Three-Receiver Broadcast Channels with Side Information

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Abstract—Three-receiver broadcast channel (BC) is of interest due to its information theoretical differences with two receiver one. In this paper, we derive achievable rate regions for two classes of 3-receiver BC with side information available at the transmitter, Multilevel BC and 3-receiver less noisy BC, by using superposition coding, Gel’fand-Pinsker binning scheme and Nair-EI Gamal indirect decoding. Our rate region for multilevel BC subsumes the Steinberg rate region for 2-receiver degraded BC with side information as its special case. We also find the capacity region of 3-receiver less noisy BC when side information is available both at the transmitter and at the receivers.

Keywords: 3-receiver broadcast channel, less noisy, Multilevel broadcast channel

I. INTRODUCTION

The k-receiver, \(k \geq 3\), broadcast channel (BC) was first studied by Borade et al. in [1] where they simply surmised that straightforward extension of Körner-Marton’s capacity region for two-receiver BCs with degraded message sets [2] to k-receiver multilevel broadcast networks is optimal. Nair-EI Gamal [3] showed that the capacity region of a special class of 3-receiver BCs with two degraded message sets when one of the receivers is a degraded version of the other, is a superset of [1], thus proving that direct extension of [2] is not in general optimal. Nair and Wang later in [4] established the capacity region of the 3-receiver less noisy BC. Channels with Side information (SI), were first studied by Shannon [5], where he found the capacity region of the Single-Input-Single-Output channel when SI is causally available at the encoder. Gel’fand and Pinsker [6] found the capacity region of a single-user channel when SI is non-causally available at the transmitter while the receiver is kept ignorant of it. Cover and Chiang [7] extended the results of [6] to the case where SI is available at both the encoder and the decoder. Multiple user channels with side information were studied in [8] where inner and outer bounds for degraded BC with non-causal SI and capacity region of degraded BC with causal SI were found. Moreover, in [9] inner and outer bounds were given to general two-user BCs with SI available at the transmitter and other special cases both for BCs and MACs were also found.

In this paper, we find the achievable rate region of Multilevel BC and 3-receiver less noisy BC both with SI non-causally available at the encoder. Our achievable rate regions reduce to that of [3] and [4] when there is no side information. We also find the capacity region of the latter when side information is also available at the receivers. The rest of the paper is organized as follows. In section II, basic definitions and notations are presented. In sections III and IV, new achievable rate regions are given for the Multilevel BC and 3-receiver less noisy BC, respectively. In section V, conclusion is given.

II. DEFINITIONS

Random variables and their realizations are denoted by uppercase and lowercase letters, respectively, e.g. \(X\) is a realization of \(X\). Let \(X, Y_1, Y_2, Y_3, and S\) be finite sets showing alphabets of random variables. The \(n\)-sequence of a random variable is given by \(X^n\) where the superscript is omitted when the choice of \(n\) is clear, thus we only use boldface letters for the random variable itself, i.e. \(\mathbf{x} = x^n\). Throughout, we assume that \(X^i\) is the sequence \((X_i, X_{i+1}, \ldots, X_n)\).

Definition 1: A channel \(X \rightarrow Z\) is said to be a degraded version of the channel \(X \rightarrow Y\) with SI if \(X \rightarrow Y \rightarrow Z\) be a Markov chain conditioned on every \(s \in S\) for all \(p(u, x|s)\).

Multilevel BC with side information, denoted by \((X, S, Y_1, Y_2, Y_3, p(y_1, y_2|X, S), p(y_3|y_2))\), is a 3-receiver BC with 2-degraded message sets with input alphabet \(X\) and output alphabets \(Y_1, Y_2, and Y_3\). The side information is the random variable \(S\) distributed over the set \(S\) according to \(p(s)\). The transition probability function \(p(y_1, y_2|x, s)\) describes the relationship between channel input \(X\), side information \(S\), and channel outputs \(Y_1\) and \(Y_2\) while the probability function \(p(y_3|y_2)\) shows the virtual channel modeling the output \(Y_2\) as the degraded version of \(Y_1\). Independent message sets \(m_0 \in \mathcal{M}_0\) and \(m_1 \in \mathcal{M}_1\) are to be reliably sent, \(m_0\) being the common message for all the receivers and \(m_1\) the private message only for \(Y_1\). Channel model is depicted in Fig. 1.