THE FIXED POINT ALTERNATIVE THEOREM 
AND SET-VALUED FUNCTIONAL EQUATIONS

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Abstract. We use the fixed point alternative theorem to prove the stability of the set-valued function equation
\[ c(x)F(h(x)) = F(x). \]
This result enable us to prove the stability of some set-valued functional equations.

Key Words and Phrases: Set-valued mappings, functional inequalities, non-expensive mappings.

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1. Introduction

One of the main topics in functional equations is Hyers-Ulam stability which was originated from a question of S. M. Ulam [25]. D. H. Hyers [12] gave the first significant partial solution to Ulam’s question. The theorem of Hyers was generalized by T. Aoki [1] for additive mappings and by Th.M. Rassias [22] for linear mappings by considering an unbounded Cauchy difference. The paper of Th.M. Rassias has provided a lot of influence in the development of what we now call Hyers-Ulam-Rassias stability of functional equations.

It should be noted that almost all proofs in this topic used Hyers method. In 1991, Baker [3] used the Banach fixed point theorem to prove Hyers-Ulam stability for a non-linear functional equation. V. Radu [21], in 2003, employed the fixed point alternative theorem [9] to establish the stability of Cauchy additive functional equation. Using such an elegant idea, several authors applied the method to investigate the stability of some functional equations, (see e. g. [5, 6, 7, 10, 13, 14, 16, 17, 19, 20]).

The theory of set-valued functions was fairly systematically developed for the first time in Berge’s book [4]. It is of interest to investigate the Hyers-Ulam stability of set-valued functional equations and inclusions. Although there are much less results of Hyers-Ulam stability for set-valued ones than those for single-valued ones, some interesting results were obtained by several mathematicians (e.g. [2, 11, 15, 18, 23, 24, 26]).