Extended Fuzzy Logic: Sets and Systems

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***Abstract*—The concepts of sets and approximate reasoning within extended fuzzy logic (FLe) provide a systematic procedure for transforming unprecisiated knowledge into a nonlinear mapping over what we define here as *f*-sets. An *f*-set differs from a fuzzy set in that it is associated with the restriction of validity in addition to that of possibility. Therefore, by *f*-set, we can simultaneously deal with two different types of uncertainties: one that is related to ill-known objects represented by incomplete information—information with its one or more aspects being imprecise/vague/partial/nonspecific/undetermined—and another that is related to truth values considering gradualness. Here, we define new concepts of ϑ-cuts and $\alpha\vartheta$-cuts, introduce the *f*-extension principle, and consider arithmetic computations within FLe. We then address other aspects of the proposed FLe system such as fuzzification and validification operations in input processing stage, set-conversion and defuzzification in output processing stage, and inferencing. In fact, in this paper, we intend to develop FLe theoretically and practically from the stands of sets and systems to extend the concept of approximate reasoning. As a consequence of this development, we assert that considering the validity degree of methods and information can lead to more reasonable and trustworthy results through capturing more uncertainty.**

***Index Terms*—Approximate reasoning, extension principle, fuzzy sets, fuzzy systems, validity.**

I. Introduction

FUZZY logic has been the subject of much research since the seminal paper on fuzzy sets in 1965 [1]. The views derived from the research suggest that fuzzy logic concepts coincide very closely with approximate reasoning, which for the first time justified its power to solve problems. However, provable validity remains a necessary condition for coping with problems when applying fuzzy logic [2]. This requirement may be satisfied in closed-world environments, such as finding the *shortest*-path to reach a destination; however, most open-world problems, such as finding the *fastest*-path to reach a destination, do not meet this criterion. Therefore, it may be inferred that fuzzy logic must be enhanced to address open-world problems. This enhancement is done by adding unprecisiated fuzzy logic (FLu) to fuzzy logic that leads to extended fuzzy logic (FLe) [2].

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FLe is still in its early stages of development; thus, relatively little research has been conducted on it. Niskanen [3], [4] considered approximate scientific explanation by using Zadeh's impossibility principle and FLe and provided insight into approximate probability. He also considered the degree of confirmation for the explanatory hypotheses. Wilke [5] proposed a kind of fuzzy geometry by considering FLe and a version of David Hilbert's axiomatic logical calculus, later used by Aliev *et al.* [6] to deal with unprecisiated information in the decision-making process. An approach named "sketching with words" based on FLe is proposed in [7] and [8] by describing geometric shapes with an exponential function with respect to their classical prototypes. In [9], it is argued that using FLe in reasoning leads to the obtaining of more approximate conclusions. In addition, to analyze FLe's theoretical foundation, a qualified description of FLe is provided in [10]. There, the authors provided a theoretical foundation for FLe by considering basic concepts such as *f*-validity and *f*-transformation. A regression analysis based on FLe is also proposed in [11] that applies to medical qualitative research. A mathematical analysis of FLe is also conducted in [12] that proves the representation theorem for the rationality of reasoning within FLe. A practical approach based on FLe, which introduces a new distribution of possibility-probability-validity, has also been proposed and applied to the actual data of judicial decision making in [13] and to the risk assessment of coronary heart disease in [14]. Furthermore, trends of inference based on FLe for different applications are addressed in [15].

Despite the various contributions of the aforementioned studies to the development of FLe, what particularly remains with respect to the pragmatic elements of reasoning is formulating approximate reasoning and methods of computation to reach comprehensive solutions to real-world problems. Therefore, our concern in this paper is to explore FLe from reasoning and computational perspectives and to address a practical basis for reaching solutions to real-world problems. More specifically, we present a way to increase the credibility of solutions in approximate reasoning by proposing FLe systems that provide a unified framework to solve problems in unprecisiated environment. In fact, the FLe system makes a very basic stand by aiming to represent more aspects of knowledge, focusing specifically on its validity.

In this paper, we intend to develop different relevant notions to FLe theoretically and mathematically from the stands of sets and systems. Specifically, our insight is extending the concept of approximate reasoning, defining numbers and sets, and providing methods of computation within FLe. To achieve this, we introduce several new concepts such as ϑ-cuts and $\alpha\vartheta$-cuts as well as extend (*f*-transform) fuzzy concepts such as fuzzy sets into *f*-sets and the extension principle into the *f*-extension principle. The concepts of ϑ-cut and $\alpha\vartheta$-cut appear to be useful tools