HAHN-BANACH THEOREM IN GENERALIZED 2-NORMED SPACES

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Abstract. In this paper we prove an extension Hahn-Banach theorem in the context of generalized 2-normed spaces.

1. Introduction.

In [4] Z. Lewandowska introduced a generalization of Gähler 2-normed space (see [2]) as follows.

Definition 1.1. Let $X$ and $Y$ be real linear spaces. Denote by $D$ a non-empty subset of $X \times Y$ such that for every $x \in X, y \in Y$ the sets $D_x = \{ y \in Y; (x, y) \in D \}$ and $D^y = \{ x \in X; (x, y) \in D \}$ are linear subspaces of the spaces $Y$ and $X$, respectively.

A function $\|.,.\|: D \rightarrow [0, \infty)$ will be called a generalized 2-norm on $D$ if it satisfies the following conditions:

1. $\|\alpha x, y\| = |\alpha| \cdot \|x, y\|$ for any real number $\alpha$ and all $(x, y) \in D$;
2. $\|x, y + z\| \leq \|x, y\| + \|x, z\|$ for $x \in X, y, z \in Y$ with $(x, y), (x, z) \in D$;
3. $\|x + y, z\| \leq \|x, z\| + \|y, z\|$ for $x, y \in X, z \in Y$ with $(x, z), (y, z) \in D$.

The set $D$ is called a 2-normed set. In particular, if $D = X \times Y$, the function $\|.,.\|$ is said to be a generalized 2-norm on $X \times Y$ and the pair $(X \times Y, \|.,.\|)$ is called a generalized 2-normed space. If $X = Y$, then the generalized 2-normed space $(X \times X, \|.,.\|)$ is denoted by $(X, \|.,.\|)$. In the case that $X = Y$, $D = D^{-1}$, where $D^{-1} = \{(y, x); (x, y) \in D\}$, and $\|x, y\| = \|y, x\|$ for all $(x, y) \in D$, we call $\|.,.\|$ a generalized symmetric 2-norm and $D$ a symmetric 2-normed set.

Recall that in Gähler definition of a 2-norm $\|x, y\| = 0$ if and only if $x$ and $y$ are linearly dependent, and this is a crucial difference between Gähler’s approach and Lewandowska’s one.