Stochastic Environmental Research and Risk Assessment
https://doi.org/10.1007/s00477-024-02716-0

ORIGINAL PAPER

Hysteresis-influenced stage-discharge rating curve based on isovel contours and Jones formula

Sajjad M. Vatanchi[1] · **Mahmoud F. Maghrebi**[1]

Accepted: 25 March 2024

Abstract

Considering the hysteresis effect, the stage-discharge rating curve (SDRC) is challenging to measure at gauge stations. Hysteresis also causes uncertainty in SDRC estimations. Therefore, an accurate method to estimate discharge is essential in such conditions. In steady flow conditions, isovel contours can be used to estimate SDRC. However, this method has not been investigated in unsteady conditions. This study uses the Jones formula and isovel contours-based SDRC to consider the hysteresis in Chattahoochee River, USA. First, this method identifies hydro-geometric parameters, such as velocity parameters extracted from isovel contours, to calculate the steady SDRC (SSDRC). Then, it is combined with the Jones formula to estimate loop SDRC (LSDRC). The SSDRC is calibrated using the Markov chain Monte Carlo (MCMC) method. MCMC calibrates model parameters by iteratively selecting parameters that best fit the observed data from the parameter space. Also, uncertainty in model output can be characterized using the posterior distribution of model parameters. The results show that LSDRC is more accurate than SSDRC based on R^2, mean absolute error (MAE), root mean square error (RMSE), and mean absolute percentage error (MAPE) metrics. Also, the proposed method outperforms the power function rating curve. The proposed model achieved a maximum MAPE of 6.6%, while the power function rating curve reached 24.6%.

Keywords Hysteresis · Isovel · Jones formula · Loop rating curve · Markov chain Monte Carlo (MCMC) · Stage–discharge rating curve

1 Introduction

An accurate stage-discharge rating curve (SDRC) is crucial for making informed decisions and ensuring the sustainability and safety of water resources (Ali and Maghrebi 2023). Continuous discharge measurements are the main objective of a hydrometric gauging station, which is done by using an SDRC. An SDRC is usually estimated by measuring the stage and converting it to discharge. Deriving a trustworthy single-valued SDRC by applying conventional regression methods to the available observed data is straightforward when the hydraulic channel control or flow regime is constant and unsteadiness does not affect the relationship (Petersen-Øverleir 2006). However, the assumption of single-valued SDRC may not hold if unsteadiness impacts (hysteresis) the relationship between stage and discharge (Petersen-Øverleir 2006). Variations in steady SDRC occur when the slope of the stream is relatively flat and discharge changes rapidly (Braca 2008). Adjustment factors that relate steady to unsteady flow must be used to develop SDRC at sites where this occurs (Holmes 2016). Hysteresis caused by an unsteady flow can make obtaining reliable stage-discharge curves at gauging stations challenging (Perret et al. 2022).

Hysteresis affects the stability of the SDRC, particularly for conventional methods such as single rating curves (Petersen-Øverleir 2006). SDRCs can exhibit hysteresis behavior under unsteady flow conditions when the stream's energy slope changes (Petersen-Øverleir 2006). Hysteresis creates a loop in the rating curve, which may vary depending on the channel's geometrical characteristics and the flow event type (intensity and duration) (Perret et al. 2022). Most of the time, the loops are small, and there is very little difference between rising- and falling-limb hydrographs (Kuhnle and Bowie 1992). Certain hydraulic conditions, however, can result in large loops that prevent steady SDRC from being used (Holmes 2016). When unsteady flows are present

✉ Mahmoud F. Maghrebi
maghrebi@um.ac.ir

1 Civil Engineering Department, Ferdowsi University of Mashhad, Mashhad 91775-1111, Iran

Published online: 20 May 2024

and significant, most standard hydrometric literature, e.g., Herschy (2009), recommends that the Jones formula (Jones 1915) be applied to correcting the steady SDRC (SSDRC). Hysteresis loop analysis, accompanied by correction methods, is essential to accurately characterize and mitigate the hysteresis effect in measuring river discharge, ensuring its reliability and precision (Paterson et al. 2018).

Two general methods for measuring river discharge are direct measurement and indirect approaches, such as SDRC. A direct discharge measurement during the entire unsteady event is the most reliable method for capturing hysteresis in natural streams (Muste et al. 2020). Measuring water flow during unsteady events is challenging and time-consuming, particularly in medium and large rivers (Muste et al. 2022). Despite attempts to conduct direct discharge measurements without assistance, continuous streamflow monitoring relies on indirect methods (Dottori et al. 2009). Various indirect methods have been developed, including corrections to the Jones formula, such as Henderson formula (Henderson 1966), Di Silvio formula (Di Silvio 1969), Fread formula (Fread 1975), Faye and Cherry formula (Faye and Cherry 1980), Fenton formula (Fenton 1999), Perumal formula (Perumal and Raju 1999) and Fenton and Keller formula (Fenton and Keller 2001). Other studies, such as Petersen-Øverleir (2006), Perumal et al. (2004), Perumal et al. (2007), Reitan and Petersen-Øverleir (2011), Dottori et al. (2009), Zakwan (2018), Muste et al. (2020), Perret et al. (2022) and Majdalani et al. (2023) have developed the mentioned methods. Using these methods, one can model and analyze the hysteresis effect of loop SDRC (LSDRC) and improve the accuracy of flow discharge estimation. This study applies the Jones formula to correct the SSDRC for estimating streamflow.

The importance of uncertainty analysis in SDRC in hydrology and water resources management is undeniable. There are different methods for estimating SSDRC. The power function is the most common equation (Herschy 2009). As another method, Maghrebi and Ahmadi (2017) have proposed a method to estimate the SSDRC, which is called "isovel contours-based SDRC." This method utilizes hydraulic parameters and velocity obtained from isovel contours and one observed data. With this method, there is no need to measure velocity at different stages. In estimating SDRC, uncertainty analyses are crucial for ensuring accuracy and reliability. There are several sources of uncertainty in SDRC that can impact the accuracy of streamflow estimation. Some methods have been proposed to analyze the SDRC's uncertainty (Kiang et al. 2018). One of these methods is a Monte Carlo simulation that generates random values based on the distribution of input variables. Multiple rating curves are constructed based on these simulated values, allowing uncertainty to be assessed. Bayesian inference can also be used to update the uncertainty distribution of the rating curve parameters using prior knowledge and observational data.

Markov chain Monte Carlo (MCMC) is one Bayesian inference method that performs model calibration and uncertainty analysis (Reitan and Petersen-Øverleir 2009). This method is beneficial when dealing with complex models or when the likelihood function is difficult to compute analytically (Marjoram et al. 2003). It allows for exploring high-dimensional parameter spaces and quantifies uncertainty by generating samples from the posterior distribution (Vlachou et al. 2023). In the context of hydrological simulation, the MCMC method can assess parameter uncertainty in models (Shi et al. 2023). Studies such as Reitan and Petersen-Øverleir (2009), Le Coz et al. (2014), and McMillan and Westerberg (2015) used the MCMC method to analyze the uncertainty of the power function rating curve. It has been shown in these studies that the MCMC can be applied to a wide range of catchments without requiring detailed at-site knowledge of hydraulic properties or the causes of epistemic error, thus being practical and sufficiently flexible for its use. The MCMC method is used for the first time for calibration and uncertainty analysis of the isovel contours-based SDRC.

An isovel contour-based SDRC is used in this study to estimate hysteresis or loop stage-discharge rating curves. First, the model requires observed data and effective hydrogeometric variables, such as wetted area, wetted perimeter, water surface width, and velocity variables based on isovel contours introduced by Maghrebi (2006). Correlation analysis is used to identify effective parameters. The subsequent step involves utilizing the MCMC method to calibrate the proposed model's powers under steady conditions. In addition to calibrating the model, the uncertainty of the parameters is also determined using the MCMC method. After that, the proposed model is corrected using Jones's formula, and the Monte Carlo simulation is used to find the bed slope parameter. As a result, the proposed model and its uncertainty due to the variation of bed slope can be determined in an unsteady state. Finally, a comparison is made between the performance of the proposed model and that of the conventional power function rating curve. The goals of this study are as follows:

- Calibrating the proposed steady stage-discharge rating curve (SSDRC) by the MCMC method and assessing its performance.
- Calibration and performance evaluation of the loop stage-discharge rating curve LSDRC.
- Uncertainty analysis of the SSDRC due to the parameter uncertainty.
- Uncertainty analysis of the LSDRC due to the variation of bed slope.
- Comparing the proposed models with the power function rating curve.

2 Material and methods

2.1 Jones formula

Of all the correction methods available in the literature, the Jones formula is considered the most famous. The Jones formula (Eq. 1) is:

$$Q_u = Q_s\sqrt{1 + \frac{1}{Sc}\frac{\partial H}{\partial t}} \tag{1}$$

$$c = \frac{\partial Q_s}{\partial A} \tag{2}$$

where the Q_u (m^3/s) is the unsteady discharge, Q_s (m^3/s) is the steady discharge, H (m) is the stage, t (s) is the time, S is the local channel slope, c (m/s) is the wave celerity, and A (m^2) is the wetted area. The Jones formula estimates Q_u by adjusting Q_s.

2.2 Stage-discharge rating curves (SDRC) formulation

2.2.1 Isovel contours-based SDRC

The SSDRC, introduced by Maghrebi and Ahmadi (2017), is used to estimate Q_s. The general form of the relationship is as follows:

$$\frac{(Q_s)_e}{(Q_s)_r} = \left(\frac{A_e}{A_r}\right)^{a_1}\left(\frac{P_e}{P_r}\right)^{a_2}\left(\frac{T_e}{T_r}\right)^{a_3}\left(\frac{U_e}{U_r}\right)^{a_4} \tag{3}$$

where the subscripts r and e are related to the referenced and estimated values, respectively. P is the wetted perimeter of the flow section, T is the width of the water surface, a_1 to a_4 are the calibration parameters, and U is the mean velocity based on isovel contours, introduced by Maghrebi (2006) (see Supplementary Information, S1).

All influential hydro-geometric variables must be calculated at all water stages to estimate $(Q_s)_e$ by Eq. 3. In other words, for an arbitrary open channel, the following variables: $A = A(H)$, $P = P(H)$, $T = T(H)$, and $U = U(H)$ should be calculated. Furthermore, only one measured discharge $(Q_s)_r$ and corresponding hydro-geometric data are required to estimate the entire rating curve.

Therefore, the proposed relationship for estimating the LSDRC by combining Eq. 1 and Eq. 3 is as follows:

$$Q_u = (Q_s)_r\left(\frac{A_e}{A_r}\right)^{a_1}\left(\frac{P_e}{P_r}\right)^{a_2}\left(\frac{T_e}{T_r}\right)^{a_3}\left(\frac{U_e}{U_r}\right)^{a_4}\sqrt{1 + \frac{1}{Sc}\frac{\partial H}{\partial t}} \tag{4}$$

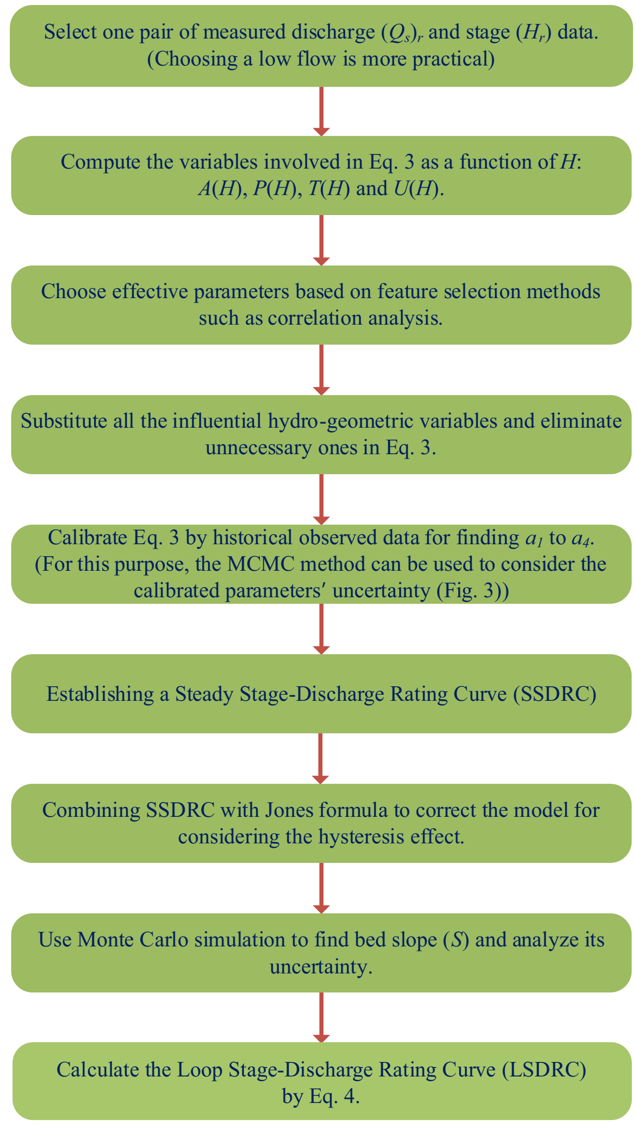

Fig. 1 Flowchart of calculating LSDRC

The steps in Fig. 1 should be followed to obtain the LSDRC.

2.2.2 Power function SDRC

Instead of using Eq. 4, the power function rating curve can also be used as follows:

$$Q_u = a\left(H - h_0\right)^b\sqrt{1 + \frac{1}{Sc}\frac{\partial H}{\partial t}} \tag{5}$$

where a and b are the rating curve constants under steady-state conditions, $Q_s = a\,(H—h_0)^b$, and h_0 is the unknown reference gauge height at zero flow. This approach is used to compare with the proposed method.

2.3 Markov Chain Monte Carlo (MCMC) method

In classical statistics, the parameters of a distribution, such as the mean or variance, are considered unknown but constant and can be estimated from sample statistics. A maximum likelihood estimation or a method of moments is typically used to estimate these parameters. Confidence intervals can be constructed once the parameters have been estimated to quantify the uncertainty associated with parameter estimates. By contrast, in the Bayesian approach, parameters are treated as random variables with their probability distributions.

Calculating the posterior distribution when implementing Bayesian theory is difficult due to the mathematical and computational challenges involved (see Supplementary Information, S2). However, it is possible to obtain the precise nature of any posterior distribution using the MCMC method by producing many samples from the posterior distribution function (Van de Schoot et al. 2021). In MCMC, two concepts are combined: a Markov chain is used to obtain a sample from the expected distribution, and the Monte Carlo calculates the posterior statistics from sampled parameters.

Many MCMC algorithms exist, but Metropolis–Hastings is one of the most popular (Wang et al. 2017). The Metropolis–Hastings algorithm needs an auxiliary PDF ($q(\theta^* | \theta^{(t+1)})$), generally called proposal distribution, to estimate the target distribution function P(θ|D). In summary, the Metropolis–Hastings algorithm proceeds as follows:

1) Set an arbitrary starting point $\theta^{(t-1)}$ for calculation at $t=1$;
2) Generate a new value θ^* from the proposal function $q(\theta^* | \theta^{(t-1)})$;
3) Calculate the probability of acceptance as follows:

$$\alpha = \min\left\{ 1, \frac{P(\theta^*|D)}{P\left(\theta^{t-1}|D\right)} \frac{q\left(\theta^{t-1}|\theta^*\right)}{q\left(\theta^*|\theta^{t-1}\right)} \right\} \tag{6}$$

4) Generate a random number u from a uniform distribution $u \sim U[0,1]$;
5) If $u < \alpha$, then $\theta^{(t)} = \theta^*$; otherwise $\theta^{(t)} = \theta^{(t-1)}$ and $t = t + 1$ and return to step (2).

Designing a well-designed proposal function can speed up convergence, improve search space exploration, and improve overall algorithm performance. According to Reis and Stedinger (2005), although theoretically, the $q(\theta^* | \theta^{(t+1)})$ can be arbitrarily chosen, the choice of it significantly influences computation efficiency. The normal distribution, the most commonly used proposal function, is utilized for this study (Reis and Stedinger 2005).

Moreover, efficiency is a crucial factor to consider when using Metropolis–Hastings algorithms. The acceptance rate measures how quickly the algorithm converges and how effectively it explores the search space. Low acceptance rates lead to significantly different random number generation from the target distribution, making the algorithm invalid for effectively exploring the search space and converging to the desired distribution. On the other hand, a high acceptance rate close to 1 means the algorithm takes small steps in the search space, leading to slower convergence, making it invalid. There are different opinions on the ideal acceptance rates for parameter estimation. Some suggest 0.45 for single and 0.23 for multiple parameters (Gelman et al. 1997), while others find 0.2 to 0.5 acceptable (Rosenthal 2014).

In this study, MCMC is applied to estimate and analyze the uncertainty of the parameters of the proposed model as follows:

1) Establish the proposed model by Eq. 3.
2) Determine a set of walkers defined by a θ vector containing the parameters used by the proposed model.
3) Determine the initial values of θ randomly from the Gaussian distribution ($\theta \sim N(\mu, \sum)$). In this study, for all parameters $\mu = 1$ and $\sum$ is determined by trial and error.
4) Every random walk will now explore the parameter space. In this process, each walk takes a "step" towards a new value of θ and generates a model with that value.
5) Compare the model results (Q_e) to the observed data (Q_o) via a simple χ^2 as follows:

$$\chi^2 = \sum_i \frac{\left(Q_{obs,i} - Q_{e,i}\right)}{2\sigma_i^2} \tag{7}$$

where σ is the estimate of the error of the data. Experimental uncertainty or just making some guesses can be used to estimate σ.

6) Constructing a likelihood function. For this purpose, a Gaussian distribution with zero medium and constant variance is commonly used:

$$\pi(D|\theta) = \exp[-\chi^2] \tag{8}$$

7) The MCMC algorithm compares new and current models with data. If the new model is better, the random walk moves there. If it is worse, the random walk tries a new direction or stays put. The acceptance ratio prevents random walks from getting stuck on peaks of high probability.
8) All random walks climb toward the regions of the highest likelihood function among the models. After all of the steps have been completed, we have what is known

as a posterior distribution. Every random walk records every accepted θ vector and its likelihood. The mean of the posterior distribution is equivalent to the optimal estimate. Calculating the 95% confidence interval (95CI) based on the posterior distribution is also possible.

The flowchart of the calibration process of the SSDRC by the Metropolis–Hastings algorithm is presented in Fig. 2. After finding the powers of the proposed model, the SSDRC can be formed. The Jones correction method can be applied to the proposed model for an unsteady state. The slope parameter (S) will be obtained using the Monte Carlo simulation. After calibrating the S, an LSDRC can be generated.

2.4 Model performance evaluation

Four error estimation criteria, namely R^2, mean absolute error (MAE), root mean square error ($RMSE$), and mean absolute percentage error ($MAPE$) for quantitative assessment have been used:

$$R^2 = \frac{\left[\sum_{i=1}^{N}\left(Q_{o,i}-\overline{Q}_o\right)\left(Q_{e,i}-\overline{Q}_e\right)\right]^2}{\sum_{i=1}^{N}\left(Q_{o,i}-\overline{Q}_o\right)^2\sum_{i=1}^{N}\left(Q_{e,i}-\overline{Q}_e\right)^2} \tag{9}$$

$$MAE(m^3/s) = \frac{\sum_{i=1}^{N}\left|Q_{e,i}-Q_{o,i}\right|}{N} \tag{10}$$

$$RMSE\left(m^3/s\right) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(Q_{e,i}-Q_{o,i}\right)^2} \tag{11}$$

$$MAPE(\%) = \frac{100}{N}\sum_{i=1}^{N}\left|\frac{Q_{e,i}-Q_{o,i}}{Q_{o,i}}\right| \tag{12}$$

where Q_o and Q_e are the measured discharge and the estimated discharge, Q_o is the mean of observed discharges, N is the number of observation data, and Q_{max} and Q_{min} are the maximum and minimum measured discharges, respectively.

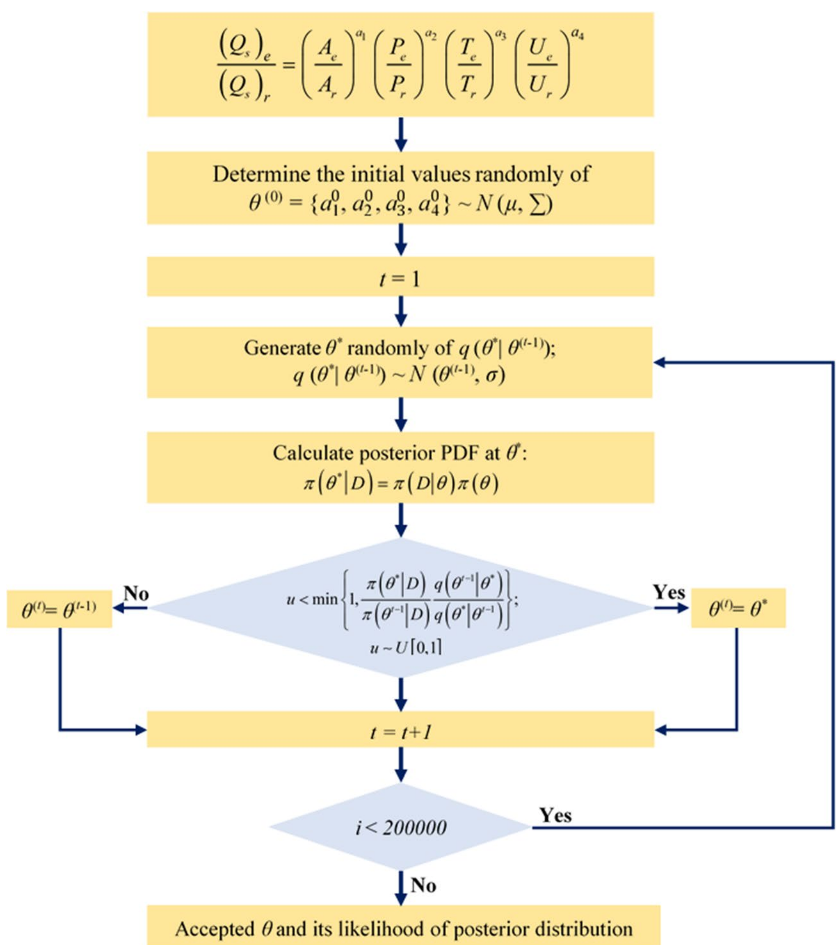

Fig. 2 Flowchart of Metropolis–Hastings algorithm for the parameter estimation using the MCMC method

Lower *MAE*, *RMSE*, and *MAPE* values indicate better accuracy in estimating discharge. Also, a higher R^2 value (closer to 1) suggests a better fit of the model to the observed data (Maghrebi et al. 2023).

3 Case study and datasets

The data sets used in this study are from the Chattahoochee River gauging station at Atlanta (USGS station No: 02336000; 84°27′16″ W, 33°51′33″ N) located in Fulton County, Georgia (Fig. 3). The Chattahoochee River is a major river in the southeastern United States, flowing through the states of Georgia, Alabama, and Florida. It is a 690-km-long water source for Atlanta and surrounding communities. The cross-section of the desired station is also presented in Fig. 3.

The USGS Water Data (U.S. Geological Survey 2023) provides current discharge conditions and gauge height. This study uses the 15-min streamflow and water level data from 2009 to 2012 (Fig. 4). The data from Storm 1 is used

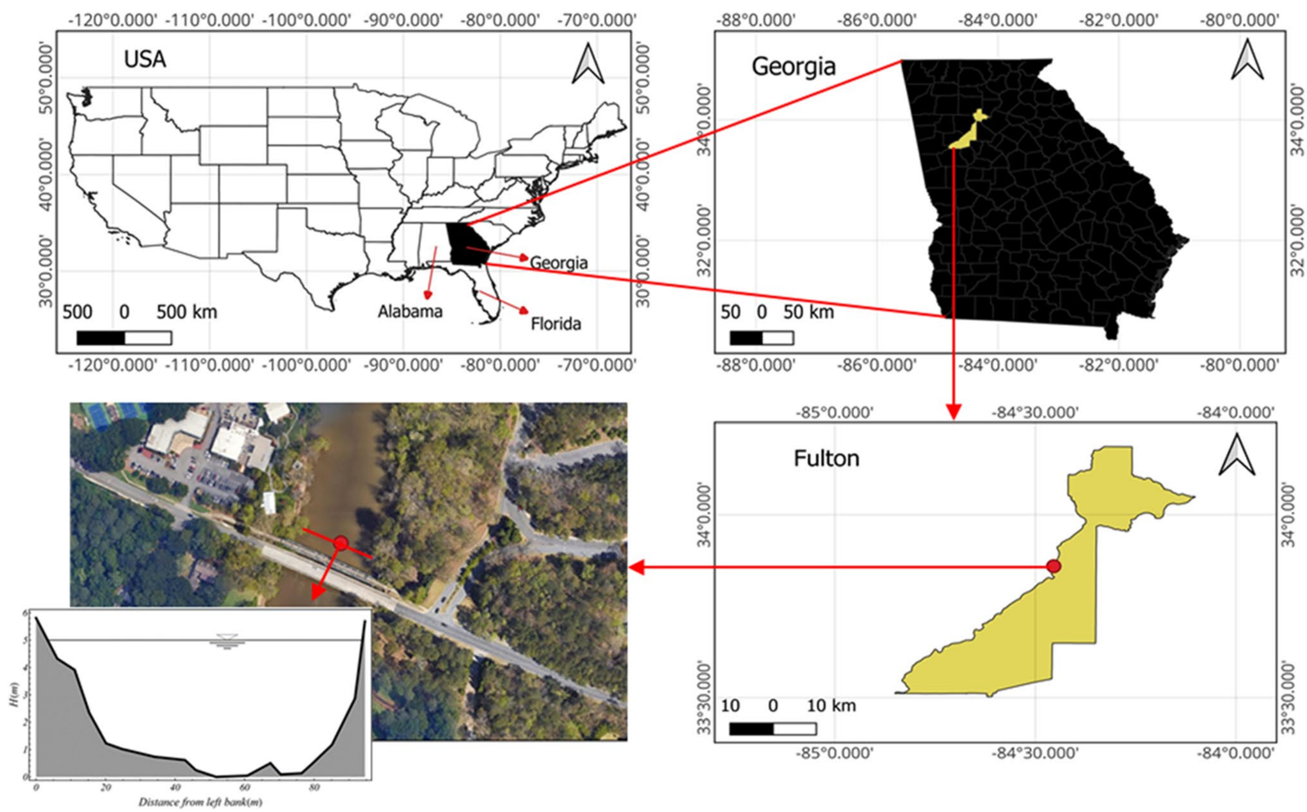

Fig. 3 Map of the study area

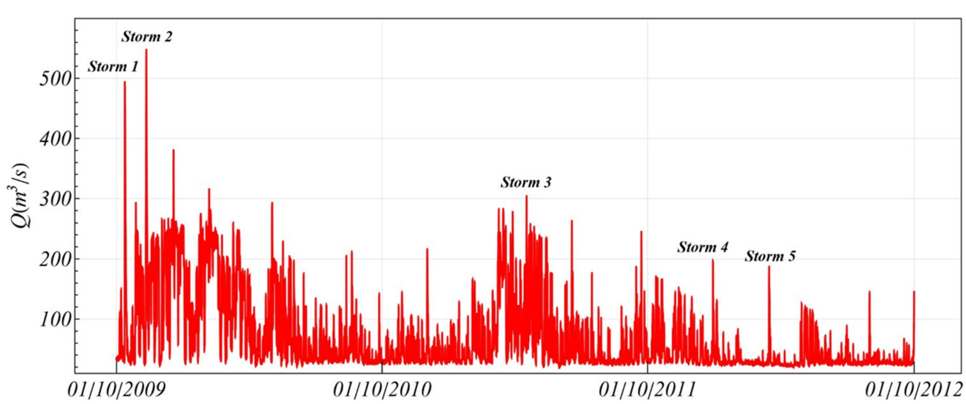

Fig. 4 Streamflow data of Chattahoochee River at Atlanta

for calibrating the proposed model. Storms 2 to 5 are also used for validation.

During this period, the maximum discharge is about 550 m^3/s, and the minimum is about 20 m^3/s.

4 Results

This section presents the calibration results of the proposed model using Storm 1 data, followed by schematic representations of model results for Storms 2 to 5. The performance evaluation of the proposed model and its comparison with the power function rating curve are presented in the discussion section.

4.1 Stage-discharge rating curve calibration

First, it is necessary to calculate the hydro-geometric parameters at different stages. As shown in Fig. 5a, these variables have changed with respect to H. As shown in Fig. 5b, features are selected using a correlation heatmap. Correlation values closer to one indicate stronger correlations. U and A variables correlate most significantly with discharge (Q). One variable should be removed among T and P variables to avoid collinearity. Since the P variable shows a higher correlation with the Q, this variable is selected, and T is omitted ($a_3=0$).

With the observational data from Storm 1, Eq. 3 can be calibrated by MCMC. For this purpose, 200,000 simulations are done. The optimal values of the power parameters and their 95% confidence interval (95CI) for the steady state can be estimated from the posterior distribution (Fig. S3). Therefore, $a_1=5.23$, $a_2=-1.86$, and $a_4=-6.15$. Now, Eq. 3 can be used to estimate the SSDRC and the streamflow time series; however, to use Eq. 4, it is necessary to calibrate the bed slope (S).

4.2 Monte Carlo simulation for calibration bed slope (S)

For this purpose, observational data from "Storm 1" is used to calibrate the bed slope. A uniform distribution between 0.0001 and 0.0005 is assumed as the prior bed slope distribution. Ten thousand simulations are run, and each run's model error is estimated. The bed slope value corresponding to the lowest $RMSE$ (Eq. 11) is determined as the optimal S. Figure S4 shows the $RMSE$ variations with the slope. The optimal value of the bed slope is 0.00025. An uncertainty analysis caused by the bed slope in the proposed model is examined in the following section.

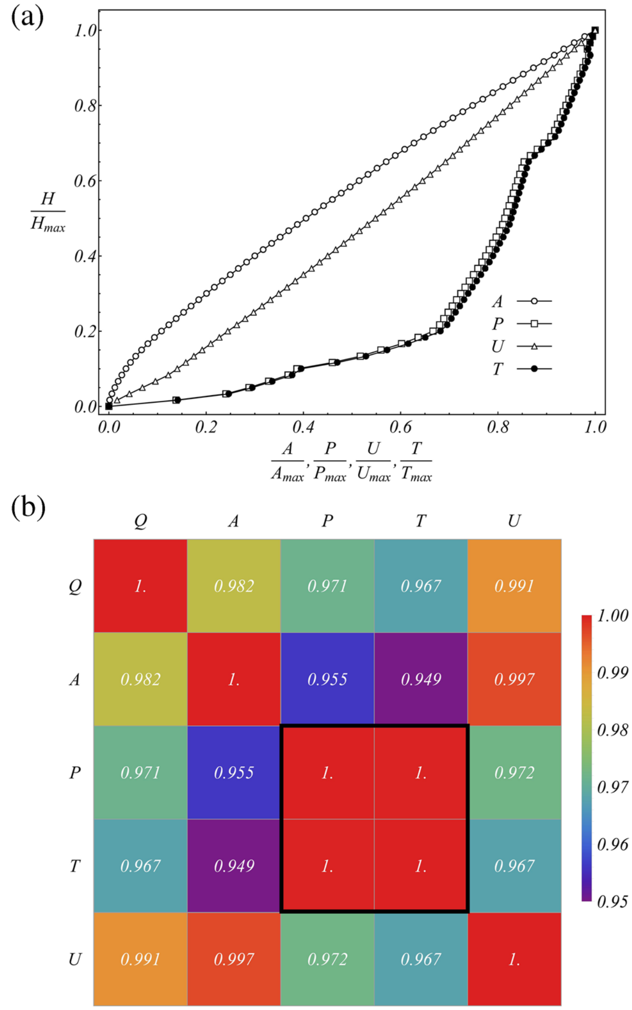

Fig. 5 **a** Variations of hydro-geometric variables with stage and **b**) correlation heatmap between hydro-geometric variables and discharge

4.3 Uncertainty analysis

Based on calibrations of power parameters and bed slope, one can compute the SDRC for steady and unsteady states from Eq. 3 and Eq. 4, respectively. Figure 6 shows the proposed model's estimated SDRC and the "Storm 1" streamflow time series. As shown in Fig. 6, two approaches have been used for uncertainty analysis. In the first approach, the uncertainty caused by the parameters in the SSDRC (Eq. 3) has been considered using the MCMC method. However, the second approach uses the LSDRC (Eq. 4) and MC simulation to analyze uncertainty caused by bed slope.

The MCMC method parameters, such as σ and $\sum$, are determined by bracketing the maximum observed data with the minimum confidence interval. It should be mentioned that using a large 95CI band can make the prediction uncertainty extremely large and meaningless for decision-making and analysis. As part of the uncertainty analysis, the results

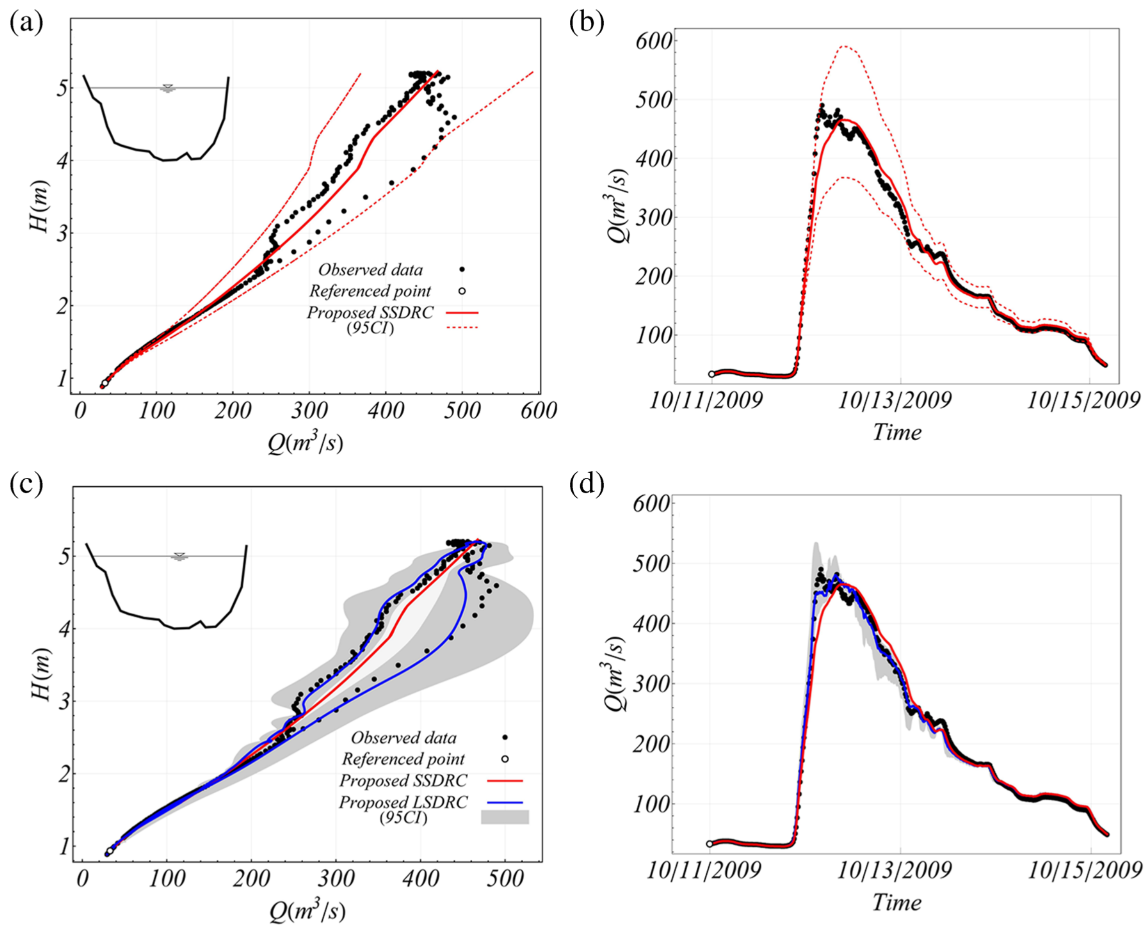

Fig. 6 **a** and **b** Estimation of SSDRC and streamflow as well as parametric uncertainty using MCMC method, and (**c** and **d**) estimated LSDRC and streamflow and uncertainty analysis due to bed slope by MC simulation

should be bracketed around the most observed data with the most minor uncertainty band possible.

Figure 6 shows less uncertainty in the low discharge (lower stages) and the rising limb. On the other hand, this uncertainty in the high release (higher stages) and the recession limb is significant. Compared to the second approach, the first approach provided better performance in the uncertainty analysis of the model because more data was bracketed. Also, several data in the second approach do not lie within the confidence interval. The maximum absolute error between the proposed SSDRC and the upper and lower limits of the confidence interval is about 92 and 117 (m^3/s), respectively. In addition, the maximum absolute error between SSDRC and LSDRC is about 75 (m^3/s). Maximum absolute errors between the proposed LSDRC and its upper and lower limits are 40 and 88 (m^3/s), respectively. The comparison between the proposed SSDRC and LSDRC with the observational data is presented in Table 1.

Table 1 Comparing the results of the proposed model in steady and unsteady states

MAPE (%)		*RMSE* (m^3/s)		*MAE* (m^3/s)		R^2		Data
LSDRC	SSDRC	LSDRC	SSDRC	LSDRC	SSDRC	LSDRC	SSDRC	
2.8	3.8	8.09	16.29	5.64	9.25	0.997	0.987	Storm 1
4.2	5.2	18.10	26.56	10.99	15.10	0.994	0.979	Storm 2
4.7	5.0	7.92	9.49	6.17	6.70	0.995	0.991	Storm 3
3.5	3.5	4.94	4.93	3.43	3.42	0.995	0.995	Storm 4
6.5	6.6	13.28	13.57	7.20	7.31	0.984	0.982	Storm 5

4.4 Estimating the proposed model

According to the previous step's calibration results and finding the proposed relationship's exponents, the streamflow time series can be obtained in posterior time steps. As shown in Fig. 7, appropriate accuracy is generally observed in estimating the streamflow time series. The accuracy decreases at higher stages as the distance from the calibration time increases. The model's accuracy is higher in the rising limb than in the recession limb. Also, more uncertainty interval is seen in the recession limb and high flows compared to the rising limb.

5 Discussion

The quantitative comparison of the results of the proposed model in steady and unsteady states is presented in Table 1. The proposed LSDRC's accuracy is generally better than the SSDRC, as shown in Table 1. However, this difference in estimation accuracy between these two proposed models decreases in storms 4 and 5. In the calibration step, Storm 1, the LSDRC performs better than the SSDRC based on all criteria. This issue is also true in storms 2, 3, and 5. However, in Storm 4, there are no differences between these models.

The upper and lower limits of the proposed model are compared with observational data in Fig. 8. Two *MAPE* and *RMSE* criteria are used for comparison. As can be seen, based on *RMSE*, the lower limit of the SSDRC in storms 4 and 5 shows a better performance than the optimal models in steady and unsteady states. However, in the upper limit of the SSDRC, the model is less accurate than the optimal value in all storms. The error difference between the upper and lower band of the uncertainty caused by the bed slope is slight and does not significantly differ from the optimal LSDRC.

Therefore, it can be said that with the distance from the calibration period, the proposed model will be overestimated, and it is necessary to recalibrate the proposed model or use the lower limit of the SSDRC to increase the estimation accuracy.

Observational data from Storm 1 must be used to calibrate the power function rating curve (Eq. 5). It is determined that

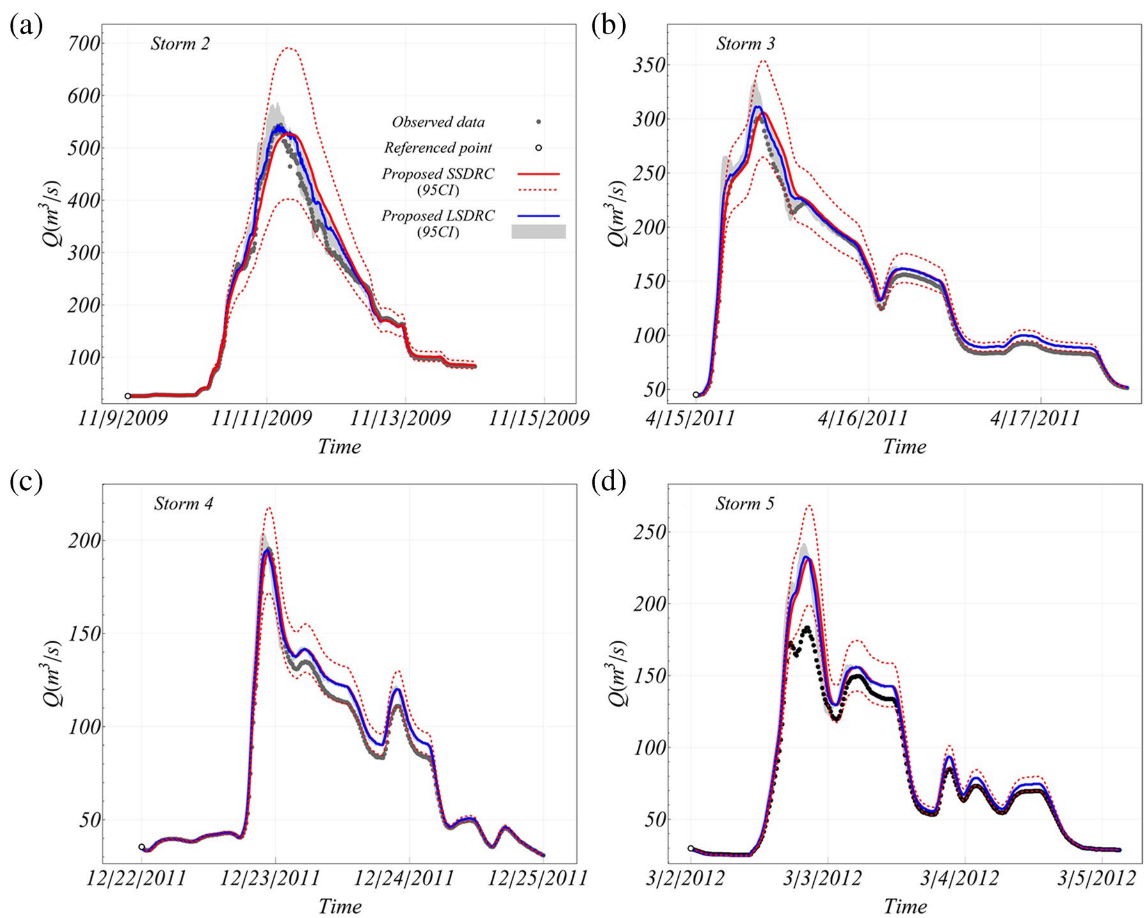

Fig. 7 Estimated streamflow time series by SSDRC and LSDRC for posterior time steps and their corresponding uncertainty

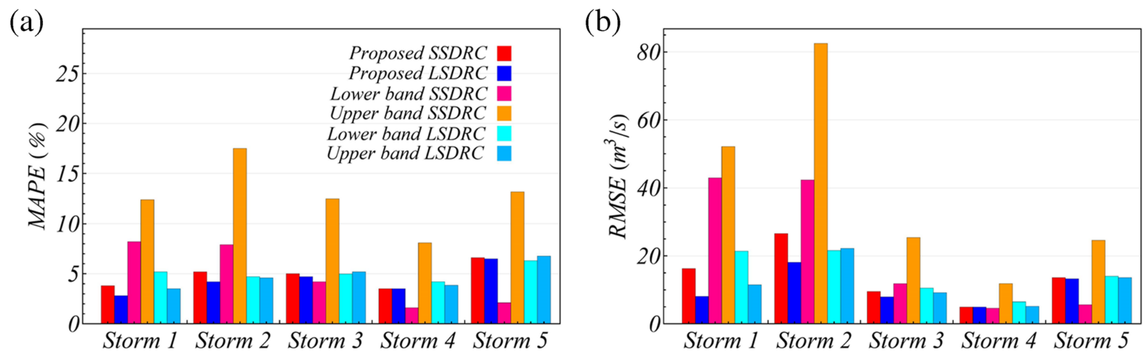

Fig. 8 Comparison of the lower and upper bands of the SSDRC and LSDRC model

the calibration values for a, b, and h_0 parameters are respectively 144.3, 0.79, and 0.8. Figure S5 shows the SDRC and streamflow in steady and unsteady states for the power function rating curve, along with the proposed model.

For the quantitative comparison between the proposed model and the power function rating curve, *MAPE* (%), *RMSE* (m^3/s), *MAE* (m^3/s), and R^2 criteria are used. As shown in Fig. 9, according to *MAPE*, the proposed model shows less error in all storms. The maximum *MAPE* of the proposed model is 6.6%, whereas the maximum *MAPE* of the power function is equal to 24.6% in Storm 5. Also, according to *RMSE*, the proposed model has performed better in all storms except for Storm 2. In Storm 2, the *RMSE* of the power function SSDRC is 25.4 m^3/s, and the LSDRC is 16.8 m^3/s, which is near the results of the proposed model.

MAE shows that the proposed model performs better than the power function rating curve in all storms. The maximum *MAE* in the proposed method in Storm 2 is 15.1 m^3/s, while the power function rating curve is 18.7 m^3/s. Also, the minimum error in the proposed method and power function equals 3.42 m^3/s and 6.28 m^3/s, respectively. Based on the R^2 criterion, both models provide almost similar results, but the proposed model is slightly better than the power function rating curve, which is evident in storms 4 and 5.

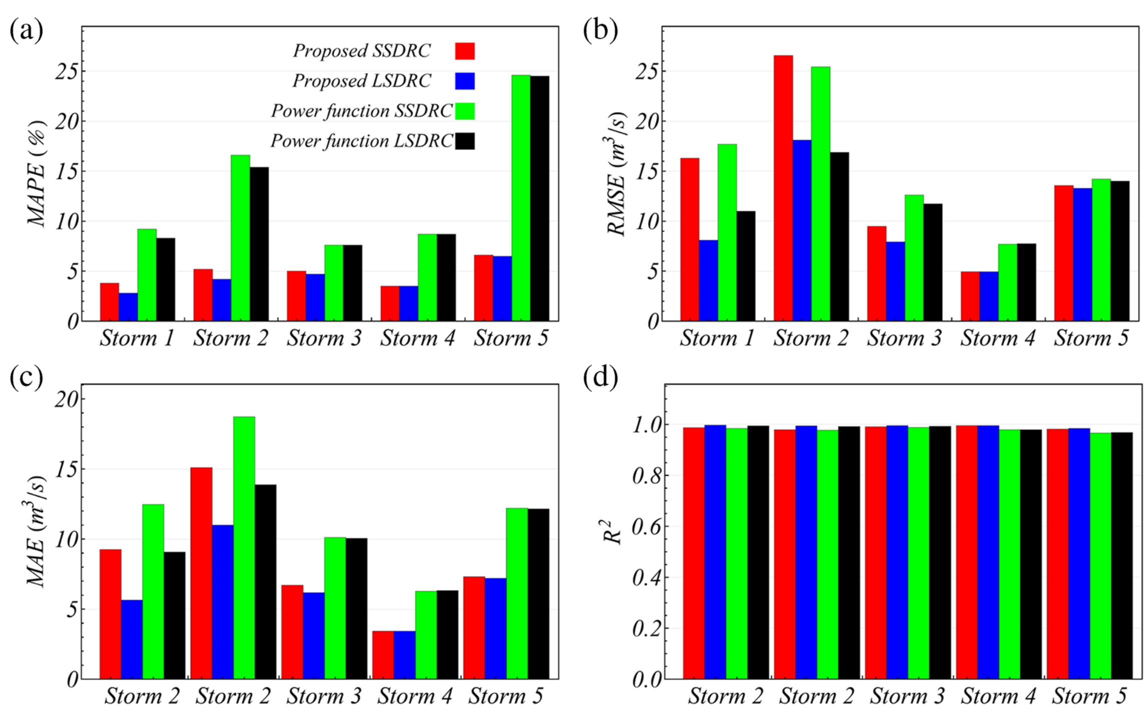

Fig. 9 Comparison of the proposed model with the power function rating curve

6 Conclusion

This study developed the isovel-based stage-discharge rating curve (SDRC) to estimate the loop SDRC (LSDRC). This model obtains the LSDRC by combining the isovel-based SDRC for steady-state conditions and the Jones formula. This model includes geometric-hydraulic variables such as the isovel-based velocity and measured discharge data. The MCMC method used observational data from one storm to calibrate the proposed model, and then posterior storms were used to validate it. The study was conducted in the Chattahoochee River, Georgia, USA. This study also examined the uncertainty of the proposed model from two perspectives. In the first approach, the parameter uncertainty of the proposed model by using the MCMC method was considered in the steady state, and the second one, the uncertainty caused by the bed slope parameter was discussed in the unsteady state.

The findings of this study are as follows:

- Generally, the LSDRC performs better than the steady SDRC (SSDRC) based on four performance criteria: $MAPE$ (%), $RMSE$ (m^3/s), MAE (m^3/s), and R^2.
- The estimation accuracy is higher during the rising limb than the recession limb, with a greater confidence interval for the recession limb and higher flows.
- The proposed model accuracy decreases over time, but a lower band of SSDRC uncertainty showed a better estimate for storms 4 and 5.
- The proposed model performed better in steady and unsteady conditions than the power function rating curve.

Therefore, the proposed model can be an alternative to the standard methods of estimating SDRC in steady and unsteady state conditions.

Supplementary Information The online version contains supplementary material available at https://doi.org/10.1007/s00477-024-02716-0.

Acknowledgements Authors acknowledged that this study has not been published or submitted to any publication outlets for publication.

Authors contribution Sajjad M. Vatanchi: Software, Data curation, Writing – original draft, Investigation, Validation. Mahmoud F. Maghrebi: Conceptualization, Visualization, Investigation, Supervision, Methodology, Writing—review & editing.

Funding The authors declare that no funds, grants, or other support were received during the preparation of this manuscript.

Data availability Data will be made available by authors on request.

Declarations

Competing interests The authors have no relevant financial or non-financial interests to disclose.

References

Ali G, Maghrebi MF (2023) A robust approach for the derivation of rating curves using minimum gauging data. J Hydrol 129609. https://doi.org/10.1016/j.jhydrol.2023.129609

Braca G (2008) Stage – discharge relationships in open channels: practices and problems. Environ Prot 24

Di Silvio G (1969) Flood wave modification along prismatic channels. J Hydraul Div 95:1589–1614. https://doi.org/10.1061/JYCEAJ.0002152

Dottori F, Martina MLV, Todini E (2009) A dynamic rating curve approach to indirect discharge measurement. Hydrol Earth Syst Sci 13:847–863. https://doi.org/10.5194/hess-13-847-2009

Faye RE, Cherry RN (1980) Channel and dynamic flow characteristics of the Chattahoochee River, Buford Dam to Georgia Highway 141

Fenton JD (1999) Calculating hydrographs from stage records. In: Proc. 28th IAHR Congress, Graz, Austria

Fenton JD, Keller RJ (2001) The calculation of streamflow from measurements of stage

Fread DL (1975) Computation of stage-discharge relationships affected by unsteady flow. JAWRA 11:213–228. https://doi.org/10.1111/j.1752-1688.1975.tb00674.x

Gelman A, Gilks WR, Roberts GO (1997) Weak convergence and optimal scaling of random walk Metropolis algorithms. Ann Appl Probab 7:110–120. https://doi.org/10.1214/aoap/1034625254

U.S. Geological Survey (2023) National water information system data available on the World Wide Web (USGS water data for the nation). URL https://waterdata.usgs.gov/nwis/inventory/?site_no=02336000. Accessed 10 Jan 2023

Henderson F (1966) Macmillan series in civil engineering

Herschy RW (2009) Streamflow measurement. CRC Press. https://doi.org/10.2113/gseegeosci.ii.4.609

Holmes Jr RR (2016) River rating complexity. In: Proceedings of the International Conference on Fluvial Hydraulics (River flow 2016), pp 679–686

Jones BE (1915) A method of correcting river discharge for a changing stage. US Geol Surv. https://doi.org/10.3133/wsp375E

Kiang JE, Gazoorian C, McMillan H, Coxon G, Le Coz J, Westerberg IK, Belleville A, Sevrez D, Sikorska AE, Petersen-Øverleir A (2018) A comparison of methods for streamflow uncertainty estimation. Water Resour Res 54:7149–7176. https://doi.org/10.1029/2018WR022708

Kuhnle RA, Bowie AJ (1992) Loop rating curves from Goodwin Creek. Publications from USDA-ARS/UNL Faculty 316

Le Coz J, Renard B, Bonnifait L, Branger F, Le Boursicaud R (2014) Combining hydraulic knowledge and uncertain gaugings in the estimation of hydrometric rating curves: A Bayesian approach. J Hydrol 509:573–587. https://doi.org/10.1016/j.jhydrol.2013.11.016

Maghrebi MF (2006) Application of the single point measurement in discharge estimation. Adv Water Resour 29:1504–1514. https://doi.org/10.1016/j.advwatres.2005.11.007

Maghrebi MF, Ahmadi A (2017) Stage-discharge prediction in natural rivers using an innovative approach. J Hydrol (Amst) 545:172–181. https://doi.org/10.1016/j.jhydrol.2016.12.026

Maghrebi MF, Vatanchi SM, Kawanisi K (2023) Investigation of stage-discharge model performance for streamflow estimating: A case study of the Gono River, Japan. River Res Appl 39:805–818. https://doi.org/10.1002/rra.4106

Majdalani S, Delestre O, Moussa R (2023) Experimental investigation of loop rating curve on a small 3D printed laboratory channel. J Hydrol (Amst) 623. https://doi.org/10.1016/j.jhydrol.2023.129741

Marjoram P, Molitor J, Plagnol V, Tavaré S (2003) Markov chain Monte Carlo without likelihoods. PNAS 23:15324–15328. https://doi.org/10.1073/pnas.030689910

McMillan HK, Westerberg IK (2015) Rating curve estimation under epistemic uncertainty. Hydrol Process 29:1873–1882. https://doi.org/10.1002/hyp.10419

Muste M, Lee K, Kim D, Bacotiu C, Oliveros MR, Cheng Z, Quintero F (2020) Revisiting hysteresis of flow variables in monitoring unsteady streamflows. J Hydraul Res 58:867–887. https://doi.org/10.1080/00221686.2020.1786742

Muste M, Kim D, Kim K (2022) Insights into flood wave propagation in natural streams as captured with acoustic profilers at an index-velocity gaging station. Water (Switzerland) 14. https://doi.org/10.3390/w14091380

Paterson GA, Zhao X, Jackson M, Heslop D (2018) Measuring, processing, and analyzing hysteresis data. Geochem Geophys 19:1925–1945. https://doi.org/10.1029/2018GC007620

Perret E, Lang M, Le Coz J (2022) A framework for detecting stage-discharge hysteresis due to flow unsteadiness: application to France's national hydrometry network. J Hydrol (Amst) 608. https://doi.org/10.1016/j.jhydrol.2022.127567

Perumal M, Raju KGR (1999) Approximate convection-diffusion equations. J Hydrol Eng 4:160–164. https://doi.org/10.1061/(ASCE)1084-0699(1999)4:2(160)

Perumal M, Shrestha KB, Chaube UC (2004) Reproduction of hysteresis in rating curves. J Hydraul Eng 130:870–878. https://doi.org/10.1061/(ASCE)0733-9429(2004)130:9(870)

Perumal M, Moramarco T, Sahoo B, Barbetta S (2007) A methodology for discharge estimation and rating curve development at ungauged sites. Water Resour Res 43:22. https://doi.org/10.1029/2005WR004609

Petersen-Øverleir A (2006) Modelling stage-discharge relationships affected by hysteresis using the Jones formula and nonlinear regression. Hydrol Sci J 51:365–388. https://doi.org/10.1623/hysj.51.3.365

Reis DS Jr, Stedinger JR (2005) Bayesian MCMC flood frequency analysis with historical information. J Hydrol 313:97–116. https://doi.org/10.1016/j.jhydrol.2005.02.028

Reitan T, Petersen-Øverleir A (2009) Bayesian methods for estimating multi-segment discharge rating curves. Stoch Environ Res Risk Assess 23:627–642. https://doi.org/10.1007/s00477-008-0248-0

Reitan T, Petersen-Øverleir A (2011) Dynamic rating curve assessment in unstable rivers using Ornstein-Uhlenbeck processes. Water Resour Res 47:1–14. https://doi.org/10.1029/2010WR009504

Rosenthal JS (2014) Optimizing and adapting the Metropolis algorithm. Chapter 6:93–108

Van de Schoot R, Depaoli S, King R, Kramer B, Märtens K, Tadesse MG, Vannucci M, Gelman A, Veen D, Willemsen J, Yau C (2021) Bayesian statistics and modelling. Nat Rev Methods Prim 1. https://doi.org/10.1038/s43586-020-00001-2

Shi P, Yang T, Yong B, Xu CY, Li Z, Wang X, Qin Y, Zhou X (2023) Some statistical inferences of parameter in MCMC approach and the application in uncertainty analysis of hydrological simulation. J Hydrol (Amst) 617. https://doi.org/10.1016/j.jhydrol.2022.128767

Vlachou E, Karras C, Karras A, Tsolis D, Sioutas S (2023) EVCA classifier: a MCMC-based classifier for analyzing high-dimensional big data. Information (Switzerland) 14. https://doi.org/10.3390/info14080451

Wang H, Wang C, Wang Y, Gao X, Yu C (2017) Bayesian forecasting and uncertainty quantifying of stream flows using Metropolis-Hastings Markov Chain Monte Carlo algorithm. J Hydrol (amst) 549:476–483. https://doi.org/10.1016/j.jhydrol.2017.03.073

Zakwan M (2018) Spreadsheet-based modelling of hysteresis-affected curves. Appl Water Sci 8. https://doi.org/10.1007/s13201-018-0745-3