Blood glucose regulation in type 1 diabetic patients using backstepping sliding mode control

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Abstract—***In this paper, based on Bergman minimal model, a robust strategy for regulation of blood glucose in type 1 diabetic patients is presented. Glucose/insulin concentration in the patient body is controlled through the injection under the patient's skin by the pump. Many various controllers for this system have been proposed in the literature. However, most of them suffer from an important disadvantage that is long settling time of the control system. Thus, the contribution of this paper in comparison with previous related works is presenting a back-stepping sliding mode control that considerably reduces the required time for glucose to reach its desired level. Due to the sliding mode design, the proposed controller is robust against external disturbances. Due to the backstepping design, convergence of each state variable of the system to its desired value can be guaranteed separately. Simulation results, verify the satisfactory performance of the proposed controller.***

Keywords- Sliding mode control, backstepping design, blood glucose regulation, Bergman minimal model

I. INTRODUCTION

Diabetes is discussed as a serious condition in which the body's production and use of insulin are impaired, causing glucose concentration level to increase in the bloodstream. Insulin is a hormone by specific cells, called beta cells, in the pancreas. In order to transfer blood glucose into cells, insulin is required. Two types of diabetes have been recognized. In type I diabetes mellitus (T1DM),the b-cells in the pancreas that are responsible for producing insulin are destroyed by the immune system of the patients .Thus, the current solution for treatment is the delivery of exogenous insulin to maintain the glucose levels close to normal.

Based on continuous glucose monitoring (CGM) systems and insulin pumps technologies, a controller that automatically monitors and regulates the blood glucose level can be designed. In other words, it can play the role of an artificial pancreas system to replace the conventional treatment strategies in T1DM. In recent decades, various approaches have been presented in the literature for intelligent control of blood glucose. In this paper, the 3rd order minimal model of Bergman [1] is adopted.

Various approaches have been presented to design a feedback controller for blood glucose regulation, such as fuzzy logic control [2-5], recurrent neural networks [6], model predictive control (MPC) [7] ,high order sliding mode control [8], optimal control [9] and back-stepping sliding mode control[10]. Also, based on fractional order control, interesting approaches have been introduced in the field of blood glucose regulation [11-15].

This paper presents a robust controller for glucose-insulin system using backstepping sliding mode design. Although various controllers for this system have been presented in the literature, most of them suffer from an important disadvantage that is the long time required for glucose to reach the desired level. For example, glucose settling time in [10] and [13] is about 350 minutes which is too long. Therefore, designing a more powerful controller with shorter glucose settling time is an important contribution of this paper. In fact, including the integral of the tracking error in the sliding surface in this paper has reduced the tracking error considerably and enhanced the glucose settling time.

This paper is organized as follows. Section 2, describes the glucose-insulin model. Section 3 develops the proposed controller and presents the stability analysis. Section 4 illustrates simulation results and comparisons. Finally, section 5 concludes the paper.

II. GLUCOSE - INSULIN DYNAMIC

Many models for describing glucose-insulin process has been presented. Bergman's minimal model has been proposed in 1980 by Doctor Richard Bergman. The main advantage of the Bergman minimal model is its simplicity. According to [16], it is the common model that is usually referenced in the literature. Bergman Minimal Model (BeM)is described as [16]:

$$\dot{x}_1 = -p_1\left[x_1 - G_b\right] - x_1 x_2 + \delta_1 + D(t) \quad (1)$$

$$\dot{x}_2 = -p_2 x_2 + p_3\left[x_3 - I_b\right] + \Delta_2 \quad (1)$$

$$\dot{x}_3 = -n\left[x_3 - I_b\right] + \gamma t\left[x_1 - G_b\right]^+ + \Delta_3 + u(t) \quad (1)$$

in which $x_1(t)$, $x_2(t)$ and $x_3(t)$ are plasma glucose concentration, the insulin influence on glucose concentration reduction, and insulin concentration in plasma respectively, $u(t) \in R$ is injected insulin rate in (mU/min), G_b is the basal pre-injection level of glucose (mg/dl), I_b is the basal pre-injection level of insulin (µU/ml), p_1 the insulin independent rate constant of glucose uptake in muscles and liver (1/min), p_2 the rate for decrease in tissue glucose uptake ability (1/min), p_3 the insulin-dependent increase in glucose uptake ability in tissue per-unit of insulin concentration above the basal level ((µU/ml)/min). The term $\gamma t\left[x(t)-G_b\right]^+$ represents the pancreatic insulin secretion after a meal in take at t = 0. It has been assumed that the parameters in (1) are nominal parameters that may be different from their actual values. Thus, the terms Δ_1, Δ_2 and Δ_3 are the lumped uncertainties originated from these mismatches. It is assumed that these uncertainties are bounded as $|\Delta_1| \le k_1$, $|\Delta_2| \le d_2$ and $|\Delta_3| \le d_3$ where k_1, d_2 and d_3 are known positive constants. As this work is focused on insulin therapy which is usually administrated to type I diabetes mellitus patients, γ is assumed to be zero to represent the true dynamic of this disease and t should also be considered zero. The parameter n is the first order decay rate for insulin in blood. This disturbance can be modeled by a decaying exponential function of the following form [16]:

$$D(t) = A\exp(-Bt) \qquad B > 0 \tag{2}$$

The pump can be modeled as a first order linear system:

$$\dot{u}(t) = \frac{1}{a}(w(t) - u(t)) + \Delta_4 \tag{3}$$

where $w(t)$ is insulin rate command in pump as input, and the parameter a is pump time constant. Also, Δ_4 is the lumped uncertainty originated from the mismatch between the actual and nominal a.

III. The Proposed Controller and Stability Analysis

Define the tracking error of glucose as

$$e_1 = x_1(t) - x_{1d}(t) \tag{4}$$

where $x_{1d}(t)$ is the desired blood glucose Also, consider the following sliding surface

$$s_1 = e_1 + \int_0^t \lambda_1 e_1 dt \tag{5}$$

in which λ_1 is a design parameter. Taking the derivative of (5) results in

$$\dot{s}_1 = \dot{e}_1 + \lambda_1 e_1 = \dot{x}_1 - \dot{x}_{1d} + \lambda_1 e_1 \tag{6}$$

Substitution of $\dot{x}_1$ from (1) into (6) and solving $\dot{s}_1 = 0$ results in

$$x_{2eq} = (x_1)^{-1}(-p_1x_1 + p_1G_b - \dot{G}_d + \lambda_1 x_1 - \lambda_1 G_d + d_1\, sign(s_1)) \tag{7}$$

in which $d_1 sign(s_1)$ has been added to the control law to compensate for the external disturbance D(t) and the lumped uncertainty $\Delta_1 = \delta_1 + D(t)$. In other words, we have, $d_1 > |\Delta_1|$.Now, applying the control law (7) into the (1) results in

$$\dot{x}_1 = \dot{x}_{1d} - \lambda_1 e_1 + D + \Delta_1 - d_1\, sign(s_1) \tag{8}$$

which can be rewritten as

$$\dot{s}_1 = D + \Delta_1 - d_1\, sign(s_1) \tag{9}$$

In order to verify that the control law (7) guarantees the stability ,consider the following positive definite function:

$$V_1 = \frac{1}{2}s_1^2 \tag{10}$$

Taking the derivative of (10) leads to

$$\dot{V}_1 = \dot{s}_1 s_1 \tag{11}$$

Substitution of (9) into (11) results in

$$\dot{V}_1 = s_1(D + \Delta_1 - d_1\, sign(s_1)) \tag{12}$$

It is obvious that

$$\dot{V}_1 \le |s_1||D + \Delta_1| - s_1 d_1\, sign(s_1) \tag{13}$$

In other words, we have

$$\dot{V}_1 \le |s_1||\Delta_1| - d_1|s_1| = |s_1|(|\Delta_1| - d_1) \tag{14}$$

Since $d_1 \ge |\Delta_1|$, it can be concluded that

$$\dot{V}_1 \le 0 \tag{15}$$

Now, consider (1-b). Define the tracking error as

$$e_2 = x_2 - x_{2d} \tag{16}$$

where x_{2d} is the desired value of x_2. Also, consider the following sliding surface

$$s_2 = e_2 + \int_0^t \lambda_2 e_2 dt \tag{17}$$

in which λ_2 is a design parameter. Taking the derivative of (17) results in

$$\dot{s}_2 = \dot{e}_2 + \lambda_2 e_2 = \dot{x}_2 - \dot{x}_{2d} + \lambda_2 e_2 \tag{18}$$

Substitution of $\dot{x}_2$ from (1) into (18) and solving $\dot{s}_2 = 0$ results in

$$x_{3eq} = \frac{p_2x_2 + p_3I_b + \dot{x}_{2d} - \lambda_2 e_2 + d_2\, sign(s_2)}{p_3} \tag{19}$$

in which $d_2 sign(s_2)$ has been considered for compensation of the lumped uncertainty Δ_2. Now, applying the control law (19) into the (1) results in

$$\dot{x}_2 = \dot{x}_{2d} - \lambda_2 e_2 + \Delta_2 - d_2 sign(s_2) \quad (20)$$

In other words

$$\dot{s}_2 = \Delta_2 - d_2 sign(s_2) \quad (21)$$

consider the following positive definite function

$$V_2 = \frac{1}{2} s_2^2 \quad (22)$$

Taking the time derivative of (22) results in

$$\dot{V}_2 = \dot{s}_2 s_2 \quad (23)$$

Substitution of (21) into (23) leads to

$$\dot{V}_2 = s_2(\Delta_2 - d_2 sign(s_2)) \quad (24)$$

It follows from (24) that

$$\dot{V}_2 \leq |s_2||\Delta_2| - s_2 d_2 sign(s_2) \quad (25)$$

which can be rewritten as

$$\dot{V}_2 \leq |s_2||\Delta_2| - d_2|s_2| = |s_2|(|\Delta_2| - d_2) \quad (26)$$

Since $d_2 > |\Delta_2|$, it can be concluded that

$$\dot{V}_2 \leq 0 \quad (27)$$

Now, consider (1). Define the tracking error as

$$e_3 = x_3 - x_{3d} \quad (28)$$

where x_{3d} is the desired value of x_3. Also, consider the following sliding surface

$$s_3 = e_3 + \int_0^t \lambda_3 e_3 dt \quad (29)$$

in which λ_3 is a design parameter. Taking the derivative of (29) results in

$$\dot{s}_3 = \dot{e}_3 + \lambda_3 e_3 = \dot{x}_3 - \dot{x}_{3d} + \lambda_3 e_3 \quad (30)$$

Substitution of $\dot{x}_3$ from (1) into (30) and solving $\dot{s}_3 = 0$ results in

$$u_d = n[x_3 - I_b] + \dot{x}_{3d} - \lambda_3 e_3 + d_3 sign(s_3) \quad (31)$$

Similar to the procedure given in (4) to (15), it can be shown that

$$\dot{V}_3 \leq |s_3|(|\Delta_3| - d_3) \leq 0 \quad (32)$$

In which

$$V_3 = \frac{1}{2} s_3^2 \quad (33)$$

$$w_d = u + a\dot{u}_d - a\lambda_4 e_4 - a d_4 sign(s_4) \quad (34)$$

In fact, it can be simply shown that this control law will result in

$$\dot{V}_4 \leq |s_4|(|\Delta_4| - d_4) \leq 0 \quad (35)$$

in which

$$V_4 = \frac{1}{2} s_4^2 \quad (36)$$

$$s_4 = e_4 + \int \lambda_4 e_4 dt \quad (37)$$

$$e_4 = u - u_d \quad (38)$$

Now the following theorem is presented.

Theorem 1: *Consider the dynamic system (1). If the control laws (7), (19), (31) and (34) are applied to this system ,then the closed-loop signals are bounded and the tracking errors e_1, e_2, e_3 and e_4 asymptotically converge to zero.*

Proof :*Define the Lyapunov function candidate as*

$$V = \sum_{i=1}^{4} V_i = \sum_{i=1}^{4} \frac{1}{2} s_i^2 \quad (39)$$

in which V_1, V_2, V_3 and V_4 are defined in (10), (22), (33) and (36), respectively. Based on (14), (26), (32) and (35) it can be concluded that

$$\dot{V} \leq -\sum_{i=1}^{4} |s_i| \underbrace{(|\Delta_i| - d_i)}_{\alpha_i} \quad (40)$$

According to (9) and (21), $\dot{s}_1$ and $\dot{s}_2$ are bounded. Similarly, it can be concluded that $\dot{s}_3$ and $\dot{s}_4$ are bounded.

Table 1 The model parameters

Bergman minimal model	
$P_1(min)^{-1}$	0
$P_2(min)^{-1}$	0.0123
$P_3(min)^{-1}$	8.2×10^{-8}
$n(min^{-1})$	0.2659
I_b	7
G_b	70
$X_1(0)$	200
$X_3(0)$	50

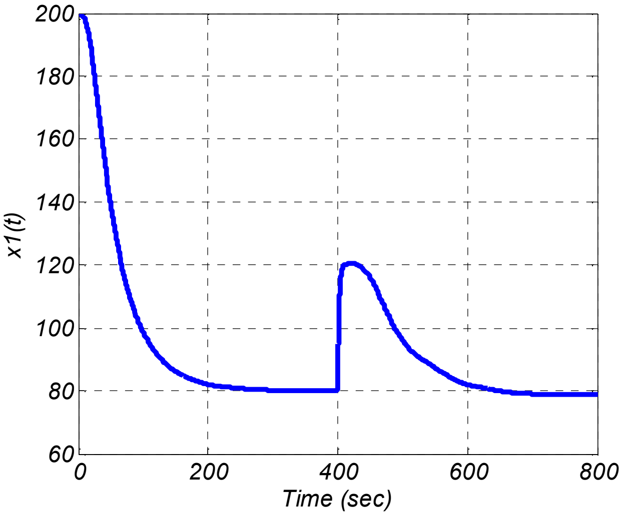

Fig.1 Glucose concentration

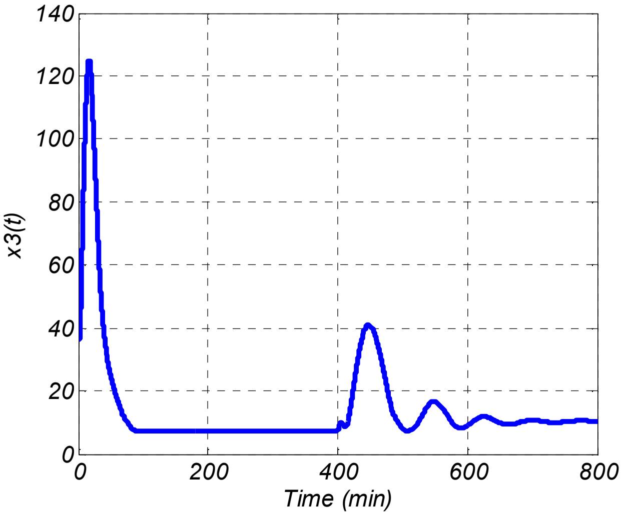

Fig. 2Plasma insulin concentration

VI. SIMULATION RESULTS

Consider the model described in [11]. Its parameters are given in Table 1.The parameter of the controller have been set to $\lambda_1 = 0.015,\ \lambda_2 = 0.05,\ \lambda_3 = 0.14,\ \lambda_4 = 0.1$. *Also, To investigate the controller robustness against parametric uncertainty, 10 percent uncertainty has been applied to* P_2 *and* P_3. *Moreover, the external disturbance* $D(t) = 20\exp(-0.5\,t)$ *affects the control system at* $t = 400$ *(min) .The blood glucose level is presented in Fig. 1. As shown in this figure, the controller can reduce the blood glucose concentration from the initial value of 200 (mg/dl) to the approximate value of 80 (mg/dl) which is our desired level. In comparison with controller designed in [10], the proposed controller is superior. The reason is that a steady state is clearly seen in the response of the proposed controller, while there is not any steady state for glucose signal presented in [10]. The Plasma insulin concentration in (mU/L) is illustrated in Fig.2. As shown in this figure, this signal shows acceptable values.*

CONCLUSION

In this paper, a robust controller based on back stepping sliding mode design for blood glucose regulation in type I diabetes patients has been presented. Uncertainties including external disturbance and parametric uncertainty have been considered in the design procedure. Including the integral of the tracking error in the sliding surface in this paper has reduced the tracking error considerably. Therefore, in comparison with some previous related works, the proposed controller is superior. Simulation results verify the satisfactory performance of the proposed controller.

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