Combining Exact and Heuristic Approaches for the Covering Salesman Problem

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Abstract—We consider a generalized version of the well known Traveling Salesman Problem called Covering Salesman problem. In this problem, we are given a set of vertices while each vertex $i$ can cover a subset of vertices within its predetermined covering distance $r_i$. The goal is to construct a minimum length Hamiltonian cycle over a subset of vertices in which vertices not visited on the tour have to be within the covering distance of at least one vertex visited on the tour. We propose a hybrid exact and heuristic approach which takes advantage of Integer Linear Programming (ILP) techniques and heuristic search to improve the quality of the solutions. Extensive computational tests on the standard benchmark instances and on a new set of large sized datasets show the effectiveness of the proposed approach.

I. INTRODUCTION

The Covering Salesman Problem (CSP) is a generalization of the Traveling Salesman Problem (TSP) in which the assumption of visiting all the vertices by the tour is not valid. Here, we are given a set of vertices while each vertex $i$ can cover a subset of vertices within its predetermined covering distance $r_i$. The goal of the CSP is to construct a minimum length Hamiltonian cycle over a subset of vertices in which those not visited by the tour must be within the covering distance of at least one visited vertex [4].

The CSP was defined and modeled by Current and Schilling [4], while they proposed a heuristic method for this problem. In their approach, the optimal TSP tour is constructed over the minimum number of vertices that makes the solution feasible. In other words, to cover all the demands, they proposed to solve the corresponding Set Covering Problem (SCP). Then, the minimum length Hamiltonian cycle is constructed over these nodes by solving the corresponding TSP to optimality. Since the associated SCP may have multiple optimal solutions, with the same number of vertices that cover all the demands, they suggested to take the minimum length tour found after applying the TSP solver over all of the optimal solutions obtained by solving the corresponding SCP.

Arkin and Hassin [1] proposed a geometric version of the CSP. Unlike the CSP, in this problem each neighborhood is a compact set in the plane in such a way that by intersecting that set all the vertices in that neighborhood will be covered. Starting from a node in a neighborhood set, the goal is to construct a minimum length tour obtained by intersecting all the neighborhoods and returning to the initial node. They proposed several simple heuristic methods for this problem [1].

Golden et al. [12] proposed two heuristic approaches for the CSP which outperform the Current and Schilling’s method in almost all of the tested instances. The proposed heuristics take advantage of different extraction and reinsertion moves to improve the tour length. Moreover, they showed that the least number of vertices, i.e., the solution of the SCP, does not necessarily lead to a shorter tour length. Finally, they proposed some generalizations of the CSP as well [12].

The concept of covering has many real world applications and has been used by many researchers in a wide area of combinatorial optimization problems. Gendreau et al. [11], proposed a generalization of the CSP, called the Covering Tour Problem (CTP). Suppose $G = (V \cup W, E)$ be an undirected graph in which the set of vertices $V \cup W$ is partitioned into three groups. Let $V$ be the set of vertices that can be visited, $T \subseteq V$ the set of vertices that must be visited and $W$ the set of vertices that must be covered. The goal of the CTP is to construct a minimum length Hamiltonian cycle over a subset of nodes in $V$, in which the tour must contain all vertices in $T$, and every vertex of $W$ has to be covered by at least one vertex visited by the tour [11]. The authors proposed a simple heuristic which takes advantage of two powerful procedures for solving TSP and SCP called GENIUS [10] and PRIMAL1 [2], respectively.

Hachicha et al. [14], proposed the Multi-Vehicle Covering Tour Problem (MVCTP) which is a generalization of the CTP in which we aim at designing $m$ Hamiltonian cycles over a subset of eligible vertices to visit or cover all of the vertices in $G$ with the side constraints defined for the CTP.

The Generalized Covering Tour Problem (GCTP) is another generalization of the CTP and consists of finding a minimum length Hamiltonian cycle over a subset of nodes in $V \cup W$, instead of considering exclusively a subset of $V$ [18].

In the Maximum Covering Tour Problem (MCTP), we attempt to identify a tour on $p$ vertices, while minimizing the tour length and the total uncovered demands. Here, each vertex has an associated demand, and a vertex is said to be covered when it lies within a given distance $r$ from a visited node. $p$ and $r$ are two parameters of the problem [5]. Another variation of the MCTP, called Mediam Tour Problem (MTP),