}}.\end{aligned} \tag{53}$$

As a matter of fact, $\|\zeta_i\|_2 \le \sqrt{n}\,\|\zeta_i\|_4$. In this problem formulation n is equal to three. Then,

$$\begin{aligned}\mathcal{L}\mathcal{V}_{3,i} \le \mathcal{L}\mathcal{V}_{2,i} &+ \zeta_i^{\mathcal{T}} \circ |\zeta_i^2|^{\mathcal{T}} [\mathbf{g}_i + \mathbf{G}_i \mathbf{Y}_i \\ &+ \phi_i(\boldsymbol{\eta}_i, \mathbf{v}_i)\hat{\boldsymbol{\xi}}_i + \boldsymbol{\tau}_i^* + \boldsymbol{\mu}_i^{-1}\boldsymbol{\chi}_i] \\ &+ \zeta_i^{\mathcal{T}} \circ |\zeta_i^2|^{\mathcal{T}} \mathbf{G}_i \mathbf{Y}_i + \zeta_i^{\mathcal{T}} \circ |\zeta_i^2|^{\mathcal{T}} \phi_i(\boldsymbol{\eta}_i, \mathbf{v}_i)\tilde{\boldsymbol{\xi}}_i \\ &+ \frac{27}{2}\beta_{6i} \|\mathbf{G}_i\|^4 \|\zeta_i\|_4^4 (\tilde{\delta}_i + \hat{\delta}_i) - \mathbf{Y}_i^{\mathcal{T}} \Gamma_{1i}^{-1} \dot{\mathbf{Y}}_i \\ &- \tilde{\boldsymbol{\xi}}_i^{\mathcal{T}} \Gamma_{2i}^{-1} \dot{\hat{\boldsymbol{\xi}}}_i - \frac{1}{\gamma_{3i}} \tilde{\delta}_i \dot{\hat{\delta}}_i + \frac{3}{8\beta_{6i}}.\end{aligned} \tag{54}$$

We know that

$$\frac{27}{2}\beta_{6i} \|\mathbf{G}_i\|^4 \|\zeta_i\|_4^4 \hat{\delta}_i = \zeta_i^{\mathcal{T}} \circ |\zeta_i^2|^{\mathcal{T}} \left(\frac{27}{2}\beta_{6i}\hat{\delta}_i \|\mathbf{G}_i\|^4 \zeta_i\right) \tag{55}$$

Now we choose the control law as

$$\begin{aligned}\boldsymbol{\tau}_i^* = &-(\mathcal{K}_i + \frac{27}{2}\beta_{6i}\hat{\delta}_i \|\mathbf{G}_i\|^4)\zeta_i - \phi_i(\boldsymbol{\eta}_i, \mathbf{v}_i)\hat{\boldsymbol{\xi}}_i \\ &- \mathbf{G}_i \mathbf{Y}_i - \mathbf{g}_i - \boldsymbol{\mu}_i^{-1}\boldsymbol{\chi}_i,\end{aligned} \tag{56}$$

where $\mathcal{K}_i$, $i = 1, \ldots, N$ are control gain matrices. The following are also derived

$$\begin{cases}\dot{\mathbf{Y}}_i = \Gamma_{1i}\mathcal{J}(\mathbf{G}_i^{\mathcal{T}}(\zeta_i \circ |\zeta_i^2|), \mathbf{Y}_i), \\ \dot{\hat{\boldsymbol{\xi}}}_i = \Gamma_{2i}\mathcal{J}(\phi_i^{\mathcal{T}}(\zeta_i \circ |\zeta_i^2|), \hat{\boldsymbol{\xi}}_i), \\ \dot{\hat{\delta}}_i = \gamma_{3i}\mathcal{J}(\frac{27}{2}\beta_{6i} \|\mathbf{G}_i\|^4 \|\zeta_i\|_4^4, \hat{\delta}_i).\end{cases} \tag{57}$$

By taking the aforementioned equations into account, we have

$$\begin{aligned}\mathcal{L}\mathcal{V}_{3,i} \le &[-\mathcal{K}_{xi} + \frac{3}{2}(\beta_{5i}^{\frac{-4}{3}} + \beta_{7i}^{\frac{-4}{3}})]s_{xi}^4 - \frac{1}{2}\sigma_{1i}\tilde{\Theta}_{1i}^2 - \frac{1}{2}\sigma_{2i}\tilde{\Theta}_{2i}^2 \\ &+ [-\mathcal{K}_{yi} + \frac{3}{2}(\beta_{5i}^{\frac{-4}{3}} + \beta_{7i}^{\frac{-4}{3}})]s_{yi}^4 - \frac{1}{2}\sigma_{3i}\tilde{\Theta}_{3i}^2 + \sum_{j=1}^{3}\frac{1}{4\beta_{4i}^4}\mathcal{B}_{qi}^4(\bullet) \\ &+ [-\mathcal{K}_{\psi i} + \frac{3}{4}(\beta_{5i}^{\frac{-4}{3}} + \beta_{7i}^{\frac{-4}{3}})]s_{\psi i}^4 + [-\frac{1}{\mu_{1i}} + \frac{1}{4}(3\beta_{4i}^{\frac{4}{3}} + \beta_{7i}^4)]\chi_{1i}^4 \\ &+ [-\frac{1}{\mu_{2i}} + \frac{1}{4}(3\beta_{4i}^{\frac{4}{3}} + \beta_{7i}^4)]\chi_{2i}^4 + \frac{3}{8\beta_{6i}} + \mathbb{A}_i + \frac{1}{2}\sigma_{1i}\Theta_{1i}^{*2} \\ &+ [-\frac{1}{\mu_{3i}} + \frac{1}{4}(3\beta_{4i}^{\frac{4}{3}} + \beta_{7i}^4)]\chi_{3i}^4 + [-\lambda_m(\mathcal{K}_i) + \frac{1}{4}\beta_{5i}^4]\|\zeta_i\|_4^4 \\ &+ \frac{1}{2}\sigma_{2i}\Theta_{2i}^{*2} + \frac{1}{2}\sigma_{3i}\Theta_{3i}^{*2} + \frac{1}{2}a_{1i}^2 + \frac{1}{4}\epsilon_{1i}^{*4} + \frac{1}{2}a_{2i}^2 + \frac{1}{4}\epsilon_{2i}^{*4} \\ &+ \frac{1}{2}a_{3i}^2 + \frac{1}{4}\epsilon_{3i}^{*4} + \frac{3}{4}l_{1i}^2 + \frac{3}{4}l_{2i}^2 + \frac{3}{4}l_{3i}^2.\end{aligned} \tag{58}$$

Consider the compact sets

$$\Omega_{di} = \{\|\bar{\boldsymbol{\eta}}_{i-1}\|^2 + \|\dot{\bar{\boldsymbol{\eta}}}_{i-1}\|^2 + \|\ddot{\bar{\boldsymbol{\eta}}}_{i-1}\|^2 \le B_{di}\}$$

and

$$\begin{aligned}\Omega_i = \{&|s_{xi}|^2 + |s_{yi}|^2 + |s_{\psi i}|^2 + \|\zeta_i\|^2 + \|\boldsymbol{\chi}_i\|^2 \\ &+ |\tilde{\Theta}_{1i}|^2 + |\tilde{\Theta}_{2i}|^2 + |\tilde{\Theta}_{3i}|^2 + \|\mathbf{Y}_i\|^2 \\ &+ \|\tilde{\boldsymbol{\xi}}_i\|^2 + \|\tilde{\delta}_i\|^2 \le 2\iota_i\}\end{aligned}$$

with positive constants B_{di} and ι_i for $i = 1, \ldots, N$. We know that all the variables of functions $\mathcal{B}_{qi}(\bullet) q = 1,2,3$ are bounded in $\Omega_{di} \times \Omega_i$, which indicates that $|\mathcal{B}_{qi}(\bullet)| \le \bar{b}_i$. With similar lines of reasoning, one concludes that $|(\frac{\partial \alpha_{1i}}{\partial s_{xi}})^4 + (\frac{\partial \alpha_{1i}}{\partial s_{yi}})^4| \le a_{1i}$, $|(\frac{\partial \alpha_{2i}}{\partial s_{xi}})^4 + (\frac{\partial \alpha_{2i}}{\partial s_{yi}})^4| \le a_{2i}$ and $|(\frac{\partial \alpha_{3i}}{\partial s_{\psi i}})^4| \le a_{3i}$.

Theorem 1: By considering Assumptions 1-3, consider a team of N USVs seeking to follow a reference signal and keep a predefined formation among themselves under stochastic exogenous disturbances as described in uncertain dynamics (12) and (19). Choose the parameters such that Ξ_{qi}, $q = 1,2,\ldots,7$ and $i = 1,\ldots,N$ are strictly positive where,

$$
\begin{cases}
\Xi_{1i} = \mathcal{K}_{xi} - \frac{3}{2}(\beta_{5i}^{\frac{-4}{3}} + \beta_{7i}^{\frac{-4}{3}}),\ \Xi_{2i} = \mathcal{K}_{yi} - \frac{3}{2}(\beta_{5i}^{\frac{-4}{3}} + \beta_{7i}^{\frac{-4}{3}}), \\
\Xi_{3i} = \mathcal{K}_{\psi i} - \frac{3}{4}(\beta_{5i}^{\frac{-4}{3}} + \beta_{7i}^{\frac{-4}{3}}),\ \Xi_{4i} = \frac{1}{\mu_{1i}} - \frac{1}{4}(3\beta_{4i}^{\frac{4}{3}} + \beta_{7i}^{4}) - \frac{3}{2}\beta_{1i}^{2}a_{1i}, \\
\Xi_{5i} = \frac{1}{\mu_{2i}} - \frac{1}{4}(3\beta_{4i}^{\frac{4}{3}} + \beta_{7i}^{4}) - \frac{3}{2}\beta_{2i}^{2}a_{2i}, \\
\Xi_{6i} = \frac{1}{\mu_{3i}} - \frac{1}{4}(3\beta_{4i}^{\frac{4}{3}} + \beta_{7i}^{4}) - \frac{3}{4}\beta_{3i}^{2}a_{3i}, \\
\Xi_{7i} = \lambda_{m}(\mathcal{K}_{i}) - \frac{1}{4}\beta_{5i}^{4}
\end{cases}
$$

Then, design the virtual inputs (38), the adaptive rules (39), and the actual control signal (56) alongside with update laws using the projection algorithm in (57). Finally, by applying the control signal $\boldsymbol{\tau}_i = \mathbf{M}_t\boldsymbol{\tau}_i^*$ to the i-th physical system, all the tracking errors, i.e.,

$$\mathbf{X}_{\mathbf{ei}} = \mathcal{C}\mathrm{ol}(s_{xi}, s_{yi}, s_{\psi i}, \tilde{\Theta}_{1i}, \tilde{\Theta}_{2i}, \tilde{\Theta}_{3i}, \chi_{1i}, \chi_{2i}, \chi_{3i}, \zeta_i)$$

converge to a ball centered at the origin in probability.

Proof: The following holds true

$$
\begin{aligned}
\mathcal{L}\mathcal{V}_{3,i} &\le \mathcal{L}\mathcal{V}_{2,i} + [-\lambda_m(\mathcal{K}_i) + \frac{1}{4}\beta_{5i}^4]\|\zeta_i\|_4^4 \\
&+ \frac{3}{8\beta_{6i}} \pm \frac{1}{2\gamma_{3i}}\tilde{\delta}_i^2 \pm \frac{1}{2}\tilde{\mathbf{Y}}_i^{\mathcal{T}}\Gamma_{1i}^{-1}\tilde{\mathbf{Y}}_i \pm \frac{1}{2}\tilde{\xi}_i^{\mathcal{T}}\Gamma_{2i}^{-1}\tilde{\xi}_i,
\end{aligned} \tag{59}
$$

which results in

$$\mathcal{L}\mathcal{V}_{3,i} \le -\frac{c_{1i}}{c_{2i}}\mathcal{V}_{3,i} + c_{3i}. \tag{60}$$

The positive constants c_{1i}, c_{2i} and $c_{3i},\ i = 1,\ldots,N$ are specified as

$$
\begin{cases}
c_{1i} = \min\{4\Xi_{1i}, 4\Xi_{2i}, 4\Xi_{3i}, 4\Xi_{4i}, 4\Xi_{5i}, 4\Xi_{6i}, \\
\qquad 4\Xi_{7i}, \sigma_{1i}, \sigma_{2i}, \sigma_{3i}, \lambda_m(\Gamma_{1i}), \lambda_m(\Gamma_{2i}), \frac{1}{2\gamma_{3i}}\}, \\
c_{2i} = \max\{\frac{1}{4}, \lambda_M(\Gamma_{1i}), \lambda_M(\Gamma_{2i}), \frac{1}{2\gamma_{3i}}\}, \\
c_{3i} = \Xi_{8i} + \frac{1}{2}\lambda_M(\Gamma_{1i}^{-1})(2\mathrm{Y}_i^{\mathbf{M}} + \kappa)^2 \\
\qquad + \frac{1}{2}\lambda_M(\Gamma_{2i}^{-1})(2\xi_i^{\mathbf{M}} + \kappa)^2 + \frac{1}{2\gamma_{3i}}(2\Lambda_i^{\mathbf{M}} + \kappa)^2.
\end{cases} \tag{61}
$$

where κ is defined in Definition 1, and

$$
\begin{aligned}
\Xi_{8i} &= \frac{1}{2}a_{1i}^2 + \frac{1}{4}\epsilon_{1i}^{*4} + \frac{1}{2}a_{2i}^2 + \frac{1}{4}\epsilon_{2i}^{*4} + \frac{1}{2}a_{3i}^2 + \frac{1}{4}\epsilon_{3i}^{*4} \\
&+ \frac{3}{4}l_{1i}^2 + \frac{3}{4}l_{2i}^2 + \frac{3}{4}l_{3i}^2 + \frac{1}{2}\sigma_{1i}\Theta_{1i}^{*2} + \frac{1}{2}\sigma_{2i}\Theta_{2i}^{*2} \\
&+ \frac{1}{2}\sigma_{3i}\Theta_{3i}^{*2} + \frac{3}{8\beta_{6i}} + \frac{3}{4\beta_{4i}^4}\bar{b}_i^4.
\end{aligned} \tag{62}
$$

Then according to Lemma 1, the closed loop system consisting has a strong solution. By applying Lemma 2 to (60), an exponential convergence of the expectation of the tracking errors to $c_{3i}c_{2i}/c_{1i}$ is achieved. Then,

$$\mathbf{E}[\mathcal{V}_{3,i}(\mathbf{X_{ei}}(t))] \le \exp(-c(t-t_0))\mathcal{V}_{3,i}(\mathbf{X_{ei}}(t_0)) + \frac{c_{3i}c_{2i}}{c_{1i}}$$ and

therefore,

$$\mathbf{E}[\|\mathbf{X_{ei}}(t)\|] \le [\|\mathbf{X_{ei}}(t_0)\|^4 \exp(-\frac{c_{1i}}{c_{2i}}(t-t_0)) + \frac{4c_{3i}c_{2i}}{c_{1i}}]^{\frac{1}{4}}. \tag{63}$$

This completes the proof. ■

IV. SIMULATION RESULTS

The simulations are performed on a vigilant networked group of $N = 3$ USVs seeking for the platoon formation among themselves. In platoon configuration, $\varpi_i = \pi$ and $\bar{\varphi}_i = 0$. Consider the stochastic dynamics of a USV group as described in (12) and (19). A desired reference trajectory is defined as

$$
\begin{cases}
\boldsymbol{\eta}_0 = [1.2t, 0, 0]^{\mathcal{T}} & t \le t_c \\
\boldsymbol{\eta}_0 = [1.2t + 60\sin t', 60(1-\cos t'), t']^{\mathcal{T}} & t > t_c
\end{cases}
$$

where, $t_c = 100$ and $t' = 0.02(t - t_c)$. The desired distance is $\bar{e}_i = 5$. The USV agents are stationary positioned at $\boldsymbol{\eta}_1 = [0,5,0]^{\mathcal{T}}$, $\boldsymbol{\eta}_2 = [0,10,0]^{\mathcal{T}}$, $\boldsymbol{\eta}_3 = [0,15,0]^{\mathcal{T}}$.

The design parameters are also chosen as $\Gamma_{1i} = 2\mathbf{I}_6$, $\Gamma_{1i} = 2\mathbf{I}_{10}$, $\gamma_{3i} = 1.5$, $\mathcal{K}_{xi} = 10$, $\mathcal{K}_{yi} = 12$, $\mathcal{K}_{\psi i} = 8$, $\mathcal{K}_i = \mathrm{diag}(9,9,6)$. The simulation is performed for $t = 400$ sec with a sample time of $T_s = 0.001$ sec.

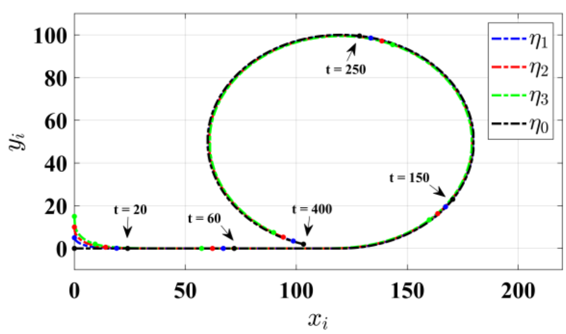

Fig. 3: The formation tracking of USV agents.

The uncertain functions are $g_{x_i} = x_i^2\ln(x_i^2 + 2)$, $g_{y_i} = y_i^2\sin(y_i)$, $g_{\psi_i} = 0.5\psi_i^2\cos(\psi_i)$. All other dynamical system parameters are taken from [29] including the system matrices. Additionally, the covariance matrices $\boldsymbol{\Lambda}_{1i}$ and $\boldsymbol{\Lambda}_{2i}$

are chosen such that $\| \mathbf{\Lambda}_i \mathbf{\Lambda}_i^T \|_\infty = 40$. The Neural Networks inputs are $\mathbf{Z}_{1i} = [x_i, x_{ri}]^{\mathrm{T}}$, $\mathbf{Z}_{2i} = [y_i, y_{ri}]^{\mathrm{T}}$, and $\mathbf{Z}_{3i} = [\psi_i, \psi_{ri}]^{\mathrm{T}}$. The centers of the NN are evenly spaced on $[-1.5,1.5]\times[-1.5,1.5]$, $[-1.5,1.5]\times[-1.5,1.5]$ and $[-0.5,0.5]\times[-0.5,0.5]$, respectively. The results are shown in Fig. 3 -Fig. 6. In Fig. 3, the tracking performance of USV agents is shown for the whole simulation time including four distinct time instants, $t = 20$, $t = 60$, $t = 150$, $t = 250$, and $t = 400$. The control inputs are also shown in Fig. 4 – Fig. 6.

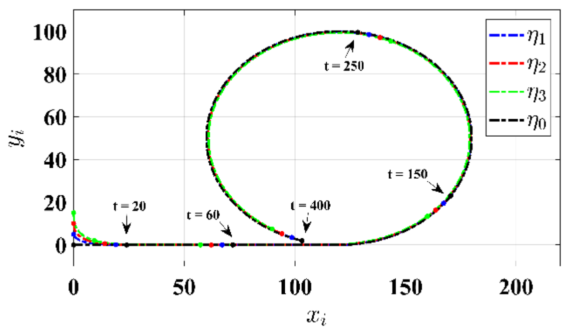

Fig. 3: The formation tracking of USV agents.

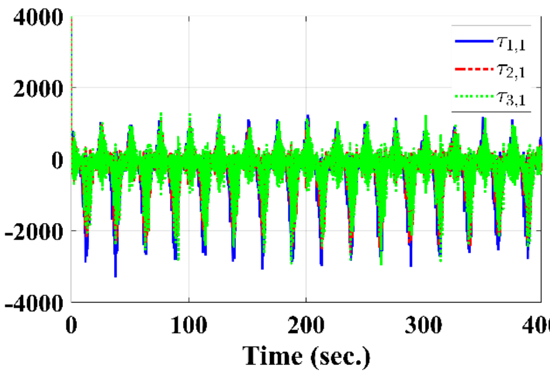

Fig. 4: The Control inputs $\tau_{i,1}$.

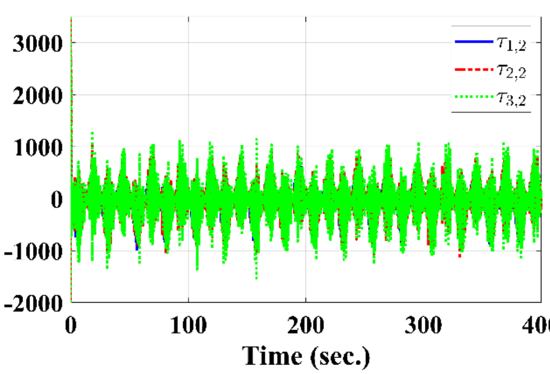

Fig. 5: The Control inputs $\tau_{i,2}$.

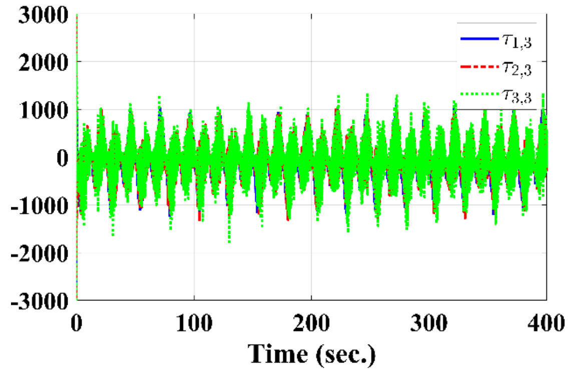

Fig. 6: The Control inputs $\tau_{i,3}$.

V. Conclusion

We studied the robust adaptive leader-follower formation of USVs under stochastic exogenous disturbances. The environmental disturbances were separated into distinguished stochastic and deterministic parts. A comprehensive model for each USV agent was then derived by stochastic differential equations. All the tracking errors are proved to converge to a ball centered at the origin in probability.

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