Abstract: We address the Capacitated m-Ring-Star Problem in which the aim is to find m rings (simple cycles) visiting a central depot, a subset of customers and a subset of potential Steiner nodes, while customers not belonging to any ring must be "allocated" to a visited (customer or Steiner) node. Moreover, the rings must be node-disjoint and the number of customers allocated or visited in a ring cannot be greater than a given capacity Q. The objective is to minimize the total visiting and allocation costs. The Capacitated m-Ring-Star Problem is NP-hard, since it generalizes the Traveling Salesman Problem. In this paper we propose a new approach which combines both heuristic and exact ideas to solve the problem. Considering the general scheme of the Variable Neighborhood Search approach, the algorithm incorporates an Integer Linear Programming based improvement method which is applied whenever the heuristic procedure is not able to enhance the quality of the current solution. Extensive computational experiments on benchmark instances of the literature have been performed to compare the proposed approach with the most effective methods from the literature. The results show that the proposed algorithm outperforms the other approach.

Keywords: Capacitated m-Ring-Star Problem, Local Search, Integer Linear Programming.

1. INTRODUCTION AND PROBLEM DEFINITION

Introduced by Baldacci et al. [3] in 2007, the Capacitated m-Ring-Star Problem (CmRSP) has many applications in telecommunication systems, in particular in the fiber optic communication networks (see, e.g. Baldacci et al. [3]).

CmRSP can be described as follows: a mixed graph \( G = (V \cup U \cup W, E) \) is given, where \( V \) is the set of nodes, \( E = \{ (i,j) \mid i,j \in V, i \neq j \} \) is the set of edges (undirected arcs) and \( A \) is the set of arcs. The node set \( V \) is defined as \( V = \{ 0 \} \cup U \cup W \), where node 0, \( U \) and \( W \) represent, respectively, the central depot, the set of customers and the set of Steiner nodes. For each customer \( i \in U \cup C \subseteq V \cup U \cup W \), denotes the subset of nodes to which customer \( i \) can be connected. The arc set \( A \) is defined as \( A = \{ (i,j) \mid i \in U \cup C, j \in C \} \). Each edge \( e \in E \) has a non negative visiting cost \( c_e \), and each arc \( (i,j) \in A \) has a non negative allocation cost \( d_{ij} \).

We refer to a simple cycle consisting of a subset of nodes and the depot as a ring. If a customer is visited by a ring or allocated to a node of a ring, we consider the customer as assigned to that ring. Two input parameters \( m \) and \( Q \) are given, representing, respectively, the number of rings and the capacity of each ring. We assume \( m, Q \geq 1 \), so that a feasible solution for the considered CmRSP instance always exists.

A solution of the CmRSP is feasible if each customer is assigned to exactly one ring, no Steiner node is used more than once, and the number of customers assigned to a ring does not exceed the capacity \( Q \). We impose as well that a Steiner node can belong to a ring only if one or more customers are allocated to it.

The goal of the CmRSP is to find \( m \) rings with the minimum global cost, given by the sum of the visiting and allocation costs. CmRSP is known to be NP-hard, since it is a generalization of the Symmetric Traveling Salesman Problem (TSP), arising when \( m = 1 \), \( Q = |V| \), \( W = \emptyset \). A hybrid metaheuristic approach, which combines GRASP and Tabu Search algorithms, has been proposed by Baldacci et al. [1].

A Branch-and-Cut (BC) approach have been proposed by Baldacci et al. [1].

Two Integer Linear Programming (ILP) formulations and a Branch-and-Cut (BC) approach have been proposed by Baldacci et al. [1].